YEAR 13 FURTHER MATHEMATICS

Students will study Further Mathematics alongside the Mathematics A Level; this acts as preparation for those students who may be considering further study in the most challenging STEM disciplines at degree level. As part of their Further Mathematics course, students will learn sophisticated mathematical ideas and techniques. In order to keep students' options as broad as possible in terms of future study, we offer modules in core pure mathematics, further statistics and further mechanics.

Y13 A Level FM		Topic	Knowledge	Skills	Assessment
AUTUMN TERM 1		Complex Numbers (Recap)	Apply all Complex Number knowledge from Y12 Core Pure Book 1 Radians Use the binomial expansion Understand the cyclic nature of trigonometric curves and 'CAST diagram' Use the exponential function De Moivre's proof requires proof by induction Rules of indices Integration of trigonometrical functions using limits Sum of an infinite geometric series	Express a complex number in exponential form Multiply and divide complex numbers in exponential form Understand de Moivre's theorem Use de Moivre's theorem to derive trigonometric identities Use de Moivre's theorem to find sums of series Solve completely equations of the form $z^n - a - ib = 0$ giving special attention to cases where a = 1 and b = 0 Use complex roots of unity to solve geometric problems	

AUTUMN TERM 1		Integration (recap)	Differentiate and integrate polynomials Use limits to find definite integrals of polynomials Use integration to find the area under a curve, between limits, for polynomial functions	Integrate standard mathematical functions including trigonometric and exponential functions and use the reverse of the chain rule to integrate functions of the form f(ax + b) Use trigonometric identities in integration Use the reverse of the chain rule to integrate more complex functions Integrate functions by making a substitution, using integration by parts and using partial fractions Use integration to find the area under a curve Use the trapezium rule to approximate the area under a curve Solve simple differential equations and model real-life situations with differential equations	
AUTUMN TERM 1		Geometric and negative binomial distributions	Calculate individual probabilities for the binomial distribution Find the mean E(X) and variance Var(X) for the binomial distribution Calculate probabilities for independent events	Understand and use the geometric distribution Calculate and use the mean and variance of the geometric distribution Understand and use the negative binomial distribution Calculate and use the mean and variance of the negative binomial distribution	
AUTUMN TERM 1		Series	Use sum of series (Y13 Pure Ch3) Use series knowledge from Y12 Core Pure Book 1 Differentiate trigonometric functions Use partial fractions Add algebraic fractions	Understand and use the method of differences to sum finite series Find and use higher derivatives of functions Express functions as an infinite series in ascending powers suing Maclaurin series expansion Find the series expansions of compound functions	

AUTUMN TERM 1		Hypothesis testing	Use the geometric distribution Understand the language and concepts of hypothesis testing	Use hypothesis tests to test for the parameter p in a geometric distribution Find the critical regions of a geometric distribution	AP1
AUTUMN TERM 1		Momentum and impulse	The impulse-momentum principle in one dimension and will have used this to solve problems. How to write the impulse-momentum principle in vector form for two dimensions. The principle of conservation of momentum in one dimension and will have used this to solve problems. How to write the principle of conservation of momentum in vector form for two dimensions.	Use the impulse-momentum principle in vector form to solve problems in two dimensions. Use the principle of conservation of momentum in vector form to solve problems in two dimensions.	
AUTUMN TERM 2		Elastic strings and springs	The relationship called Hooke's law and understand the modulus of elasticity of an elastic string or spring How to calculate the extension even when a spring is compressed How to resolve forces into components to apply principles of equilibrium to problems in two dimensions The conditions for maximum velocity/speed and the conditions for instantaneously at rest The relationship called Hooke's law That the elastic potential energy stored in an elastic string or spring equals the work done to stretch (or compress) the elastic string or spring That when no external forces (other than gravity) act on a particle, then the sum of kinetic energy, gravitational potential energy and elastic potential energy remains constant	Use Hooke's law to solve equilibrium problems involving elastic strings or springs Use Hooke's law to solve dynamics problems involving elastic strings or spring Find the energy stored in an elastic string or spring Solve problems involving elastic energy using the principle of conservation of mechanical energy and the work-energy principle	
AUTUMN TERM 2		Chi-squared tests	Understand and use the geometric distribution Understand and use hypothesis testing and goodness-of-fit testing for discrete distributions	Apply goodness-of-fit tests to geometric distributions	

