Addressing Longevity Inequality: How Retirement Age Differentiation Can Be Implemented

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Abstract. Differences in life expectancy across socioeconomic groups create a serious problem of inequality within the public part of the pension system. This paper considers two actuarially sound ways of addressing longevity inequality. The first is to allow low life-expectancy workers to retire earlier and delay the retirement of the high life-expectancy workers so that the two groups receive the same amount, equal to the expected discounted value of future pension benefits received by the average worker under the current system. The second, and more radical, is to delegate to occupational pension funds the task of paying out the public pension benefits to each retiree, based on a lump sum transfer from the government to the pension funds of an amount for each retiree equivalent to the payout in the first scenario.

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1. The issue

Population ageing, caused by increased longevity along with decreased fertility, is a worldwide
phenomenon with complicated and widespread challenges. From an economic perspective, this
demographic shift has a pronounced impact on the sustainability of public finances and pension
systems. One approach to mitigating these effects is to link the official retirement age to life expectancy.
This has in fact been implemented in Scandinavian countries to varying degrees, as well as in the
Netherlands (Jensen et al., 2020).

However, there is a fundamental weakness in linking retirement to average life expectancy, which
is that it ignores the variation in life expectancy, health, and ability to work at each age across the
population. While some workers leave school early and join the labour market in their late teens or early
twenties, others remain in school until their late twenties or even early thirties. Moreover, the physical
toll of working differs from one industry to another. Thus blue-collar workers may find it more difficult
to work into their late sixties or early seventies than white-collar workers, having spent more years in
the labour market performing more physically difficult tasks.

While it has long been known that women tend to live longer than men, it is also well-documented
that there is significant inequality in life expectancy across socioeconomic groups. Indeed, we know
that the more education an individual has, the longer the lifespan, yet the causality is not obvious. One
possibility is that with shorter education, an individual experiences a higher degree of wear and tear and
therefore is subject to a shorter lifespan. Another possibility is that a different set of values and attitudes
makes people both spend more time in school and take better care of their health, since both decisions
involve incurring sacrifices in the present in order to invest more in human capital.

Whatever the reasons, the pattern is clear in the data (see Figure 1). The length of education is
evidently important for our expected lifetime, and there is no indication that the gap will close anytime
soon. This means that if we all have the same retirement age, people with longer education will enjoy
more years in retirement than those with fewer years of education. Unsurprisingly, this has given rise
to ideas of retirement age differentiation, by allowing earlier retirement for those with less education
who have, on average, spent more years working. While straightforward in theory, this may be more
difficult to put into practice.

In this article, we propose two alternative ways of allowing differences in life expectancy to
influence the retirement age. Focusing on benefits paid out through pillar one of the pension system,
our proposed models abide by a few key principles: First, they must not lead to unwanted demand for
early withdrawals. Second, the state pension should be based on an actuarial-like principle so that the
total pension benefits an individual receives throughout retirement are not significantly affected by the
time of retirement. Third, the models must be consistent with the principle of linking retirement age to
life expectancy.
Both of our proposals are based on a calculation of the present value of future benefits for an individual with average life expectancy. Denoting this sum by $S$, the first model offers workers flexibility and discretion about when to retire, subject to the constraint $S$. Workers have their own estimated life expectancy based on information provided by the state, their pension fund, or other authoritative sources. The point is that workers with low life expectancy would typically be allowed to retire earlier than the official retirement age, even if they have no private savings.

The second, and more radical, model assumes that the sum $S$ is transferred from the state to occupational pension funds, which are then tasked with paying out the pension benefit based on the average life expectancy of its members. In effect, this model differs from the first model in that the pension funds are given the role of paying the pillar one pension with money received from the state. Put differently, while the first model is purely public, the second model is based on public-private partnership. In the remainder of the paper, we outline the principles behind the two alternative models.

2. Public models of retirement age differentiation

A possibility would be to “repair” the existing system by implementing means testing on the ability of the individual to remain in the workforce. Typically, a health care professional would perform this. If people consider a visit to a nurse or a medical doctor to be intimidating, an alternative solution would be to base eligibility for early pension on some kind of “objective” criterion, such as the number of years an individual has been active in the labour market.

However, such a criterion might not, in general, serve as a good proxy for identifying worn-out workers with low life expectancy. If not, one could think of a system of self-selection, leaving decisions about early retirement to individuals themselves, and designed to limit the incentives for individuals to
withhold information about their true health status. Effectively, this means that the pension benefit received would need to be relatively low.

In view of the difficulties associated with screening based on health assessment, we consider two alternative schemes for retirement age differentiation, both based on official estimates of life expectancy of different socioeconomic groups.

The first scheme assumes that a low life-expectancy worker, $L$, can retire at $R_L$, which is earlier than the official retirement age, $R^O$. Similarly, the high life-expectancy worker, $H$, would have to retire later, at $R^H$, instead of $R^O$. The $L$ and $H$ workers are expected to die at time $T_L$ and $T^H$, respectively. In Figure 2, the solid blue line shows the current pension income path for the $L$ worker, and the broken blue line shows the path under the proposed regime. Similarly, the solid grey line shows the current path for the $H$ worker and the broken grey line the proposed path.

**Figure 2.** Retirement length and pension benefits under current and proposed pension schemes

In Figure 2, the solid blue line shows the current pension income path for the $L$ worker, and the broken blue line shows the path under the proposed regime. Similarly, the solid grey line shows the current path for the $H$ worker and the broken grey line the proposed path.

