Repeat Sales Indexes: Estimation Without Assuming that Errors in Asset Returns Are Independently Distributed

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Abstract

This paper proposes an alternative specification for the second stage of the Case-Shiller repeat sales method. This specification is based on serial correlation in the deviations from the mean one-period returns on the underlying individual assets, whereas the original Case-Shiller method assumes that the deviations from mean returns by the underlying individual assets are i.i.d. The methodology proposed in this paper is easy to implement and provides more accurate estimates of the standard errors of returns under serial correlation. The repeat sales methodology is generally used to construct an index of prices or returns for unique, infrequently traded assets such as houses, art, and musical instruments, which are likely to be prone to exhibit serial correlation in returns. We demonstrate our methodology on a dataset of art prices and on a dataset of real estate prices from the city of Amsterdam.

Keywords: repeat sales, heteroskedasticity, serial correlation

JEL classifications: C13, C29, G12

1. Introduction

The repeat sales methodology has become an important technique to determine price trends and returns for idiosyncratic assets, including real estate, art, and antique musical instruments. The basic principle is to improve upon hedonic regression techniques by using pairs of observations on the same asset. In effect, this is using fixed effects estimation with the usual advantage of controlling for unobserved characteristics of individual assets.

Bailey, Muth, and Nourse [1963] were the first to propose a methodology for repeat sales regressions, simply using ordinary least squares. Case and Shiller [1987] improved on this with a three-stage generalized least squares (GLS) methodology. Under the assumption that deviations from the mean single-period returns by the underlying assets are independently and identically distributed, the variance of returns grows linearly when single-period returns are aggregated over the holding period of an asset. Thus, the errors in the repeat sales regression are heteroskedastic. In order to correct for this heteroskedasticity, one proceeds by first estimating OLS regressions using dummy variables for time periods between sales (or -1 and +1 indicators for first and second sale dates if one is estimating index levels). Then, the squared residuals are regressed against the length of the holding period.[1] The estimates from the second stage regression are then used to form weights for the third-stage GLS regressions. While both the OLS and GLS regressions provide unbiased estimates of the return coefficients for each period, the

[1] The second stage regressions also include a constant. Since the repeat sales approach purges estimates of "house-specific" errors, the constant represents the variance of a "transaction-specific random error" which is independent of the holding period. With this explanation, the constant must be positive.

GLS regressions will be more efficient.[2] Our goal in this paper is to study the implications of non-i.i.d. errors on the specification of the second-stage regression. We show that, in many cases, the i.i.d. hypothesis on the individual asset returns is rejected and that efficiency can be easily improved by a more general specification of the second stage. We demonstrate our method using two different datasets: art sold in Amsterdam between 1780 and 2007, and real estate sold along the Herengracht canal in Amsterdam between 1630 and 1972.

There has been considerable research since Case and Shiller on repeat sales techniques. It is well-known that the standard specification of logarithmic returns produces estimates of the geometric mean of property returns. The estimate of the arithmetic mean in a period depends on the geometric mean return and the cross-sectional variance of the per-period return of the individual assets. Thus, any biases in estimates of the variance affect point estimates of the arithmetic mean return. Goetzmann [1992] proposed using the second stage of the Case-Shiller method to estimate the cross-sectional variance. We show that under any assumptions other than i.i.d., the Goetzmann correction provides a biased estimate of the variance in per-period returns, and thus a biased estimate of the per period arithmetic return.

A major criticism of repeat sales indexes is that the items that are frequently traded are not a random sample of all goods. Hence, with repeat sales indexes, sample selection biases can be serious. A further criticism of repeat sales indexes is that improvements to assets can result in an increase in value -- the item that sold is not

[2] It is well-known that the last few periods in the sample have only a small fraction of transactions spanning them. The estimated returns in these last periods may have large standard errors, making the GLS efficiency improvements particularly critical.

identical to the item that is purchased. The analysis is this paper does not address these two criticisms; repeat sales indices, despite these shortcomings, are widely used in practice. Furthermore, our modifications of the estimation procedure can augment corrections for sample selection and assets that have changed in quality over time.

In Section 2, we explore work that has used the Case-Shiller method and some of their findings. In Section 3 we detail the Case-Shiller methodology and look at its effects on our two applications. Section 4 explores different assumptions regarding the asset return errors. Section 5 proposes a non-parametric alternative to the standard Case-Shiller method. In Section 6 we discuss further applications and extensions and in Section 7 we conclude our analysis.

2. The Importance of Repeat Sales Indexes and the Case-Shiller Method

In the current economic environment and the resulting subprime crisis, economists are paying very close attention to estimates of house price movements. Efficient and consistent estimates are important for gauging the state of the economy.

Both the Office of Federal Housing Enterprise Oversight (OFHEO) and the S&P/ Case-Shiller home price indexes use a variation of the Case-Shiller method to calculate their indexes. While the Case-Shiller method proposes using a linear specification for the second-stage regressions, regressing the squared errors from the first stage on the time between sales (which theoretically results from the i.i.d. assumption on the individual asset returns), the OFHEO approach (see Calhoun [1996] for details) fits a quadratic equation—regressing the squared error on time between sales and the square of time between sales. Abraham and Schauman [1991] discussed fitting a quadratic term in the holding period, but they did not discuss the theoretical implications. Calhoun [1996] states that, in practice, the constant term in the second-stage regression is often negative, which is inconsistent with the Case-Shiller explanation. Calhoun suggests forcing the constant to zero and re-estimating, which is the approach taken by OFHEO in their regressions. OFHEO presents the coefficients from their second stage regression so that users may go from the geometric to the arithmetic mean.

Case and Shiller [Standard and Poor's, 2006] directly estimate an arithmetic index but still use the standard Case-Shiller correction to correct for heteroskedasticity. The addition of non-linear terms to the Case-Shiller correction, as discussed below, could possibly improve the efficiency of their estimates.

In the literature on estimating returns to art, Goetzmann [1993] and Mei and Moses [2002] use the second stage regression coefficients in the Case-Shiller estimation scheme to estimate the variance of the cross-sectional return and continue to make use of the i.i.d. assumption on the errors in the single-period returns. These papers do not present the second-stage regressions, so readers obtain little evidence on the fit in this regression.

Quigley [1995] proposes estimation of a hybrid model using both repeat sales and hedonic estimates for properties sold once and fits the squared residuals to a quadratic function of elapsed time without a constant. He refers to this as the implication of a random walk, with little further explanation. In a subsequent paper, Hwang and Quigley [2004, p. 165] specify an error distribution "to include mean reversion as well as a random walk" in which they model autoregression in the errors in price levels.

Assuming serial independence in errors may be limiting and could have an effect on the efficiency of the index estimation. We therefore start by dropping the i.i.d.

assumption and deriving the statistical properties of the effect of the holding period on the variance of returns. Unlike Hwang and Quigley, we consider violations of independence in the errors in returns, rather than errors in price levels. It turns out to be relatively straightforward to model the effects of short-term violations of independence (such as first-order moving average processes) on the variances at different holding periods. In most cases, it is not possible to identify the statistical properties of the error process on the individual asset returns from the pattern of residuals.

As noted above, in a correctly specified equation OLS estimates, as well as the Case-Shiller method, are unbiased so the value of a more efficient GLS procedure will not reveal itself in changes in the estimated coefficients. If there are significant nonlinearities in the second-stage regression, the Case-Shiller approach will lead to incorrectly estimated standard errors but unbiased coefficients in the third stage. We discuss the importance of correctly estimated standard errors after we demonstrate differences in the Case-Shiller estimated errors and our estimates using a more flexible GLS procedure.

3. The Case-Shiller Method and Heteroskedasticity

Assume that there are N observations of repeat sales in a data set. Each observation consists of the purchase (buy) date, b_i, the purchase price, B_i, the sale date, s_i, and the sale price, S_i. The purchase and sales dates span the interval from t = 0 to t = T.[3] .

Define the length of the holding period as $\tau_i = s_i - b_i$. Let $y_i = \log\left(\frac{S_i}{B_i}\right)$ be the log of the

[3] Our presentation adopts some of the notation in Goetzmann and Peng [2002]. We use their notation as they work in returns, which is the approach that we have adopted. Case and Shiller originally worked in levels, and then differenced the levels, which resulted in an index being estimated, rather than returns.

compound return on property i. We can write this as the sum of the returns to property i in each period between purchase and sale, or $y_i = \sum_{t=b_i}^{s_i} r_{i,t}$ where $r_{i,t} \equiv \log\left(\frac{P_{i,t}}{P_{i,t-1}}\right)$, and $P_{i,t}$ is the price of property i in period t (only observed for t = s_i and b_i).