AUTUMN TERM 2		Methods in differential equations	Solve to find the general solution to a first order differential equations using method of separating the variables Find a particular solution to the above using boundary conditions Integrate functions to give answers in natural log form Integrate trigonometric functions Understand how the discriminant relates to roots of an equation Complex roots Differentiation of functions using product rule	Solve first-order differential equations using an integrating factor Solve second-order homogeneous differential equations using the auxiliary equation Solve second-order non-homogeneous differential equations using the complimentary function and the particular integral Find particular solutions to differential equations using given boundary conditions	
AUTUMN TERM 2		Elastic collisions in 2D	How to find the coefficient of restitution between a sphere and a smooth wall when it collides normally to the fixed surface How to resolve forces/velocity/motion into components that are parallel and perpendicular to the fixed surface How to calculate and use the speed of approach and speed of rebound And understand the term “angle of deflection” And be able to use the scalar product of two vectors to find the angle between them That the components of the velocities of the spheres perpendicular to the line of centres are unchanged in the impact And be able to apply Newton’s law of restitution to the components of the velocities to the spheres parallel to the line of centres And be able to apply the principle of the conservation of momentum to motion parallel to the line of centres How to solve problems involving oblique impact of a smooth sphere with a fixed surface (see notes above) And be aware that the coefficient of restitution of different surfaces can be different And be able to resolve vectors into different perpendicular directions for each impact where necessary	Solve problems involving the oblique impact of a smooth sphere with a fixed surface Solve problems involving the oblique impact of two smooth spheres Solve problems involving successive oblique impacts of a sphere with smooth plane surfaces.	AP2

AUTUMN TERM 2		Central limit theorem	Calculate probabilities for the normal distribution Find percentage points on a normal curve Find the expected value and variance of a discrete random variable Understand and use the negative binomial distribution	Understand and apply the central limit theorem to approximate the sample mean of a random variable, $\bar{X}$ Apply the central limit theorem to other distributions	
SPRING TERM 1		Polar coordinates	Radians Be able to draw loci of eg |z-3-4i|=5 from Core Pure Book 1 Know sec x, cosec x, cot x Know trigonometrical identities from Y13 Pure Book 2 Integrate with limits using identities Differentiate products	Understand and use polar coordinates Convert between polar and Cartesian coordinates Sketch curves with r given as a function of θ Find the area enclosed by a polar curve Find tangents parallel to, or at right angles to, the initial line	

SPRING TERM 1		Methods in calculus	The difference between convergent and divergent series/sequences How to determine if a function has one limit/both limits that are infinite How to determine if a function is not defined at some point within a given interval And be able to use limit notation and be able to show the limiting process clearing in their working out How to split integrals over a given range and be able to choose appropriate points at which to do so And understand and be able to calculate the arithmetic mean of a finite set And understand that a function takes an infinite number of values within a given range And understand that we can represent the sum of an infinite number of values of a function within a given range as the integral of that function over that range How transformations of a function impact the integral of the transformed function The standard results for integrals of trigonometric functions The standard results for integrals of inverse trigonometric functions How to complete the square to get functions in the form $a(x \pm b)^2 \pm c^2$ The relevant trigonometric substitution to use to create functions in the form of, $\frac{1}{a^2+x^2}$ or $\frac{1}{\sqrt{a^2-x^2}}$ How to write a fraction as partial fractions both with linear and quadratic denominators How to recognize improper fractions when numerators and denominators involve polynomials How to write and algebraic improper fraction as a mixed number How to use partial fractions to integrate rational functions	Evaluate improper integrals Understand and evaluate the mean value of a function Integrate rational functions using trigonometric substitution Integrate using partial fractions combined with the use of inverse trigonometric integrals	