In the current case, workers retire at the same age, regardless of longevity, and receive the same pension benefits each month. In this case, the $H$ worker’s total lifetime pension income is higher than that of the $L$ worker. In the proposed case, the workers can expect to receive the same total sum over their lifetime, which implies that pension benefits each month will differ between the two groups (see Figure 3).

While in the above proposal the retirement age was mandated, a more flexible scheme would give workers the option of retiring and starting to receive pension benefits earlier or later. Worker $L$ could retire earlier than the current official retirement age and receive the same monthly income, as is depicted in the first model above. Alternatively, worker $L$ could choose to retire even earlier, perhaps for health reasons, and receive a lower monthly pension, or retire later than the current retirement age and receive...
a higher monthly pension. In contrast, the high life-expectancy worker H must retire later than in the current system in order to receive the same pension income per month. Worker H also has flexibility about when to retire. He could retire even later and receive a higher pension, or retire earlier for a lower pension.

**Figure 3.** Total lifetime pension benefits under current and proposed pension schemes

Both the version where the L and the H workers are given separate retirement dates for their public pension and the version where they are offered flexibility involve a transfer of money from the average H worker to the average L worker. In practice, these arrangements assume that individuals receive credible information on their life expectancy from official authorities such as statistical bureaus. However, the information could also be provided by pension funds so that the public pension can be based on workers’ own occupational group life expectancy. In fact, the next model for retirement age differentiation is based on a close interaction between the state (responsible for pillar one of the pension system) and occupational pension funds (responsible for pillar two of the pension system). Clearly, it is only viable to consider such a proposal in countries with a well-developed pillar two of the pension system.¹

3. **A model based on public-private partnership (PPP)**

The main idea in this proposal is that a lump sum is transferred to every single individual. That amount is equal to what the average individual is entitled to under the current system, from the year individuals reach the official pension age until the expected year of death. This is the sum S, corresponding to the amount that every individual is to receive, on average, as a payment from pillar one in the pension system.

¹ It is now widely accepted that a well-designed pension system requires a combination of public (pay-as-you-go, defined benefits) and labour market (funded, defined contributions) pensions; see, e.g., World Bank (1994). Indeed, occupational pensions are critical to avoid a sharp fall in living standards after retirement (consumption smoothing), which requires a high level of saving before retirement.
system. Compared to the existing design of pillar one, our proposal is thus still based on a public
defined-benefit (DB) payment. But instead of ongoing periodic (monthly) payments, the amount is paid
out as a one-off (lump-sum) payment.

It is important to stress that the amount is earmarked for the pension. Therefore, it must immediately
be placed in a pension fund, where it is turned into annuities. This ensures a steady benefit for the rest
of an individual’s life, regardless of how old the individual becomes. A key input to the calculation of
the annuity payment is the specific pension fund’s assumption about the member’s life expectancy.

A factor that speaks in favour of such a public-private partnership (PPP) model is that many
countries have built up pension funds (pillar two) where various funds represent different professional
or educational groups in the labour market. For example, where should an unskilled male worker place
his pension sum, S? He may already have his savings in an occupational pension fund, so he must decide
whether to continue as a member of the same fund. Otherwise, he must choose a pension fund that
operates with a life expectancy matching his profile in terms of education, health status, and so forth.

In more general terms, a pattern will quickly emerge. A pension fund where members typically
have short education and, on average, a shorter expected lifetime will not accept members with long
education and a long expected lifetime. Similarly, a person with less education will not apply for
admission to a pension fund established for, say, schoolteachers, where members are expected to have
long lives and a correspondingly low annuity for a given contribution.

To simplify somewhat, it can be said that with this model, the satisfaction of any criteria for early
retirement, such as the health screening aspect in the public system, is privatized. Instead of leaving this
to health care professionals, such as medical doctors, the task will be delegated to pension funds. An
important aspect of the proposed model is thus that society benefits from the pension funds’
accumulated knowledge of their members’ life expectancy.

So far, we have presented the overall characteristics of the model; i.e., that the individual pays the
amount S into their occupational pension scheme upon reaching the official pension age, which is
adjusted in line with increases in average life expectancy. We next turn to addressing the model’s ability
to manage the challenge of retirement age differentiation. Again, we take the case of the low-educated
and presumably worn-out male worker.

In order to retire early, he should gain access to his pension sum, S, a few years before the official
pension age and transfer the amount to a pension fund. For example, if the official pension age is 67,
he might be entitled to cash in on S as soon as age 63. According to the assumptions of the actuarial
principle, the earlier it happens, the lower the monthly annuity will be. Nevertheless, the annuity offered
will in all probability be significantly higher than the annuity offered to the individual with tertiary
education or someone else who, due to his education, is expected to live somewhat longer. This is
already known by the relevant pension fund, so there is no need for an initial visitation.

Sudden and unexpected changes in interest rates and life expectancy would complicate matters, as
they do in the current pillar two system. If interest rates rise suddenly, the present discounted value of
future pillar one benefits – that is, sum S – would fall, thus creating a difference between individuals retiring at different times. For fairness reasons, the sum S may be determined based on an average of past interest rates. The same would apply to sudden and unexpected changes to life expectancy.