3.1 The Basic Case-Shiller Model (i.i.d. errors on individual returns)

The standard assumption is that $r_{i,t} = \mu_t + \varepsilon_{it}$, where ε_{it} is independent and identically normally distributed. Then, $y_i = \sum_{t=b_i}^{s_i} \mu_t + \sum_{t=b_i}^{s_i} \varepsilon_{it}$.[4]

Case and Shiller [1987] assumed that $\log(P_{i,t}) = C_t + H_{i,t} + N_{i,t}$ where C_t is the value of the index in period t, $H_{i,t}$ is the value of a random walk process for property i at time t, and $N_{i,t}$ is the "sale-specific random error". This is equivalent to writing the price of property i in period T as $P_{i,T} = \exp\left(\sum_{1}^{T} \mu_t + \sum_{1}^{T} \varepsilon_{i,t} + \psi_i + \upsilon_{i,T}\right)$ where $\upsilon_{i,T} \neq 0$ only if a transaction occurs in period T and ψ_i is a property-specific value. Taking logs and differencing prices from two different transactions, we obtain

$\ln\left(P_{i,T}\right) - \ln\left(P_{i,T-k}\right) = \sum_{T-k}^{T} \mu_t + \sum_{T-k}^{T} \varepsilon_{i,t} + \upsilon_{i,T} - \upsilon_{i,T-k}$. Then let $\kappa_i = \sum_{T-k}^{T} \varepsilon_{i,t} + \upsilon_{i,T} - \upsilon_{i,T-k}$ be the residual for property i in the first-stage regression. Hence, $E\left(\kappa_i\right)^2 = E\left(\sum_{1}^{\tau} \varepsilon_{i,t}\right)^2 + 2\sigma_\upsilon^2$ where σ_υ^2 is the expectation of $\left(\upsilon_{i,t}\right)^2$, under the assumption that the $\upsilon_{i,t}$ are i.i.d. Case and Shiller thus suggested first estimating an OLS repeat sales regression. Then, the

[4] In this case, the variance of the error term grows linearly with the length of the holding period. Under this assumption, one can skip the three-stage procedure and simply use $\left(s_i - b_i\right)^{\frac{-1}{2}}$ as the weights for GLS.

squared residuals are regressed against the length of the holding period and a constant in the second stage regression. Estimates from the second stage provide weights for the third-stage GLS regressions.

3.2 Applications

We start by looking at how well the Case-Shiller method corrects for heteroskedasticity empirically with two distinct repeat-sales datasets. The first dataset we use comprises 1,468 observations of repeat sales of art sold in Amsterdam between the years 1780 and 2007 and was put together by Rachel Pownall using catalogues at the Rijksmuseum in Amsterdam (see Pownall & Kraeussl [2009]). We will subsequently refer to this as the Amsterdam art dataset. The second dataset consists of 3,577 repeat sales of houses that were sold along the Herengracht canal in Amsterdam between the years 1630 and 1972. This dataset is analyzed in Eichholtz [1997].

During the Golden Age in the Netherlands, Amsterdam was a hub of trading activity in the markets for real estate and art. The Netherlands has a long history as a trading center, and records have been kept on both art and real-estate transactions since this period. Both these data sets are unique in nature and of particular interest in analyzing the effect of any dependence in the distribution of errors. The cross-sectional quality of buildings and old master paintings included in the data sets could be considered to be held fairly constant over time.

The Herengracht represents the most fashionable and beautiful of all the canals in Amsterdam, and it is likely that there is much greater homogeneity in the transaction data than the art market data, which is influenced significantly by the popularity of the artist of the day. Changes in quality (through restoration and provenance) as well as taste and fashion are likely to influence the repeat sales regressions to a greater extent than with the housing price data. The use of these 2 unique databases over such a long period provide us with an interesting case with which we are able to relax the assumption of independently distributed errors in the asset returns on these 2 markets.

We employ the Case-Shiller method as follows. In the first stage regressions we regress the difference in the logs of the price change on dummy variables $X_{i,j}$. When asset i is purchased in period b_i and sold in period s_i, $X_{i,j}$ take on the value 1for $b_i < j \leq s_i$ and zero otherwise. In the second stage of the regressions, we regress the square of the residuals from the first stage on the holding period and a constant. For the 3rd stage regressions, we construct a weighted least squares regression by dividing the regressand and the regressors from the first stage by the square root of the predicted values from the second stage. For the Amsterdam art dataset, following Goetzmann [1993] we estimate the model using 10-year periods. For the Herengracht dataset, following Eichholtz [1997] we estimate the model using 2-year periods.

We present summary results in Table 1 (Appendix Tables 1 and 2 present full results of the first and third stage regressions). In Table 1 we also present the test results from the Koenker-Basset test for heteroskedasticity [Gujarati, 2003, p. 415]. In this test, the squared residuals from the regression model ($\hat{u}_i^2$) are regressed on the squared estimated predicted values of the dependent variable ($\hat{Y}_i^2$) and a constant: $\hat{u}_i^2 = \alpha_1 + \alpha_2(\hat{Y}_i^2) + v_i$. The null hypothesis is that $\alpha_2 = 0$. If this is rejected, then one has evidence of at least one form of heteroskedasticity, that is, with respect to the predicted value of the regressor.

As shown in Table 1, the stage I R^2 for the Amsterdam art dataset is about .68 and about .60 for the Herengracht dataset, which is typical of repeat sales datasets. For the Amsterdam art dataset, we marginally cannot reject that there is no heteroskedasticity, but for the Herengracht dataset, we reject the hypothesis of no heteroskedasticity. The coefficient on time between sales is significant in both datasets. The Stage III R-squareds fall slightly relative to OLS. Again we cannot reject that there is no heteroskedasticity in the art dataset, but we continue to reject the hypothesis of no heteroskedasticity in the Herengracht dataset. Thus, the Case-Shiller GLS correction does not adjust for all the heteroskedasticity that is present.

3.3 Evidence of Non-i.i.d. Errors

Table 2 and Table 3 present different specifications of the second stage of the Case-Shiller method. The dependent variable is the squared residual from the first stage (OLS) regression and τ represents time between sales. For the Amsterdam art dataset, the standard Case-Shiller specification (shown in column 1) is dominated using the adjusted R^2 criterion and some other measures by all other specifications that include a constant. For the Herengracht dataset, the standard Case-Shiller specification does not appear to be unequivocally dominated. Nonetheless, according to the Koenker-Basset test, the Case-Shiller method fails to get rid of the heteroskedasticity, as shown in Table 1 above.[5]

Yet more evidence of non-i.i.d. errors is presented in Table 4. If the errors are i.i.d., the variance of the return error for an individual property grows linearly with time. Hence, the coefficient on time between sales when estimating two year period returns

[5] Graddy and Hamilton [2010] find that the polynomial terms in the second stage regression are significant and the Case-Shiller specification is unequivocally dominated by polynomial specifications for a repeat sales data set on violins used in Graddy and Margolis [forthcoming].

should be twice the coefficient on time between sales when estimating one year period returns. Likewise, the coefficient on time between sales when estimating 5 and 10 year periods should be respectively five and ten times the coefficient on the time between sales for one year periods. Looking at the art dataset, this is clearly not the case. Ten times the coefficient on time between sales when estimating yearly returns is statistically significantly less than the coefficient on the time between sales when estimating ten year returns. In contrast, in the Herengracht dataset, ten times the coefficient on the time between sales for the one year return does not appear to be statistically significantly different than the coefficient on the time between sales for the ten year returns.

4. Individual Asset Errors that Are Serially Correlated Across Periods

A possible reason that the Case-Shiller correction may not be working as it should is that the return errors are not i.i.d. We now drop the assumption that return errors are i.i.d. For Goetzmann's [1992] study of the behavior of repeat sales regressions using stock market data, the i.i.d. assumption seems appropriate. Considerable evidence finds few deviations from market efficiency. Market liquidity, low trading costs, and the ability to sell shares short all contribute to rapid transmission of new information into asset prices. In contrast, for many of the asset classes studied in repeat sales regressions, these features are not present.[6] Houses and individual works of art have idiosyncratic features, making simple observations of prices of other assets in the class only signals of

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[6] Shiller [2007] discusses the serial dependence in housing price aggregates. Even when repeat sales indexes incorporate a large number of properties, they usually combine data on diverse subgroups within the asset class (such as all single-family homes in a large metropolitan area). Since these submarkets may be quite thin and prices across the submarkets may not be linked closely, serial dependence in the errors also seems quite likely.

the “true price” of an asset. Trading costs are also significant (5-6% commissions plus transactions taxes and other costs for houses in the U.S. and a 10%-20% buyer’s commissions plus seller’s commissions for art sold at auction), and short sales are essentially impossible. Price data are also only available with some lag for houses (the interval between contract date and closing date at a minimum). These features could create serial dependence as well as idiosyncratic transaction errors.