SPRING TERM 1		Probability generating functions	Calculate individual probabilities for the Poisson distribution Find the mean and variance for the Poisson distribution Calculate individual probabilities for the geometric distribution and the negative binomial distribution Find the expected value and variance for the geometric distribution and the negative binomial distribution. Differentiate functions using the chain rule and product rule	Understand the use of probability generating functions Use probability generating functions for standard distributions Use probability generating functions to find the mean and variance of a distribution Know the probability generating function of the sum of independent random variables	
SPRING TERM 1		Modelling with differential equations	Good understanding and knowledge of Methods in differential equations required Knowledge of Simple Harmonic Motion from Physics helpful	Model real-life situations with first-order differential equations Use differential equations to model simple harmonic motion Model damped and forced oscillations using differential equations Model real-life situations using coupled first-order differential equations	AP3
SPRING TERM 2		Quality of tests	Carry out a hypothesis test for the mean of a normal distribution Find the critical region for a Poisson distribution and a geometric distribution	Know about Type I and Type II errors Find Type I and Type II errors using the normal distribution Calculate the size and power of a test Draw a graph of the power function of a test	

SPRING TERM 2		Volumes of revolution	How to evaluate definite integrals for a given range and integrate with respect to x And be able to determine the appropriate limits for the integral to be used How to find the coordinates where functions cross the x axis How to find the coordinates where two curves meet Formula for area of a circle and how this applied to volume of revolution As above but in terms of y: How to evaluate definite integrals for a given range and integrate with respect to y How to find the coordinates where functions cross the y axis As above and including the following: How to apply the chain rule to adjust formulae for volume of revolution How to integrate with respect to the parameter instead of x or y and impact that this has on the limits of the integral As above and including the following: How to determine what part of the function will be used to generate the volume involved in the situation How to determine the limit values to use for the integral	Find volumes of revolution around the x-axis Find volumes of revolution around the y-axis Find volumes of revolution for curves defined parametrically Model real-life applications of volumes of revolution	

SPRING TERM 2		Hyperbolic functions	Know the exponential functions $y = e^x$ and $y = e^{-x}$ and be able to combine them to create the basic hyperbolic functions for sinh x and cosh x Know how to combine sinh x and cosh x to get tanh x Know the reciprocal functions for hyperbolics and their names; cosech x, sech x and coth x Know the range and domain for each of the six hyperbolic function described above Know what happens to the values of each hyperbolic function as $x \to \pm\infty$ And understand the terms "odd" and "even" functions How to rearrange an exponential function into a logarithmic function How to create and solve a quadratic function in the variable e^x Trigonometric identities including double angles, addition and factor formulae Understand and be able to apply Osborne's rule to trigonometric identities to create equivalent hyperbolic identities Know the basic derivatives of the hyperbolic functions sinh x, cosh x and tanh x Know the basic derivatives of the inverse hyperbolic functions arsinh x, arcosh x and artanh x Use the basic derivative to determine the corresponding basic integrals involving hyperbolic and inverse hyperbolic functions How to complete the square to get functions in the form $a(x \pm b)^2 \pm c^2$ The relevant integral to use for functions in the form of, $\frac{1}{\sqrt{a^2+x^2}}$ or $\frac{1}{\sqrt{x^2-a^2}}$ And know the relevant substitution to use in these types of questions	Understand the definitions of hyperbolic functions Sketch the graphs of hyperbolic functions Understand and use the inverse hyperbolic functions Prove identities and solve equations using hyperbolic functions Differentiate and integrate hyperbolic functions	AP4
SUMMER TERM 1			Revision		External A Level Further Maths exams