While straightforward in principle, our PPP proposal is not without potential flaws. One challenge is how to deal with adverse selection. Clearly, there is asymmetric information about everyone’s life expectancy. Indeed, individuals have an incentive to join a fund with lower average life expectancy to receive a higher annuity. The adverse selection problem arises when people with longer life expectancy manage to join a pension fund whose members have shorter average life expectancy. If this happens, the actual life expectancy of members will increase, which requires the fund to reduce the annuity paid to each member.

One example might be a highly educated individual who decides to switch to an unskilled job later in life. A more concrete example would involve an academically educated economist who, throughout his career, has been a member of a pension fund with university-educated members and who chooses to switch to a job as cleaning assistant close to retirement age. Clearly, the pension fund representing the less educated individuals has no interest in letting this happen. The rules applying to pillar two benefits – based on the defined contribution principle – might be applicable in this setting. To take an example, if a worker who has paid into a pension fund belonging to the high life-expectancy group of workers for 35 years wants to end his working life paying into a pension fund for low life-expectancy workers, the sum S would be divided between the two funds based on the accrued pillar two rights. This would reduce the adverse selection problem.

There is also the issue of how to deal with inflation. The current pillar one system provides insurance against inflation through taxation. In the proposed system, unexpected inflation would erode the purchasing power of the lump sum transferred to the occupational pension funds. This could be dealt with through government issuance of indexed bonds that the pension funds could buy in order to hedge against inflation.

Furthermore, there is an issue related to how public finances are affected by our proposal. Indeed, a reform of pillar one of the pension system from paying benefits in terms of “flows” to paying them out in terms of “stocks” constitutes a major shock to public finances in the early stages of the reform. This is likely to conflict with existing fiscal rules, such as the EU’s Fiscal Compact. In practice, the government will cover its financing requirements by issuing government bonds. Some of these bonds will be purchased by pension funds, which will also channel capital to business investments, housing, infrastructure, etc. This could lead to a positive spiral, including increases in firm productivity (Beetsma et al., 2022)

No transfer of money is required if the pension funds buy the government bonds. The pension funds will then add the bonds to the assets side of their balance sheets to match the obligation to pay pillar one benefits on the liabilities side. The government will correspondingly add debt to the liabilities side of its balance sheet and lower its pillar one obligations to the same amount. Each year, the government
will then pay the principal and interest to the pension funds instead of paying pillar one pension benefits, and the pension funds will receive the payments and distribute them to their members as pension benefits.

4. Efficiency considerations

In a “Beveridgean” pension system, benefits are independent of contributions, while contributions depend on income. This is the system used in countries such as Denmark, the Netherlands, and the United Kingdom. In contrast, in the “Bismarckian” pension systems used in the large continental European countries, benefits are linked to past earnings.

In the flat-rate Beveridgean system, such as the public pension system in Denmark, the system has a negative effect on labour supply, in contrast to a system where benefits are related to contributions, as shown by Poutvaara (2007) and Koethenbuerger (2008). High-income workers pay a higher tax to fund the system but receive the same benefits. This flat-rate pillar one system provides insurance against old-age poverty. As such, the pillar one system reduces inequality at some cost to efficiency. In contrast, the occupational pillar two pension system provides old-age pension income that is linked to previous contributions – and hence earnings – through a defined-contribution system.

We have not proposed tying the public pension system to contributions. Instead, our proposed changes to the pillar one system involves a transfer of pension wealth from high life-expectancy to low life-expectancy workers. It follows that the low life-expectancy workers can retire at an earlier age, while receiving a public pension. The pension payments are flat-rate pensions financed through general taxation and independent of previous tax payments or contributions to the occupational pillar two pension system. These changes affect incentives. First, the transfer reduces the lifetime income of highly educated workers and raises the lifetime income of the less educated. This reduces the incentive to acquire higher education. However, note that the present discounted value of the transfer at the time when individuals decide on their education path is small.\(^2\) Furthermore, keep in mind that the monetary rewards of acquiring education are only a part of the total rewards, which include the rewards of learning, of working on what interests the individual, and, in the case of higher education, a longer life.

As is described by Poutvaara (2007), the organization of the pension system may also affect migration between countries. A country that reforms its public pension system in the way we propose might then expect to lose some of its highly educated workers to other countries that give them a higher discounted sum by penalizing the lower life-expectancy groups, as in the current Danish system. This is a problem of adverse selection. To offset this factor, the right to acquire a public pension always depends on how many years of working life a worker has resided in the country, which reduces the incentive to migrate if potential migrants have few years remaining in the labour force.

\(^2\) With a 5% discount rate, the discount factor for a monetary sum to be received after 40 years (the difference between 20 and 60 years) is 0.14.
5. An illustration: The Danish pension system

In Denmark, a broad majority of the parliament has decided that the official retirement age must increase with life expectancy. The fundamental idea behind the agreement is that the number of years a person can be active in the labour market increases with life expectancy. In the following, we first sketch a recent reform within the pillar one scheme of the pension system, and then we outline how our PPP proposal could be framed in a Danish context.

(a) Introducing de facto retirement age differentiation

With effect from January 2022, people with a long history in the labour market are entitled to an early retirement scheme that does not require means testing based on a visit to a health professional. The agreement is based on the implicit assumption that having spent many years in the labour market is associated with workers being worn out. The right to an early state pension applies to people who have been in the labour market for up to 44 years and are at least 61 years old. Those who satisfy this condition can apply to retire three years before the state pension age. An individual who has been in the labour market for 42 or 43 years, respectively, can apply to retire one or two years earlier.