Note that the statistical issue is whether the error term on the individual asset returns is correlated between periods. In repeat sales data, only the residuals on the aggregated time periods are observed. We shall see that this prevents us from uncovering much of the fine structure of the time series processes of the error returns. The specific form of the covariance structure depends on the unknown model of serial dependence. We cannot easily identify the model of serial dependence from the data as only the summed residuals are observed. We can, however, theoretically derive the effect under difference covariance structures.

In what follows, we shall drop the subscript i for the individual property since all calculations are with respect to a single property. Let the errors follow the general moving average process, $\varepsilon_t = \sum_{i=0}^{k} \mu_i \eta_{t-i}$, where $k \in [1,\infty)$, η_t is white noise and $\mu_0 = 1$.[7]

Then $\kappa_\tau \equiv \sum_{t=1}^{\tau} \varepsilon_t = \sum_{i=0}^{k} \mu_i \sum_{t=1}^{\tau} \eta_{t-i} = \sum_{i=0}^{k} \mu_i \zeta_{\tau i}$ is the sum of return errors over

τ periods, and $E\left(\kappa_\tau\right)^2 = \sum_{i=0}^{k} \mu_i^2 E\left(\zeta_{\tau i}\right)^2 + 2\sum_{i=0}^{k} \sum_{j>i}^{k} \mu_i \mu_j E\left(\zeta_{\tau i} \zeta_{\tau j}\right)$. If the process is stationary, then $\sum_{i=0}^{k} \mu_i^2$ is finite. Thus, the first sum equals $\tau \sigma_\eta^2 \sum_{i=0}^{k} \mu_i^2$. Letting,

[7] Since AR and ARMA processes can be represented as infinite-order MA processes, the case where k = ∞ includes them.

$s = j - i$, the second sum equals $2\sum_{i=0}^{k}\sum_{s=1}^{\tau-1}(\tau - s)\mu_i\mu_{i+s}\sigma_\eta^2$, which is also finite for stationary processes. $E(\kappa_\tau)^2$ can be written as a term which is a constant times τ and a term which is a nonlinear function of τ and μ. For particular time series processes on the errors, we can be more specific.

4.1 Specific Examples

The MA Process

Suppose that in the above general process, $\mu_0 = 1$, $\mu_1 = \theta$, and $\mu_i = 0$ for i ≥ 2. The errors then follow a first-order moving average (MA(1)) process, $\varepsilon_t = \eta_t + \theta\eta_{t-1}$, where $-1 < \theta < 1$. In this case by substitution, $E\left(\sum_1^{\tau}\varepsilon_t\right)^2 = \tau\left(1+\theta^2\right)\sigma_\eta^2 + 2(\tau-1)\theta\sigma_\eta^2 = \tau\left(1+\theta\right)^2\sigma_\eta^2 - 2\theta\sigma_\eta^2$. Thus, regressing the square of the residual $\left(\sum_1^{\tau}e_t\right)^2$ on τ and a constant yields $\hat{\alpha} = -2\theta\sigma_\eta^2$ and $\hat{\beta} = \left(1+\theta\right)^2\sigma_\eta^2$. This provides a different explanation for the constant term than Case and Shiller [1987]. Here, $\hat{\alpha} < 0$ is not an anomaly, but arises whenever $\theta > 0$ (unlike first-order autoregressive processes, there is no presumption that $\theta > 0$). Thus, a negative constant term may be evidence of a non-i.i.d. error process. If we assume that the ε_t follow an MA(1) process without transaction errors, we can identify point estimates of θ and σ_η^2 from $\hat{\alpha}$ and $\hat{\beta}$. If we allow for transaction error (as in Case and Shiller [1987]), we are unable to identify θ because there are two error variances in the constant term.

We can extend this approach to higher-order MA processes. For the MA(2) process, $\varepsilon_t = \eta_t + \theta\eta_{t-1} + \gamma\eta_{t-2}$ (or for the general process, $\mu_0 = 1$, $\mu_1 = \theta$, $\mu_2 = \gamma$ and

$\mu_i = 0$ for i > 2), so by substitution, $E\left(\sum_1^{\tau}\varepsilon_t\right)^2 =$

$\tau\left(1+\theta^2+\gamma^2+2\theta+2\theta\gamma+2\gamma\right)\sigma_\eta^2 - 2\sigma_\eta^2\left(\theta+\theta\gamma+\gamma\right)$. Similar calculations reveal that all MA(k) processes with k < τ have an intercept term and a constant multiplying τ, but no terms multiplying higher powers of τ. The slope term will be positive, but the sign of the intercept depends on the parameters of the process. For k > 1, we cannot identify the parameters of the MA process since we observe only a slope and intercept.

The AR Process

Suppose instead that $\varepsilon_t,\ t=1,\ \tau,$ follows an AR(1) process, $\varepsilon_t = \eta_t + \rho\varepsilon_{t-1}$. In the general MA process, this is equivalent to k = ∞ and $\mu_i = \rho^i$ for i = 0, k. Using the fact that

$E\left[\varepsilon_t\varepsilon_{t-k}\right] = \rho^k \dfrac{\sigma_\eta^2}{1-\rho^2}$, we find that $E\left(\sum_1^{\tau}\varepsilon_t\right)^2 =$

$\tau\sigma_\varepsilon^2 + 2\sigma_\varepsilon^2\left[\tau\left(\dfrac{\rho-\rho^\tau}{1-\rho}\right) - \dfrac{\rho-\rho^\tau(\tau-1)}{1-\rho} - \rho\left[\dfrac{\rho-\rho^{\tau-1}}{(1-\rho)^2}\right]\right]$. See Appendix A for a derivation.

As τ grows, for $\rho > 0$, this expression increases at an increasing rate and asymptotes to an increasing straight line. For $\rho < 0$, it increases at a decreasing rate. Thus, only negative first-order serial correlation is consistent with a positive coefficient on time between sales and a negative coefficient on its square, which is a common finding. Since negative autocorrelation is not common in economic data, it seems unlikely that an AR(1) error process explains the commonly observed pattern. Higher-order AR processes also result in $E\left(\sum_1^{\tau}\varepsilon_t\right)^2$ being a nonlinear function of the holding period.

The ARMA Process

An ARMA(p, q) process has pth-order autoregression and qth-order moving average. For p = q = 1, we can write the process as $\varepsilon_t = \rho\varepsilon_{t-1} + \eta_t + \theta\eta_{t-1}$. In the general MA process, this is equivalent to $k = \infty$, $\mu_0 = 1$, $\mu_1 = \rho + \theta$, and $\mu_i = \rho^i + \rho^{i-1}\theta$ for *i=2, k.* In this case,

$$E\left(\sum_{t=1}^{\tau}\varepsilon_t\right)^2 = \tau\sigma_\eta^2\left[\left(\frac{1+\theta^2+2\rho\theta}{1-\rho^2}\right)+2\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)\right] -2\sigma_\eta^2\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)$$

$$+2\sigma_\eta^2\frac{\left(\rho^\tau-(\tau-1)\rho^2+(\tau-2)\rho\right)}{(1-\rho)^2}\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right).$$

See Appendix B for the derivation. As with the MA process, there is an intercept (which is easily negative) and a constant coefficient on the time horizon. As with the AR process, there is also a term which decays exponentially.

For $\rho > 0$ (the "normal" case), the expectation of the square of the sum of the residuals increases with the holding period length. For $\rho > |\theta|$, it increases as an increasing rate, while for $0 < \rho < |\theta|$, it increases as at decreasing rate. This last possibility is consistent with a positive coefficient on the linear term and a negative coefficient on the quadratic term. It also seems to be the "minimal" assumption on the return error process to generate such concavity. As with AR processes, higher-order ARMA processes result in $E\left(\sum_{t=1}^{\tau}\varepsilon_t\right)^2$ being a nonlinear function of the holding period.

One could fit a nonlinear regression in τ to $E\left(\sum_{1}^{\tau}\varepsilon_t\right)^2$. As with the MA(1) process, one can only identify parameters of the stochastic process conditional on the assumption about the order of the ARMA process, but of course the order of the ARMA process cannot be easily recovered because we do not observe the errors in the individual asset returns, but only the summed residuals.