Importantly, the state pension age continues to be linked to life expectancy. Thus, as life expectancy increases, so does the state pension age, as well as the retirement age of those with a right to an early state pension. Approved applicants for early retirement can obtain a maximum monthly benefit of DKK 13,550 (at 2020 levels, USD 1,951) before tax. However, there may be some crowding-out effects from personal pension income (pillar two), as well as earnings.

According to official estimates in Denmark, approximately 41,000 full-time employees will obtain the right to early retirement in 2022, and 24,000 of them will exercise it. The cost of the pension will be approximately DKK 200 million in 2021 (USD 29 million) and then increase through 2025, after which annual expenses are projected at DKK 3.5 billion (USD 504 million) permanently.

While the early retirement agreement indisputably addresses a problem that is linked to inequality in life expectancy, it is not guaranteed that the early pension is given to workers who are worn out. A long history in the labour market do not necessarily mean that a worker is worn out. Similarly, someone with shorter work experience may also have health problems and can argue that these problems are due to strenuous work.

(b) Sketching our PPP proposal in a Danish context

To illustrate how our PPP proposal would work in Denmark, we assume that the official pension age is 66 years and that average life expectancy is 81 years. This gives an average retirement period of 15

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3 The notion of linking retirement age to life expectancy originates in the Danish welfare agreement of 2006.
4 This essentially means that if the remaining lifetime of a, say, 60-year-old individual rises from 21 years in 2020 to 28 years in 2050, a 72-year-old person in 2050 will have the same ability to work as a 65-year-old today.
years. To be able to calculate the lump sum payment, S, that must be transferred to each individual when they reach the state pension age, we need to know the average benefit rate for the pillar one pension. This amount can be calculated by dividing the state’s total expenditure on pillar one pensions (approximately USD 23.4 billion in 2021) by the total number of pillar one pension recipients (approximately 1,710,000 individuals in 2021).

The amount transferred to each person is the annual amount multiplied by the number of pension years. In our “baseline,” it is assumed that the sum of the basic amount and the pension supplement is USD 21,500 annually, which (with a zero interest rate for the sake of simplicity) gives a total retirement amount of approximately USD 320,000 (15 years at USD 21,550 per year). Approximately 65,000 Danes turned 65 in 2021, and if they all transfer USD 320,000 from the public sector to the pension funds, the total expense will equal roughly USD 21 billion, or 0.5% of Denmark’s GDP.

A closer look at how pension funds handle differences in life expectancy requires, first of all, an examination of how the funds treat assumptions about their members’ remaining lifetime. As is noted above, if contributions at the start of the pension period are the same (USD 320,000), the pension funds whose members have the shortest life expectancy will be able to pay a higher annuity than pension funds whose members have a longer life expectancy. Therefore, it is interesting to take a closer look at how large the differences are in the pension funds’ assumptions about their members’ life expectancy.

The principle is that the calculation of life expectancy of a specific fund’s members is based on an “industry standard,” which is anchored in the Danish Financial Supervision Authority’s life expectancy model (see Appendix for details). Utilizing that model makes it possible to assess whether the observed lifetime in a specific Danish pension fund differs from the expected lifetime of all of the funds; i.e., a benchmark.

Life expectancy assumptions do indeed differ across the Danish pension funds, see Figure 4. Here, estimates of mortality are calculated for five different funds: (A) The Pension Fund for Lawyers, Economists and Engineers (P+) and The Medical Doctors’ Pension Fund (MDPF), both with highly educated members; (B) PensionDanmark (PD) and PenSam, both with relatively low-educated members; and (C) The Pension Fund of Early Childhood Teachers (PFECT), included to represent an intermediate case.

It appears that there is a clear connection between the level of education of the different pension funds’ members and the assumptions of mortality – and thus life expectancy – with which the fund operates. If members have higher education, then the pension funds expect them to live longer, and if the members are unskilled, the pension fund expects them to have a shorter life span.5

The fact that the pension funds operate with different life expectancy assumptions in practice provides an opportunity to address this source of inequality in a new way. With the current arrangement of pillar one in the Danish pension system, there is a serious risk that low-income groups might not

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5 For more details, see the Appendix.
benefit as much from the system, since high-educated people can receive state pension over many years, and much longer than less educated workers.

**Figure 4** Mortality intensities for men across pension funds

This point can also be illustrated with a numerical example from a recent research publication of ours (Jensen et al., 2021). In this paper, we found that under plausible assumptions, highly educated workers’ pension benefits will be reduced by almost 14% if they join a pension fund that also has highly educated members than if they join a pension fund whose membership includes only unskilled workers, see Figure 5.

**Figure 5.** Monthly benefits (USD) on annuities across pensions funds

Note: The values in Figure 5 are calculated using the Danish FSA’s lifetime model for various pension funds (see Appendix). We assume that individuals have DKK 1,000,000 (USD 144,000) in their pension account and that annuities must pay the same nominal amount each month. We assume an interest rate of 0%. However, the calculations are highly sensitive to interest rate changes, as can be seen in Table A5 in the Appendix.
The current design of pillar one does not utilize information that is available in the market – namely, the pensions funds’ knowledge of Danes’ life expectancy. Under our three alternative models, including the most radical PPP model, this information would be internalized so as to utilize the pension funds’ information advantage when the market product (i.e., annuity) is launched. Due to their accumulated knowledge of life expectancy, the pension funds can offer an annuity that ensures the lowest-educated individuals the highest possible benefit through their retirement.