5. Flexible GLS

In order to correct for heteroskedacticity when the error term on the individual asset returns is correlated between periods, we propose a flexible approach in which third stage weights are constructed by regressing the squared residuals from the first stage regression on dummy variables representing the length of the holding period for each asset (duration). Hence, in the second stage, we propose regressing $\hat{u}_i^2$ on a matrix which has a row of dummy variables for each asset in the sample. The dummy variable Z_{ij} takes on the value 1 if s_j - b_j equals j - i and zero otherwise. As before, in the third stage we divide the regressors and the regressand by the square root of the predicted value from the second stage. Table 5 presents the regression results from the new second and third stage regressions along with the Koenker-Basset test for heteroskedasticity in the third stage. We also plot the coefficient estimates and the standard error estimates for each of the three regressions and for each dataset in Figures 1 through 4.

Figures 1 through 4 plot the estimated coefficients and estimated standard errors using OLS, Case and Shiller, and our flexible GLS estimator for the Amsterdam art dataset and the Herengracht datasets. The top panel of each figure plots the levels of the coefficients and errors, and the bottom panel plots the percentage difference in the Case and Shiller and the flexible GLS estimates, defined as (Case and Shiller estimates-flexible GLS estimates) /Case and Shiller estimates. As shown in figure 1, for the Amsterdam art dataset, other than in the early years, the coefficients are similar to one another. There are large differences, however, in the first part of the dataset, which is not surprising given it is well-known that coefficients for the early time periods are imprecisely estimated (see Goetzmann 1992). In Figure 3 in the Herengracht dataset, while differences occur more frequently in the early years, coefficient differences persist in the later years. This could be due to either imprecision of the estimates or mispecification of the model, as it is well known that OLS, Case and Shiller, and flexible GLS coefficient estimates should be unbiased even in the presence of heteroskedasticity, and coefficients that change under a GLS estimator can indicate misspecification.

Figures 2 and 4 plot the estimated standard errors using OLS, Case and Shiller, and our flexible GLS estimator for the Amsterdam art dataset and the Herengracht datasets. As is evident from Panel B in both figures, the Case and Shiller standard errors are consistently larger than the flexible GLS standard errors. The difference is most pronounced in the earlier years of both datasets.

Our non-parametric approach is a significant improvement in size of estimated standard errors, both over OLS and over standard Case-Shiller. On average, the non-parametric approach has decreased the estimated standard errors on the coefficients by about 19% for the Amsterdam art dataset, and by about 7% for Herengracht dataset. As shown in Table 5, the fit has also improved, going from an OLS R-squared of .68 to a Flexible GLS R-squared of .69 for the Amsterdam art dataset, and going from an OLS R-squared of .60 in the Herengracht dataset to a flexible GLS R-squared of .66.

While researchers often do not focus on the standard errors of their point estimates of period returns, there are several reasons to examine them. First, there is a general pattern that standard errors at the beginning and the end of the sample period will exceed those in the middle because of fewer datapoints in the beginning and end of a repeat sales sample (see Goetzmann's [1992] discussion of Webb's unpublished work) . When a researcher is looking to see if returns change over time, it is important to be aware of the changes in the precision of estimates. Second, the true standard errors should be a better indicator than inefficient estimated standard errors of the expected magnitudes of revision errors near the end of the sample period (when new data become available, repeat sales index estimates for previous periods will change). One should also note that flexible GLS is not guaranteed to have smaller estimated standard errors than the Case-Shiller approach—they are different weighting systems and using "more accurate" weights could lead to larger (but "more precise") standard errors for some coefficients, as is demonstrated for some years in our samples.

For the Amsterdam art dataset, the mean nominal return calculated from the coefficients in Appendix Table 1 over the period 1780-2007 was 3.13%. The mean nominal return calculated from the coefficients in Appendix Table 2 for the Herengracht data was just slightly over 1% over the period 1630 to 1972. If we compare the two Dutch assets for the overlapping time period of 1780 to 1970, we find a 3.41% nominal return for art, and a 1.19% return for the Herengracht real estate, demonstrating that the returns to art sold in Amsterdam heartily outperformed the Herengracht canal real estate market during that time period!

6. Further Applications and Extensions

6.1 Real Estate Indexes

Calhoun's [1996] discussion of the second stage regression was part of a technical description of the Office of Federal Housing Enterprise Oversight's (OFHEO) methodology in constructing price indexes, a methodology they are still currently using.

The OFHEO estimation restricts the constant term to be zero, but includes a quadratic term in the second stage. Appendix Table 3 presents their published results for 3rd quarter 2007. Although the significance levels are not published, the OFHEO report stated that for both terms, significance resulted in most regions and states. The pattern of a positive coefficient on the linear term and a negative coefficient on the quadratic term is present throughout – but note our data also displayed that pattern when the constant was restricted to be zero (see column 7 of Tables 2 and 3).

Our analysis suggests that not only should the constant be included in the regressions, but that a non-parametric second stage regression would improve their analysis. If the constant term is negative, it is likely to be indicative of an ARMA process, given the quadratic term.

The above analysis also suggests that a non-parametric second stage could sensibly be used in the construction of housing price indexes to allow for the possibility that the errors follow an AR process or ARMA process. The non-parametric second stage could lead to more precise estimates of the index.[8]

6.2 Art Indexes

Repeat sales regressions are used in numerous articles that estimate price indexes from art auctions including Pesando [1993], Goetzmann [1993], Mei and Moses [2002] and Pesando and Shum [2008]. Ashenfelter and Graddy [2003, 2006] provide a survey. Most of the articles on art auctions do not provide the results from the second stage regressions and Pesando [1993] simply uses OLS. As art is an infrequently traded asset with many of the same properties as musical instruments and housing, it is very

[8] Andrew Leventis [2007] has a nice discussion of the differences between the OFHEO and S&P/Case-Shiller House Price Indexes.

likely that many of these studies could benefit from the non-parametric specification in the second stage of the repeat sales regressions.

6.3. Implications of Serial Dependence

It is well-known that the logarithmic specification of the dependent variable results in a geometric mean across assets for each time period of the index. Goetzmann [1992] suggested a way to calculate the arithmetic mean, using the second stage of the Case-Shiller regressions. He suggested that the coefficient on the time between sales should be used as an estimate of the cross section variance to give the following formula for the arithmetic mean:

$$\mu^a \cong \exp\left(\mu^g + \frac{\sigma^2}{2}\right) - 1$$

where μ^a and μ^g are the arithmetic and geometric means and σ^2 is the cross-section variance. Goetzmann [1993] and Mei and Moses [2002] utilized this for art price indexes. This correction becomes problematic in the case of non-i.i.d. errors.

Without a specific assumption on the errors in the returns of the individual assets, the single period return variance in an asset cannot be identified from the second stage of the Case-Shiller regression results.[9] Calhoun [1996] proposes using:

$$\sigma_t^2 = At + Bt^2$$

(where A and B are the linear and quadratic coefficients from the second stage—with no constant) as the variance in the geometric to arithmetic correction formula (in index form). There is a problem with this simple adaptation of the Goetzmann approach once

[9] The S&P/Case-Shiller® Home price index directly estimate an arithmetic index in order to circumvent this problem.

the second stage includes more than a simple linear term: the estimated variance for any property becomes a function of the holding period. Even using the variance per period ($A + Bt$) depends on the holding period. Any estimate of the arithmetic return must really depend on the planned holding period if the return errors are not i.i.d.

The geometric index has a simpler specification, and is intuitively appealing to many in the finance literature because of the continuous compounding interpretation of the log returns. Depending on the purpose of the regression, one can simply recognize that this is a geometric mean across assets, or a lower bound on the arithmetic mean. While this is an unintuitive interpretation, for many standard assets such as stocks, the geometric and the arithmetic mean can be directly computed. Thus, one could compare a geometric index of a standard asset directly with the geometric index of the alternative asset.

Some of the basic ideas in our paper have been utilized in other contexts. For example, studying the effects of holding periods on return variances is one standard technique to identify mean reversion in asset returns.[10] Furthermore, variance-ratio tests are commonly used to test for non-i.i.d. returns. We are, in effect, using a form of the variance-ratio test to test for non-i.i.d. deviations from the mean one-period returns by the underlying assets. As suggested in other contexts, nonlinearities with respect to the holding period in the second-stage regressions may diagnose longer-horizon violations of independence as well.

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[10] See, for example, Poterba and Summers [1988] and Lo and MacKinlay [1988].   That approach generally uses a nonparametric framework with respect to the error process.

Variance ratio tests calculate the variance of returns over different holding periods. If returns are i.i.d., then the variance of annual returns should be 12 times the variance of monthly returns. Likewise, if the deviations from mean returns for the underlying assets are i.i.d., than the variance in the deviations in annual returns should be 12 times the monthly variance. This would precisely match the Case-Shiller modeling of the second-stage regression on repeat sales without the "house-specific error". The fact that a polynomial specification is indicated in the Amsterdam art data (based on adjusted R-squared) reveals that deviations from mean returns for the individual assets violate serial independence. Abraham and Schauman [1991] report that the variance in the errors in returns for a housing data set is maximized at a holding period of twenty to thirty years, so they find concavity in the relationship as well. An essential difference between mean reversion and the processes modeled in Section 4 is that mean reversion can operate over a much longer time horizon than conventional AR and MA processes—it is effectively a more general version of serial dependence.