Finally, it should be mentioned that a comparison of the distributional characteristics of the proposed reform and the existing design of pillar one should consider pension benefits in a much more thorough way than has been possible here. Since benefits from pillar one are income- and wealth-dependent, it provides an opportunity to target benefits specifically towards the most disadvantaged. The fact remains, however, that the most important thing has been to address differences in life expectancy in a way that will, by itself, reduce inequalities in the Danish pension system.

6. **Concluding remarks**

We have addressed the problem of inequality in pillar one of the pension system and have proposed solutions to it that are actuarially sound. Our first proposed change to the system involved allowing low life-expectancy workers to retire earlier and delaying the retirement of high life-expectancy workers so that the expected discounted sum of future benefits for the two groups would be equal, and equal to the amount the average worker receives under the current system.

Our second, and more radical, proposal involved assigning the occupational pension funds the task of paying out pillar one benefits in return for a lump sum transfer of funds from the government for each retiree. This solution does not require that the worker satisfy any “objective criterion” such as a visit to the doctor or a history in the labour market. In our case study, we demonstrate that the pension funds do in fact possess information that is not utilized in the current pillar one pension system.

Each of the two models should be mandatory for every individual, including those with longer life expectancy. In the existing system, each individual regarded with a “good” life can continue receiving public pension benefits until a very old age. The suggested model will lead to a negative redistribution from individuals with a high life expectancy. Those with a higher life expectancy will have to accept a lower annual pillar one pension based on the higher life expectancy used by its pillar two pension fund, while those with a lower life expectancy will receive a higher pillar one pension.

It is worth mentioning that this implied redistribution only corrects the injustice built into the current system under which workers with low life expectancy in effect subsidize the retirement of workers with high life expectancy. The high life-expectancy group of individuals comprises people who are well off – people with higher education and larger pillar two pensions. In addition, while workers can choose to “redeem” a worn-out body for early retirement, many individuals without health problems can and will continue to work beyond the official retirement age.
The question may arise whether the same principle applies to the genders, since women have a longer life expectancy than men (see Figures A1 and A2 in the Appendix). These differences are already reflected in the gender composition of members of different pension funds. Thus average life expectancy for members of a pension fund varies directly with the proportion of women among the membership. Moreover, differentiating between men and women within a pension fund is not legal in Denmark. Finally, the desirability of such differentiation – paying women a lower monthly pension income – is doubtful, given the gender wage gap in the labour market.

Seen from a traditional welfare state perspective, it may appear quite far-reaching to wave goodbye to key parts of pillar one in the pension system. Of course, there is nothing to prevent the outlined reform from being combined with supplementary schemes, including redistributive measures. Pension systems are complicated, and major interventions should not be undertaken arbitrarily. Thus there is a need to study this in greater detail, partly to design a new system and partly to uncover its marginal effects on GDP, public finances, employment, redistribution, and so on.

Overall, it is crucial that pension systems enjoy broad support from the people in the country and the majority of parliament. If the introduction of more lenient access to early retirement for the frail is necessary for continued support, it is probably something worth pursuing.

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Appendix: The Danish Financial Supervisory Authority's (FSA) benchmark model

To determine whether the pension funds address the prominent inequality associated with educational attainment in life expectancy, we examine mortality rates for men and women separately. A mortality rate describes the distribution of $T$ (which denotes the uncertain lifespan of an individual) in relation to the conditional probability that an individual dies after being alive at time $t$ (Ramlau-Hansen & Steffensen, 2016). Mathematically, this can be described as:

$$
\mu(t)\Delta t \approx P(T < t + \Delta t|T > t)
$$

Mortality rates can be calculated using several models, one of them the well-known Gompertz-Makeham mortality model. This model can be described as follows:

$$
\mu = \alpha + \beta e^{\gamma t}
$$

The model depends on three parameters: $\alpha$, $\beta$, and $\gamma$, where the $\alpha$-component can be interpreted as “non-age-accelerated mortality” and the $\beta e^{\gamma t}$-component denotes “age-accelerated mortality” (Ramlau-Hansen & Steffensen, 2016). This brief introduction is provided primarily because the Gompertz-Makeham model is the basis for many of the foundations of the Danish pension funds, including G82 and G93.

Moreover, it is possible to calculate death intensities based on the Danish FSA’s benchmark model, the objective of which is to provide pension funds and life insurance companies with a standardized tool to assess whether the population's mortality deviates significantly from the model. According to the Danish FSA (2011, 2021), the functional form of the model is given by:

$$
M^k_{0,k}: \quad \mu^k_{x,t} = \exp[\beta^k_1 r_1(x) + \beta^k_2 r_2(x) + \beta^k_3 r_3(x)] \bar{\mu}^k_{x,t}
$$

$$
\bar{\mu}^k_{x,t} = \mu_{k,m}(x, 2019) \cdot (1 - R_K(x))^{t-2019}
$$

where $\mu^k_{x,t}$ and $\bar{\mu}^k_{x,t}$ are population mortalities (the death intensities calculated for various pension providers/pension companies) and the benchmark mortality for age $x$ in year $t$. In addition, $r_1(x), r_2(x)$ and $r_3(x)$ denote regressors given by:

$$
r_m(x) = \begin{cases} 
1 & \text{for } x \leq x_{m-1} \\
\frac{x_m - x}{x_m - x_{m-1}} & \text{for } x_{m-1} < x < x_m \\
0 & \text{for } x \geq x_m
\end{cases}
$$

where $m=1,2,3$ and $(x_0, x_1, x_2, x_3) = (40, 60, 80, 100)$. In order to assess whether pension fund members’ mortality differs significantly from the FSA’s model, one can perform a parametric test based on a Poisson model. If the funds detect a deviation, a series of parametric tests must be performed to determine which age range the funds themselves must use to estimate mortality (Finanstilsynet, 2011). It should also be noted that the model is designed so that there is always similarity between the population mortality and the benchmark mortality ($\bar{\mu}^k_{x,t}$) from age 100 upwards.
Furthermore, the model has a natural reduction factor specified as: \( (1 - R_K(x))^{t-2019} \), which is intended to correct for the fact that people are living longer, on average. In the calculations for this paper, the reduction factor is estimated to be 1 year. Finally, \( \mu_{K,m}(x, 2019) \) indicates the benchmark for the current observed mortality of women and men.

The pension funds hypothesize that the population mortality is equal to the benchmark mortality; i.e., \( \mu_{x,t}^k = \bar{\mu}_{x,t}^k \):

\[
H_0^k = \beta_1^k = \beta_2^k = \beta_3^k = 0
\]

A likelihood ratio test with a null hypothesis \( H_0^k \) is performed on the \( M_0^k \) model. If the hypothesis is accepted at a 5% significance level, there is no evidence to conclude that the population mortality differs significantly from the benchmark mortality. If the hypothesis is rejected, the pension funds must perform several partial hypotheses for partial coincidence, which we do not include in this paper, as the intuition behind the model is more important than the statistical tests themselves.\(^6\)

In this paper, death intensities are calculated using a model developed by the Danish FSA, as its data are publicly available for all pension funds and life insurance companies, thereby allowing a meaningful comparison across the different funds. These are the beta coefficients obtained from the technical basis of the selected funds and are estimated based on the individual fund’s lifetime analysis. This leads to the following mortality intensities, with similarities to the benchmark model as it follows around age 100:

**Figure A1. Mortality rate for men across pension funds**

Source: Own calculations based on data from the Danish FSA.

Note: The Y-axis indicates the logarithm with base 10. The logarithm is taken from mortality rates based on the Danish FSA’s model. This is because mortality curves in the log scale are almost linear curves since the mortality rate estimates are calculated using exponential functions.

---

\(^6\) Further information about this subject can be found in the Danish FSA’s test hierarchy (2011).
**Figure A2.** Mortality rates for women across pension funds

![Figure A2](image)

Source: Own calculations based on data from the Danish FSA.

Note: The Y-axis indicates the logarithm with base 10. The logarithm is taken from mortality rates based on the Danish FSA’s model. This is because mortality curves in the log scale are almost linear curves since the mortality intensity estimates are calculated using exponential functions.

Additionally, death rates can be converted into expected remaining lifespans. A person’s remaining expected lifetime is the time they can expect to live after reaching age x. Mathematically, this can be expressed as

$$e_x = \frac{1}{\mu_x} (1 - \exp(-\mu_x)) + p_x, e_{x+1}, e_{111} = 0$$

where $e_{111} = 0$ means that we expect a 111-year-old to have 0 more years to live. Furthermore $p_x$ can be described as:

$$p_x = \exp(-\mu_x)$$

This shows that the probability that a person at age x reaches age x+1 is equal to the exponential function of the negative value of the death intensities. Roughly speaking, one can therefore describe the remaining expected lifetime as the lifetime an individual is expected to have left, conditional on the individual’s having reached age x. Based on our death rate estimates, we have calculated the following remaining lifetimes for men and women.
Table A1. Overview of expected remaining lifetime for men across pension funds

<table>
<thead>
<tr>
<th>Age</th>
<th>P+</th>
<th>The Medical Doctors’ Pension Fund</th>
<th>Pension Fund of Early Childhood Teachers</th>
<th>PenSam</th>
<th>PensionDanmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>25.83</td>
<td>25.41</td>
<td>23.57</td>
<td>22.64</td>
<td>23.24</td>
</tr>
<tr>
<td>61</td>
<td>24.93</td>
<td>24.48</td>
<td>22.75</td>
<td>21.84</td>
<td>22.41</td>
</tr>
<tr>
<td>62</td>
<td>24.05</td>
<td>23.55</td>
<td>21.95</td>
<td>21.05</td>
<td>21.60</td>
</tr>
<tr>
<td>63</td>
<td>23.14</td>
<td>22.64</td>
<td>21.15</td>
<td>20.27</td>
<td>20.80</td>
</tr>
<tr>
<td>64</td>
<td>22.25</td>
<td>21.74</td>
<td>20.37</td>
<td>19.50</td>
<td>19.99</td>
</tr>
<tr>
<td>65</td>
<td>21.39</td>
<td>20.84</td>
<td>19.59</td>
<td>18.74</td>
<td>19.21</td>
</tr>
<tr>
<td>66</td>
<td>20.53</td>
<td>19.95</td>
<td>18.82</td>
<td>18.00</td>
<td>18.44</td>
</tr>
<tr>
<td>67</td>
<td>19.68</td>
<td>19.08</td>
<td>18.06</td>
<td>17.27</td>
<td>17.67</td>
</tr>
<tr>
<td>68</td>
<td>18.84</td>
<td>18.21</td>
<td>17.31</td>
<td>16.53</td>
<td>16.93</td>
</tr>
<tr>
<td>69</td>
<td>18.00</td>
<td>17.35</td>
<td>16.57</td>
<td>15.82</td>
<td>16.18</td>
</tr>
</tbody>
</table>

The Danish FSA’s life expectancy benchmark forms a basis for comparison of pension funds’ life expectancy assumptions.