7. Conclusions and Issues for Further Study

Nonlinearities with respect to the holding period in estimated variances of property returns may be indicative of a failure of the assumption that errors in property returns are statistically independent over time. For assets where the usual conditions which induce market efficiency are not present, this should come as little surprise. More efficient estimates can result if researchers take account of these nonlinearities in determining GLS weights for the final stage. Researchers should also take account of the failure of independence in using the calculated index values to interpret market returns.

An alternative approach to repeat sales estimation would be to explicitly derive the likelihood function for the repeat sale model with non i.i.d. errors. Kuo [1997] provides an example when the price levels follow an AR(1) process. Francke [2010] extends this approach to take account of the covariance between successive pairs of repeat sales of the same property. We focus on the three-stage GLS procedure because variants of it are used in a wide range of applications and it is easy to implement.

In further work, we plan to extend our exploration of the implications of serial dependence in the deviations from mean returns on the standard errors of the return coefficients. Through Monte Carlo simulation, we can estimate true standard errors under a known non-i.i.d. process and compare it to the estimates of returns using the conventional procedures and our non-parametric method. We can also derive estimates of the revision changes for the returns in the last periods of the estimation. Because the estimated coefficients for the last few periods depend on only a small number of transactions, any biases in computing standard errors may matter most here.

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Standard and Poor's 2006 S&P/Case-Shiller® Metro Area Home Price Indices. McGraw Hill Companies. available at http://www2.standardandpoors.com/spf/pdf/index/SPCS_MetroArea_HomePrices_Methodology.pdf.

Appendix A: Calculations for the AR Process

Using the fact that $E\left[\varepsilon_t \varepsilon_{t-k}\right] = \rho^k \dfrac{\sigma_\eta^2}{1-\rho^2}$., we find that

$$\left(\sum_1^\tau \varepsilon_t\right)^2 = \sum_1^\tau \left(\varepsilon_t\right)^2 + 2\sum_1^{\tau-1} \varepsilon_{t+1}\varepsilon_t + 2\sum_1^{\tau-2} \varepsilon_{t+2}\varepsilon_t + \ldots + 2\sum_1^{\tau-(\tau-1)} \varepsilon_{t+\tau-1}\varepsilon_t .$$

Thus, $E\left(\sum_1^\tau \varepsilon_t\right)^2 = \tau\sigma_\varepsilon^2 + 2\rho(\tau-1)\sigma_\varepsilon^2 + 2\rho^2(\tau-2)\sigma_\varepsilon^2 + \ldots + 2\rho^{\tau-1}\sigma_\varepsilon^2$

$= \tau\sigma_\varepsilon^2 + 2\sigma_\varepsilon^2\left[\rho(\tau-1) + 2\rho^2(\tau-2) + \ldots + 2\rho^{\tau-1}\right]$

where $\left[\rho(\tau-1) + 2\rho^2(\tau-2) + \ldots + \rho^{\tau-1}\right] = \sum_1^{\tau-1} \rho^k(\tau-k) = \tau\sum_1^{\tau-1}\rho^k - \sum_1^{\tau-1}\rho^k k$.

The first term in this expression equals $\tau\left(\dfrac{\rho-\rho^\tau}{1-\rho}\right)$, using

$Z = \rho + \rho^2 + \ldots + \rho^{\tau-1}$ and $\rho Z = \rho^2 + \ldots + \rho^\tau$,

while the second term equals $-\left[\dfrac{\rho - \rho^\tau(\tau-1) + \rho\left[\dfrac{\rho-\rho^{\tau-1}}{1-\rho}\right]}{1-\rho}\right]$, using

$Y = \rho + 2\rho^2 + (\tau-1)\rho^{\tau-1}$ and $\rho Y = \rho^2 + 2\rho^3 + (\tau-1)\rho^\tau + \tau\rho^\tau$, and

$Y - \rho Y = \left(\rho - \rho^\tau(\tau-1)\right) + \left(\rho^2 + \rho^3 + \ldots + \rho^{\tau-1}\right)$

$= \left(\rho - \rho^N(N-1)\right) + \rho\left(\dfrac{\rho-\rho^{N-1}}{1-\rho}\right).$

Hence, $E\left(\sum_1^\tau \varepsilon_t\right)^2 = \tau\sigma_\varepsilon^2 + 2\sigma_\varepsilon^2\left[\tau\left(\dfrac{\rho-\rho^\tau}{1-\rho}\right) - \dfrac{\rho-\rho^\tau(\tau-1)}{1-\rho} - \rho\left[\dfrac{\rho-\rho^{\tau-1}}{(1-\rho)^2}\right]\right].$

Calculations for the ARMA process

For p = q = 1, an ARMA(1, 1) process can be written as:

$$\varepsilon_t = \rho\varepsilon_{t-1} + \eta_t + \theta\eta_{t-1}, \text{ t = 1, } \tau .$$

The expected values of the variances and covariances of errors equal:

$$E\left(\varepsilon_t^2\right) = \sigma_\eta^2\left[\frac{1+\theta^2+2\rho\theta}{1-\rho^2}\right],\ E\left(\varepsilon_t\varepsilon_{t-1}\right) = \rho\sigma_\varepsilon^2 + \theta\sigma_\eta^2 = \frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\sigma_\eta^2,$$

$$\text{and } E\left(\varepsilon_t\varepsilon_{t-k}\right) = \rho^k\hat{\beta} \text{ where } \hat{\beta} = \frac{\sigma_\eta^2}{\rho}\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2} \text{ for k} \geq 2.$$

Then, we can write the expected value of the square of the sum of the residuals as:

$$E\left(\sum_{t=1}^{\tau}\varepsilon_t\right)^2 = E\left(\sum_{t=1}^{\tau}\varepsilon_t^2\right) + 2E\left(\sum_{t=1}^{\tau-1}\varepsilon_t\varepsilon_{t+1}\right) + 2E\left(\sum_{t=1}^{\tau-2}\varepsilon_t\varepsilon_{t+2}\right) + ... + 2E\left(\sum_{t=1}^{\tau-(\tau-1)}\varepsilon_t\varepsilon_{t+(\tau-1)}\right)$$

$$= E\left(\sum_{t=1}^{\tau}\varepsilon_t^2\right) + 2E\left(\sum_{t=1}^{\tau-1}\varepsilon_t\varepsilon_{t+1}\right) + 2\left\{\sum_{J=1}^{\tau-2}JE\left(\varepsilon_t\varepsilon_{t+(\tau-J)}\right)\right\}.$$

Taking the expectations, we obtain:

$$E\left(\sum_{t=1}^{\tau}\varepsilon_t\right)^2 = \tau\sigma_\varepsilon^2 + 2(\tau-1)\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\sigma_\eta^2 + 2\hat{\beta}\rho^\tau\sum_{J=1}^{\tau-2}J\rho^{-J} .$$

For the last term, let $K = \tau - 2$ and $\omega = \frac{1}{\rho}$. Then let $Z = \sum_{J=1}^{K}J\omega^J$.

Now $Z = \omega + 2\omega^2 + 3\omega^3 + ... + K\omega^K$ and $\omega Z = \omega^2 + 2\omega^3 + ... + K\omega^{K+1}$, so

$$Z - \omega Z = \omega + \omega^2 + \omega^3 + ... + \omega^K - K\omega^{K+1} .$$

Let $Y = \omega + \omega^2 + \omega^3 + ... + \omega^K$. Then $\omega Y = \omega^2 + \omega^3 + ... + \omega^{K+1}$, so $Y - \omega Y = \omega - \omega^{K+1}$, and

$Y = \frac{\omega - \omega^{K+1}}{1-\omega}$. Substituting this into the earlier formula,

$$Z-\omega Z=\frac{\omega-\omega^{K+1}}{1-\omega}-K\omega^{K+1}=\frac{\omega-(K+1)\omega^{K+1}+K\omega^{K+2}}{1-\omega}.$$

Hence, $Z=\dfrac{\omega-(K+1)\omega^{K+1}+K\omega^{K+2}}{(1-\omega)^2}$

Thus, we have $E\left(\sum_{t=1}^{\tau}\varepsilon_t\right)^2=\tau\sigma_\varepsilon^2+2(\tau-1)\left(\dfrac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)\sigma_\eta^2+2\hat{\beta}\rho^\tau\sigma_\eta^2$.