Based on the life expectancy model and the individual pension funds’ annual reports, one can calculate the life expectancy for the average member of various pension funds. The pension funds are not obliged to use the benchmark model for their life expectancy assumptions if the assumptions of their own model are deemed reasonable. One example is the life expectancy model of ATP, where different databases are used to project different life expectancies. As the benchmark is a non-parameterized smoothing of data, it cannot be considered a parameterized equation.

Based on the data, one can calculate the observed mortality rate; i.e., the probability that a randomly selected person for a given cohort survives one year. This is made relatively simple by observing how many survived in one year and how many died. Doing this for different socioeconomic groups, one can calculate average life expectancy, assuming stationarity for the age distribution. This assumption is not adequate for the actual expected lifetime, of course, but it gives good insight into how the actual observable lifetime differs in the data.

Table A2. Overview of expected remaining lifetime for women across pension funds

<table>
<thead>
<tr>
<th>Age</th>
<th>P+</th>
<th>The Medical Doctors’ Pension Fund</th>
<th>Pension Fund of Early Childhood Teachers</th>
<th>PenSam</th>
<th>PensionDanmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>27.48</td>
<td>27.36</td>
<td>27.09</td>
<td>26.88</td>
<td>26.63</td>
</tr>
<tr>
<td>61</td>
<td>26.54</td>
<td>26.43</td>
<td>26.18</td>
<td>25.98</td>
<td>25.75</td>
</tr>
<tr>
<td>62</td>
<td>25.61</td>
<td>25.51</td>
<td>25.28</td>
<td>25.09</td>
<td>24.83</td>
</tr>
<tr>
<td>63</td>
<td>24.70</td>
<td>24.60</td>
<td>24.38</td>
<td>24.21</td>
<td>24.00</td>
</tr>
<tr>
<td>64</td>
<td>23.78</td>
<td>23.70</td>
<td>23.50</td>
<td>23.33</td>
<td>23.15</td>
</tr>
<tr>
<td>65</td>
<td>22.88</td>
<td>22.80</td>
<td>22.62</td>
<td>22.47</td>
<td>22.30</td>
</tr>
<tr>
<td>67</td>
<td>21.09</td>
<td>21.02</td>
<td>20.88</td>
<td>20.76</td>
<td>20.63</td>
</tr>
<tr>
<td>68</td>
<td>20.20</td>
<td>20.15</td>
<td>20.01</td>
<td>19.91</td>
<td>19.80</td>
</tr>
<tr>
<td>69</td>
<td>19.33</td>
<td>19.29</td>
<td>19.16</td>
<td>19.07</td>
<td>19.97</td>
</tr>
</tbody>
</table>
Table A3. Expected remaining lifetime for a 65-year-old in selected pension funds, under the assumption of stationary life expectancy

<table>
<thead>
<tr>
<th></th>
<th>P+</th>
<th>Pension Danmark</th>
<th>MDPF</th>
<th>PenSam</th>
<th>PFECT</th>
<th>FSA Benchmark</th>
<th>Denmark statistics avg. life expectancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td>22.93</td>
<td>22.35</td>
<td>22.85</td>
<td>22.52</td>
<td>22.66</td>
<td>22.52</td>
<td>21.13</td>
</tr>
</tbody>
</table>

It is very clear how observed mortality varies across the different pension funds, although there is also a significant difference between men and women. Note that the results show that the difference between men and women is greater than the difference across the various pension funds.

When predicting the future, pension funds must make assumptions about mortality rates, since a deceased individual is no longer entitled to annuity benefits. The observed mortality rates are useful for comparison across socioeconomic scales but are not suitable for actuarial exercises. An example of this could be a 20-year-old in 2020 whose life expectancy can be calculated from observed mortality rates in 2020. On the other hand, we know that the life expectancy for a given age group also increases over time, and the 20-year-old person in 2020, who will be 60 in 2060, has a significantly lower risk of death in 2060 than a similar 60-year-old in 2020. To account for this, the Danish FSA’s benchmark model includes a discounting of mortality over time – to compensate for future lifetime improvements that appear in the data. Using the discounted mortality, the lifespan of individuals becomes significantly longer, but the likelihood of its being more accurate increases as well. Calculating the remaining life expectancies with discounted mortality rates generates the following life expectancies for 65-year-olds in 2020:

Table A4. Expected remaining lifetime for a 65-year-old in selected pension funds, under the assumption of rising life expectancy

<table>
<thead>
<tr>
<th></th>
<th>P+</th>
<th>Pension Danmark</th>
<th>MDPF</th>
<th>PenSam</th>
<th>PFECT</th>
<th>FSA Benchmark</th>
<th>Statistics Denmark, average life expectancy</th>
</tr>
</thead>
</table>

Compared to the figure without discounting, the life expectancies are a bit higher (around 10%), which is to be expected.

By taking a starting point where the life expectancies are calculated based on the Danish FSA’s lifetime model for various pension funds, the benefits from a simple annuity pension can be calculated across different funds. We start by assuming that individuals in either $P^+$, *Lægernes Pension*, *Pensam*, *Pension Danmark* or *Pædagogernes pension* have DKK 1,000,000 left in their pension savings. Next, we assume that annuities must pay the same nominal amount each month. If this is done, we receive
the following payments per month, shown in Figure 6. As is seen in the figure, there is a significant
difference in the payments across the various pension funds – a male member of Pensam can expect to
receive up to 13.5% more per month than a male member of P+. Women, in general, can expect to be
paid a little less per month due to their greater longevity. On the other hand, the variation across funds
is smaller among women, due to a smaller variation in life expectancy. We have assumed an interest
rate of 0%. However, the calculations are very sensitive to interest rate changes, as can be seen in the
table below:

Table A5. Annuity payments (in DKK) at different interest rates

<table>
<thead>
<tr>
<th>Interest rate</th>
<th>P+</th>
<th>Pension Danmark</th>
<th>MDPF</th>
<th>PenSam</th>
<th>PFECT</th>
<th>FSA Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>3,628</td>
<td>3,717</td>
<td>3,640</td>
<td>3,691</td>
<td>3,669</td>
<td>3,691</td>
</tr>
<tr>
<td>1%</td>
<td>4,133</td>
<td>4,232</td>
<td>4,146</td>
<td>4,203</td>
<td>4,178</td>
<td>4,203</td>
</tr>
<tr>
<td>2%</td>
<td>4,671</td>
<td>4,781</td>
<td>4,686</td>
<td>4,749</td>
<td>4,721</td>
<td>4,749</td>
</tr>
<tr>
<td>3%</td>
<td>5,241</td>
<td>5,361</td>
<td>5,257</td>
<td>5,326</td>
<td>5,296</td>
<td>5,326</td>
</tr>
<tr>
<td>4%</td>
<td>5,840</td>
<td>5,970</td>
<td>5,857</td>
<td>5,932</td>
<td>5,900</td>
<td>5,932</td>
</tr>
<tr>
<td>Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>3,901</td>
<td>4,323</td>
<td>3,997</td>
<td>4,428</td>
<td>4,244</td>
<td>4,148</td>
</tr>
<tr>
<td>1%</td>
<td>4,412</td>
<td>4,860</td>
<td>4,507</td>
<td>4,972</td>
<td>4,779</td>
<td>4,673</td>
</tr>
<tr>
<td>2%</td>
<td>4,954</td>
<td>5,427</td>
<td>5,048</td>
<td>5,546</td>
<td>5,345</td>
<td>5,229</td>
</tr>
<tr>
<td>3%</td>
<td>5,527</td>
<td>6,023</td>
<td>5,617</td>
<td>6,149</td>
<td>5,940</td>
<td>5,814</td>
</tr>
<tr>
<td>4%</td>
<td>6,128</td>
<td>6,645</td>
<td>6,214</td>
<td>6,778</td>
<td>6,562</td>
<td>6,426</td>
</tr>
</tbody>
</table>

Annuity payments (in USD) at different interest rates

<table>
<thead>
<tr>
<th>Interest rate</th>
<th>P+</th>
<th>Pension Danmark</th>
<th>MDPF</th>
<th>PenSam</th>
<th>PFECT</th>
<th>FSA Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>522</td>
<td>535</td>
<td>524</td>
<td>532</td>
<td>528</td>
<td>532</td>
</tr>
<tr>
<td>1%</td>
<td>595</td>
<td>609</td>
<td>597</td>
<td>605</td>
<td>602</td>
<td>605</td>
</tr>
<tr>
<td>2%</td>
<td>673</td>
<td>688</td>
<td>675</td>
<td>684</td>
<td>680</td>
<td>684</td>
</tr>
<tr>
<td>3%</td>
<td>755</td>
<td>772</td>
<td>757</td>
<td>767</td>
<td>763</td>
<td>767</td>
</tr>
<tr>
<td>4%</td>
<td>841</td>
<td>860</td>
<td>843</td>
<td>854</td>
<td>850</td>
<td>854</td>
</tr>
<tr>
<td>Men</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>562</td>
<td>623</td>
<td>576</td>
<td>638</td>
<td>611</td>
<td>597</td>
</tr>
<tr>
<td>1%</td>
<td>635</td>
<td>700</td>
<td>649</td>
<td>716</td>
<td>688</td>
<td>673</td>
</tr>
<tr>
<td>2%</td>
<td>713</td>
<td>781</td>
<td>727</td>
<td>799</td>
<td>770</td>
<td>753</td>
</tr>
<tr>
<td>3%</td>
<td>796</td>
<td>867</td>
<td>809</td>
<td>885</td>
<td>855</td>
<td>837</td>
</tr>
<tr>
<td>4%</td>
<td>882</td>
<td>957</td>
<td>895</td>
<td>976</td>
<td>943</td>
<td>925</td>
</tr>
</tbody>
</table>

One reason for this sensitivity to interest rates is the long-time horizons that actually affect
compound interest rates. As a reference, a 2% interest rate can accumulate to almost a 50% increase
over 20 periods.