Substituting, $E\left(\sum_{t=1}^{\tau}\varepsilon_t\right)^2=\tau\left(\dfrac{1+\theta^2+2\rho\theta}{1-\rho^2}\right)\sigma_\eta^2+2(\tau-1)\left(\dfrac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)\sigma_\eta^2$

$$+2\sigma_\eta^2\frac{\left(\rho^\tau-(\tau-1)\rho^2+(\tau-2)\rho\right)}{(1-\rho)^2}\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)$$

$$=\tau\sigma_\eta^2\left[\left(\frac{1+\theta^2+2\rho\theta}{1-\rho^2}\right)+2\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)\right]-2\sigma_\eta^2\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right)$$

$$+2\sigma_\eta^2\frac{\left(\rho^\tau-(\tau-1)\rho^2+(\tau-2)\rho\right)}{(1-\rho)^2}\left(\frac{\theta+\rho+\theta^2\rho+\theta\rho^2}{1-\rho^2}\right).$$

Table 1
OLS and Case and Shiller Results

	Amsterdam Art Dataset	Herengracht Dataset
Number of Transaction Pairs	1468	3577
Stage I, R^2	0.684	0.604
Stage I, Koenker Basset α_2	0.041	0.063
	(1.80)	(4.73)
Stage II, R^2	0.007	0.004
Stage II, Constant	2.21	0.157
	(7.34)	(15.31)
Stage II, Coefficient	0.201	0.002
	(3.36)	(3.71)
Stage III, R^2	0.641	0.597
Stage III, Koenker Basset α_2	-0.019	0.065
	-(0.59)	(4.62)

T-statistics are noted in parentheses.

Table 2
Second Stage Regression Results: Amsterdam Art Dataset

	1	2	3	4	5	6	7
	coef	coef	coef	coef	coeff	coef	coef
τ	0.2008	0.4885	0.9607			0.5242	0.9939
	(3.36)	(3.20)	(2.59)			(12.72)	(11.99)
τ^2		-0.0191	-0.0912				-0.0444
		(0.01)	(0.05)				-(6.50)
τ^3			0.0026				
			(1.40)				
ln(τ)				1.05			
				(4.08)			
Dummies					22		
Cons	2.2060	1.6253	1.0075	1.9223	1.8723		
	(7.34)	(3.93)	(1.67)	(5.94)	(0.39)		
F-Stat	11.26	7.74	5.81	16.65	1.81	161.74	104.28
Prob>F	0.000	0.001	0.001	0.000	0.014	0.000	0.000
Adj R^2	0.007	0.009	0.010	0.011	0.011	*	*
AIC	10197	10195	10195	10192	10210	10248	10208
BIC	10208	10211	10216	10202	10327	10243	10219
Obs	1468	1468	1468	1468	1468	1468	1468

Notes: The dependent variable is the squared results from the first-stage (OLS) regression.
τ = time between sales.
t-statistics are in parentheses.

Table 3

Second Stage Regression Results: Herengracht Dataset

	1	2	3	4	5	6	7
	coef	coef	coef	coef	coeff	coef	coef
τ	0.0015	0.0024	0.0029			0.0059	0.0115
	(3.71)	(2.41)	(1.52)			(19.71)	(18.33)
τ^2		-1.27E-05	-2.71E-05				-1.04E-04
		-(0.98)	-(0.53)				-(10.07)
τ^3			1.000E-08				
			(0.29)				
$\ln(\tau)$				0.02442			
				(3.62)			
Dummies					105		
Cons	0.1565	0.1486	0.1459	0.1254	0.2740		
	(15.31)	(11.41)	(9.16)	(7.15)	(0.63)		
F-Stat	13.75	7.36	4.93	13.09	1.33	388.42	250.34
Prob>F	0.0002	0.0006	0.0020	0.0003	0.0157	0.0000	0.0000
Adj R^2	0.0036	0.0035	0.0033	0.0034	0.0095	*	*
AIC	4197	4198	4200	4198	4279	4422	4325
BIC	4210	4217	4225	4210	4934	4429	4337
Obs	3577	3577	3577	3577	3577	3577	3577

Notes: The dependent variable is the squared results from the first-stage (OLS) regression.
τ = time between sales.
t-statistics are in parentheses.

Table 4
Second-Stage results with Varying Time Periods

	Amsterdam Art Dataset			
	10-year	5-year	2-year	1-year
τ	0.209	0.074	0.014	0.000
	(3.435)	(2.700)	(1.356)	(0.084)
Cons	2.182	2.236	2.265	2.290
	(7.201)	(8.208)	(9.099)	(9.865)
Obs	1463	1463	1463	1453
R^2	0.008	0.005	0.001	0.000

	Herengracht Dataset			
	10-year	5-year	2-year	1-year
τ	0.008	0.004	0.002	0.001
	(3.508)	(3.870)	(3.694)	(3.409)
Cons	0.183	0.163	0.152	0.144
	(14.434)	(13.389)	(13.354)	(14.041)
Obs	3199	3199	3199	3199
R^2	0.209	0.074	0.014	0.000

T-statistics are noted in parentheses.

Table 5
OLS and Flexible GLS Results

	Amsterdam Art	Herengracht
Number of Transaction Pairs	1468	3577
Stage I, R^2 (OLS)	0.680	0.604
Stage I, Koenker Basset α_2	0.041	0.063
	(1.80)	(4.73)
Stage II, R^2	0.026	0.039
Stage II: F(21,1446); F(105,3471)	1.81	1.33
Stage III, R^2 (Flexible GLS)	0.690	0.659
Stage III, Koenker Basset α_2	-0.008	0.002
	-(0.61)	(0.57)

The F-statistic tests for joint significance of the dummy variables. T-statistics are noted in brackets.

Appendix Table I

Estimated Coefficients and Errors for Amsterdam Art Dataset

period	OLS			Case and Shiller			Flexible GLS		
	coef	std error	t-stat	coef	std error	t-stat	coef	std error	t-stat
1780	0.752	0.882	0.85	0.375	0.949	0.40	0.927	0.662	1.4
1790	-0.069	0.676	-0.10	-0.255	0.735	-0.35	-0.140	0.525	-0.27
1800	0.381	0.703	0.54	0.024	0.771	0.03	-0.230	0.583	-0.39
1810	-0.655	0.792	-0.83	-0.223	0.880	-0.25	0.142	0.682	0.21
1820	0.724	0.922	0.79	1.135	1.061	1.07	1.010	0.766	1.32
1830	-0.051	0.791	-0.06	-0.296	0.835	-0.35	-0.443	0.639	-0.69
1840	-0.469	0.991	-0.47	-0.748	1.018	-0.74	-0.562	0.964	-0.58
1850	0.215	1.142	0.19	0.353	1.249	0.28	0.272	1.118	0.24
1860	1.286	1.057	1.22	1.231	1.276	0.96	0.470	0.829	0.57
1870	-1.201	0.888	-1.35	-1.122	1.084	-1.04	-0.201	0.678	-0.3
1880	1.329	0.587	2.26	1.297	0.668	1.94	1.055	0.494	2.13
1890	-1.111	0.485	-2.29	-1.008	0.523	-1.93	-0.825	0.480	-1.72
1900	1.218	0.421	2.89	1.090	0.461	2.37	1.014	0.463	2.19
1910	0.798	0.350	2.28	0.800	0.380	2.10	0.808	0.380	2.13
1920	0.005	0.243	0.02	0.016	0.258	0.06	0.004	0.256	0.02
1930	-0.679	0.242	-2.80	-0.662	0.253	-2.62	-0.647	0.253	-2.56
1940	1.461	0.259	5.64	1.455	0.265	5.50	1.514	0.270	5.61
1950	-0.460	0.245	-1.87	-0.452	0.244	-1.85	-0.475	0.253	-1.88
1960	1.431	0.215	6.66	1.457	0.209	6.96	1.453	0.219	6.64
1970	1.271	0.162	7.86	1.255	0.154	8.12	1.234	0.155	7.96
1980	0.462	0.099	4.66	0.446	0.092	4.82	0.424	0.086	4.91
1990	0.617	0.088	7.04	0.624	0.082	7.57	0.633	0.076	8.31
2000*	-0.456	0.103	-4.45	-0.445	0.099	-4.49	-0.438	0.095	-4.59
Average std error		0.537			0.589			0.475	
R^2		0.684			0.641			0.691	
Obs		1468			1468			1468	

*Estimate based on incomplete data for the decade.

Appendix Table 2
Estimated Coefficients and Errors for Herengracht Dataset

	OLS			Case and Shiller			Flexible GLS		
period	coef	std error	t-stat	coef	std error	t-stat	coef	std error	t-stat
1630	0.390	0.376	1.04	0.504	0.418	1.21	0.354	0.435	0.82
1632	0.457	0.328	1.39	0.509	0.344	1.48	0.683	0.277	2.47
1634	-0.573	0.366	-1.57	-0.620	0.389	-1.59	-0.777	0.316	-2.46
1636	-0.002	0.317	-0.01	0.045	0.340	0.13	0.167	0.310	0.54
1638	0.152	0.220	0.69	0.081	0.236	0.34	-0.049	0.218	-0.23
1640	0.309	0.190	1.62	0.359	0.201	1.78	0.351	0.158	2.22
1642	0.269	0.188	1.43	0.294	0.194	1.51	0.367	0.146	2.50
1644	-0.084	0.169	-0.50	-0.135	0.173	-0.78	-0.167	0.161	-1.04
1646	0.100	0.159	0.63	0.125	0.159	0.79	0.133	0.155	0.86
1648	-0.092	0.195	-0.47	-0.089	0.197	-0.45	-0.038	0.193	-0.20
1650	0.099	0.180	0.55	0.109	0.181	0.60	0.049	0.178	0.28
1652	-0.165	0.209	-0.79	-0.146	0.210	-0.70	-0.138	0.212	-0.65
1654	-0.030	0.224	-0.13	-0.036	0.226	-0.16	-0.038	0.225	-0.17
1656	0.268	0.185	1.45	0.264	0.191	1.38	0.211	0.185	1.14
1658	0.151	0.203	0.74	0.111	0.204	0.54	0.112	0.202	0.55
1660	0.102	0.176	0.58	0.104	0.176	0.59	0.165	0.165	1.00
1662	-0.168	0.154	-1.09	-0.154	0.166	-0.93	-0.209	0.133	-1.57
1664	0.007	0.337	0.02	0.058	0.380	0.15	-0.099	0.209	-0.47
1666	-0.360	0.372	-0.97	-0.345	0.413	-0.83	-0.268	0.260	-1.03
1668	0.110	0.242	0.46	0.060	0.252	0.24	0.186	0.233	0.80
1670	0.126	0.182	0.69	0.139	0.184	0.75	0.133	0.177	0.75
1672	-0.213	0.182	-1.17	-0.229	0.186	-1.24	-0.226	0.168	-1.35
1674	-0.149	0.176	-0.84	-0.158	0.182	-0.87	-0.124	0.152	-0.82
1676	-0.472	0.229	-2.06	-0.415	0.246	-1.68	-0.577	0.201	-2.87
1678	0.323	0.236	1.37	0.302	0.250	1.21	0.428	0.219	1.96
1680	0.347	0.159	2.18	0.316	0.161	1.96	0.286	0.155	1.85
1682	-0.422	0.137	-3.08	-0.381	0.141	-2.71	-0.371	0.133	-2.78
1684	0.053	0.128	0.41	0.048	0.131	0.37	0.124	0.122	1.01
1686	0.257	0.137	1.87	0.234	0.139	1.69	0.219	0.116	1.90
1688	-0.065	0.174	-0.37	-0.071	0.180	-0.39	-0.136	0.151	-0.90
1690	0.138	0.174	0.80	0.118	0.180	0.65	0.168	0.161	1.05
1692	-0.195	0.143	-1.36	-0.188	0.143	-1.31	-0.236	0.137	-1.72
1694	0.164	0.157	1.04	0.173	0.156	1.11	0.242	0.149	1.63
1696	-0.024	0.177	-0.14	-0.014	0.181	-0.08	-0.148	0.164	-0.90
1698	-0.027	0.164	-0.17	-0.027	0.171	-0.16	0.068	0.150	0.45
1700	0.111	0.130	0.85	0.121	0.134	0.90	0.098	0.122	0.80
1702	-0.064	0.121	-0.53	-0.060	0.124	-0.49	-0.013	0.117	-0.11
1704	0.022	0.139	0.16	0.011	0.143	0.08	-0.070	0.126	-0.55
1706	-0.107	0.154	-0.70	-0.104	0.159	-0.66	-0.087	0.149	-0.58

1708	0.034	0.139	0.24	0.030	0.143	0.21	0.026	0.143	0.18
1710	0.003	0.118	0.03	0.006	0.119	0.05	0.033	0.116	0.28
1712	0.110	0.114	0.97	0.103	0.114	0.90	0.130	0.102	1.28
1714	-0.078	0.110	-0.71	-0.083	0.113	-0.73	-0.133	0.080	-1.67
1716	0.097	0.099	0.98	0.107	0.100	1.07	0.091	0.077	1.17
1718	-0.098	0.107	-0.92	-0.097	0.108	-0.90	-0.105	0.082	-1.28
1720	0.242	0.117	2.07	0.225	0.119	1.90	0.293	0.095	3.08
1722	0.119	0.109	1.10	0.134	0.110	1.21	0.093	0.105	0.88
1724	0.188	0.105	1.79	0.200	0.108	1.85	0.249	0.099	2.52
1726	-0.054	0.109	-0.50	-0.058	0.112	-0.52	-0.076	0.103	-0.74
1728	-0.079	0.101	-0.78	-0.079	0.101	-0.77	-0.100	0.095	-1.05
1730	0.133	0.106	1.26	0.126	0.107	1.18	0.146	0.101	1.44
1732	0.045	0.114	0.40	0.043	0.113	0.38	0.069	0.111	0.62
1734	-0.013	0.111	-0.12	-0.012	0.110	-0.11	-0.081	0.106	-0.76
1736	0.095	0.126	0.75	0.082	0.126	0.65	0.124	0.116	1.06
1738	-0.180	0.137	-1.31	-0.157	0.140	-1.12	-0.163	0.133	-1.23
1740	0.135	0.116	1.16	0.128	0.120	1.06	0.120	0.116	1.03
1742	-0.536	0.148	-3.63	-0.556	0.147	-3.77	-0.543	0.143	-3.79
1744	0.193	0.148	1.30	0.213	0.148	1.44	0.213	0.143	1.49
1746	0.035	0.128	0.28	0.059	0.130	0.46	-0.004	0.123	-0.03
1748	0.016	0.138	0.12	-0.037	0.139	-0.27	0.001	0.130	0.01
1750	-0.132	0.111	-1.19	-0.108	0.111	-0.97	-0.113	0.104	-1.09
1752	0.303	0.119	2.54	0.282	0.122	2.32	0.288	0.116	2.48
1754	-0.080	0.120	-0.66	-0.042	0.123	-0.34	-0.039	0.119	-0.32
1756	-0.115	0.110	-1.05	-0.132	0.110	-1.20	-0.129	0.109	-1.18
1758	-0.042	0.105	-0.40	-0.038	0.105	-0.36	-0.026	0.102	-0.26
1760	0.001	0.104	0.01	-0.001	0.104	-0.01	-0.001	0.095	-0.01
1762	0.044	0.103	0.42	0.047	0.103	0.46	0.028	0.096	0.29
1764	0.132	0.090	1.46	0.131	0.091	1.44	0.067	0.072	0.92
1766	0.046	0.100	0.46	0.047	0.101	0.46	0.116	0.081	1.42
1768	0.071	0.095	0.75	0.079	0.095	0.83	0.070	0.089	0.78
1770	-0.015	0.095	-0.16	-0.019	0.096	-0.20	-0.006	0.092	-0.07
1772	-0.015	0.095	-0.16	-0.023	0.095	-0.24	-0.025	0.094	-0.27
1774	-0.119	0.106	-1.12	-0.111	0.107	-1.03	-0.131	0.100	-1.30
1776	0.067	0.101	0.67	0.066	0.102	0.64	0.092	0.093	1.00
1778	0.204	0.095	2.15	0.182	0.095	1.92	0.160	0.091	1.76
1780	-0.241	0.100	-2.40	-0.210	0.101	-2.09	-0.153	0.099	-1.55
1782	0.199	0.104	1.91	0.185	0.105	1.77	0.148	0.103	1.44
1784	0.024	0.099	0.24	0.020	0.100	0.20	-0.006	0.095	-0.06
1786	-0.101	0.100	-1.01	-0.093	0.100	-0.93	-0.095	0.095	-1.01
1788	-0.220	0.107	-2.05	-0.199	0.108	-1.84	-0.148	0.104	-1.43
1790	0.048	0.104	0.46	0.040	0.105	0.38	-0.011	0.105	-0.10
1792	0.065	0.099	0.66	0.047	0.101	0.46	0.079	0.099	0.80
1794	-0.045	0.096	-0.47	-0.046	0.098	-0.47	-0.074	0.094	-0.79
1796	-0.278	0.106	-2.62	-0.298	0.109	-2.74	-0.288	0.102	-2.83

1798	-0.134	0.109	-1.23	-0.112	0.112	-0.99	-0.107	0.105	-1.03
1800	-0.106	0.098	-1.08	-0.094	0.100	-0.94	-0.115	0.095	-1.21
1802	0.118	0.083	1.41	0.105	0.083	1.26	0.108	0.082	1.32
1804	0.015	0.103	0.15	0.019	0.103	0.19	0.007	0.096	0.07
1806	-0.094	0.125	-0.75	-0.088	0.126	-0.70	-0.068	0.117	-0.59
1808	0.227	0.108	2.10	0.223	0.110	2.04	0.224	0.102	2.20
1810	-0.210	0.099	-2.13	-0.228	0.100	-2.27	-0.219	0.093	-2.36
1812	-0.356	0.182	-1.95	-0.339	0.175	-1.94	-0.340	0.166	-2.04
1814	-0.196	0.190	-1.03	-0.203	0.182	-1.11	-0.247	0.176	-1.41
1816	0.150	0.135	1.12	0.148	0.136	1.09	0.166	0.133	1.25
1818	0.065	0.133	0.49	0.074	0.133	0.55	0.072	0.125	0.58
1820	0.139	0.128	1.08	0.127	0.129	0.99	0.151	0.120	1.26
1822	-0.097	0.127	-0.77	-0.072	0.128	-0.56	-0.064	0.123	-0.52
1824	0.203	0.130	1.56	0.165	0.132	1.25	0.162	0.129	1.26
1826	-0.126	0.159	-0.79	-0.109	0.158	-0.69	-0.124	0.155	-0.80
1828	-0.192	0.153	-1.26	-0.182	0.151	-1.20	-0.197	0.148	-1.33
1830	0.002	0.136	0.02	-0.022	0.134	-0.16	0.016	0.129	0.12
1832	0.012	0.146	0.08	0.032	0.145	0.22	0.011	0.136	0.08
1834	-0.021	0.126	-0.17	-0.021	0.126	-0.17	0.002	0.121	0.02
1836	0.215	0.119	1.80	0.207	0.118	1.75	0.204	0.117	1.75
1838	0.128	0.105	1.22	0.129	0.105	1.23	0.126	0.103	1.22
1840	0.048	0.080	0.60	0.062	0.081	0.77	0.057	0.078	0.74
1842	-0.052	0.072	-0.72	-0.056	0.072	-0.77	-0.064	0.068	-0.94
1844	-0.031	0.074	-0.42	-0.032	0.074	-0.43	-0.046	0.070	-0.66
1846	0.012	0.080	0.15	0.006	0.079	0.08	0.050	0.076	0.66
1848	-0.019	0.083	-0.23	-0.027	0.082	-0.33	-0.048	0.078	-0.61
1850	-0.032	0.087	-0.37	-0.021	0.086	-0.24	-0.036	0.083	-0.44
1852	0.084	0.088	0.95	0.086	0.088	0.97	0.092	0.085	1.08
1854	0.174	0.078	2.22	0.176	0.078	2.25	0.177	0.073	2.41
1856	-0.048	0.075	-0.63	-0.061	0.075	-0.81	-0.062	0.069	-0.89
1858	-0.079	0.076	-1.04	-0.074	0.076	-0.97	-0.071	0.069	-1.03
1860	0.053	0.073	0.73	0.055	0.073	0.75	0.055	0.066	0.83
1862	0.185	0.071	2.59	0.188	0.071	2.65	0.178	0.068	2.63
1864	0.042	0.072	0.59	0.049	0.071	0.69	0.052	0.068	0.76
1866	0.042	0.072	0.59	0.037	0.071	0.52	0.030	0.067	0.45
1868	-0.011	0.073	-0.15	-0.004	0.073	-0.05	-0.005	0.070	-0.07
1870	0.000	0.069	0.00	-0.012	0.068	-0.17	0.004	0.067	0.06
1872	0.115	0.063	1.82	0.110	0.062	1.78	0.101	0.060	1.67
1874	0.209	0.061	3.45	0.211	0.059	3.56	0.218	0.055	3.99
1876	0.019	0.062	0.31	0.027	0.060	0.46	0.028	0.055	0.51
1878	0.075	0.062	1.22	0.074	0.060	1.24	0.073	0.058	1.27
1880	0.042	0.060	0.71	0.041	0.058	0.71	0.044	0.055	0.80
1882	0.017	0.065	0.26	0.023	0.064	0.36	0.016	0.063	0.25
1884	-0.128	0.070	-1.83	-0.132	0.069	-1.91	-0.111	0.068	-1.64
1886	0.001	0.076	0.01	0.001	0.075	0.02	-0.009	0.073	-0.12
1888	-0.088	0.078	-1.13	-0.096	0.076	-1.26	-0.106	0.074	-1.44

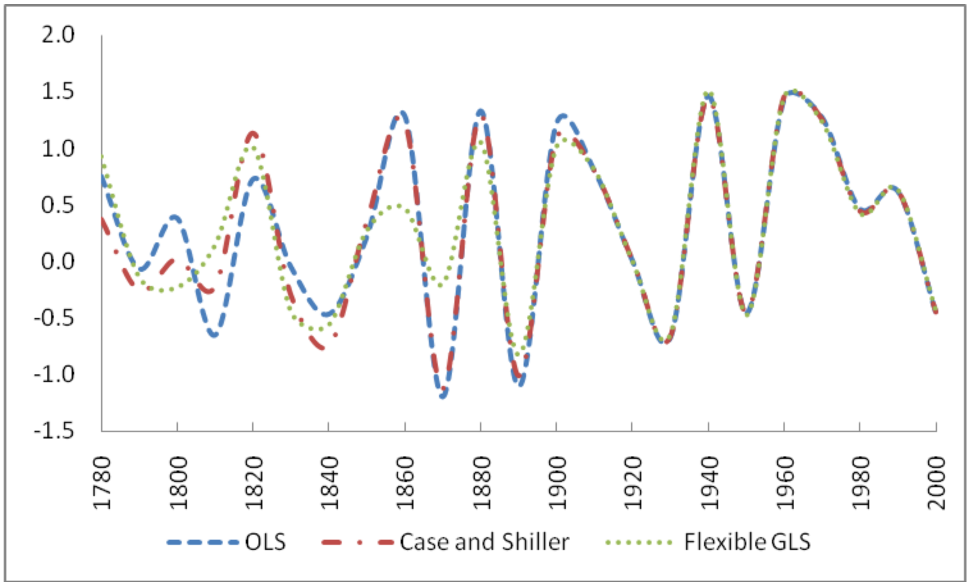
1890	-0.057	0.074	-0.77	-0.053	0.071	-0.74	-0.072	0.069	-1.05
1892	0.090	0.086	1.04	0.088	0.084	1.05	0.101	0.081	1.26
1894	-0.156	0.084	-1.87	-0.156	0.081	-1.92	-0.156	0.079	-1.98
1896	-0.028	0.069	-0.40	-0.025	0.067	-0.38	-0.009	0.060	-0.14
1898	0.094	0.071	1.31	0.101	0.070	1.45	0.100	0.064	1.57
1900	0.064	0.068	0.94	0.058	0.066	0.87	0.052	0.065	0.80
1902	0.063	0.063	1.00	0.058	0.061	0.94	0.054	0.060	0.91
1904	-0.073	0.066	-1.12	-0.066	0.064	-1.04	-0.063	0.063	-1.01
1906	-0.050	0.072	-0.70	-0.050	0.070	-0.72	-0.069	0.068	-1.01
1908	-0.012	0.074	-0.16	-0.008	0.072	-0.11	0.004	0.070	0.06
1910	-0.012	0.077	-0.15	-0.020	0.075	-0.27	-0.043	0.070	-0.61
1912	0.129	0.080	1.62	0.142	0.077	1.83	0.156	0.073	2.14
1914	-0.054	0.080	-0.68	-0.068	0.077	-0.88	-0.064	0.075	-0.85
1916	0.253	0.073	3.45	0.251	0.070	3.56	0.243	0.069	3.52
1918	0.283	0.053	5.32	0.294	0.052	5.71	0.304	0.051	6.00
1920	0.363	0.052	7.03	0.361	0.050	7.24	0.359	0.046	7.74
1922	-0.356	0.063	-5.66	-0.361	0.061	-5.96	-0.366	0.057	-6.45
1924	0.068	0.072	0.94	0.075	0.069	1.08	0.050	0.068	0.73
1926	-0.253	0.072	-3.50	-0.250	0.070	-3.59	-0.214	0.068	-3.13
1928	0.081	0.077	1.04	0.082	0.074	1.11	0.072	0.071	1.01
1930	-0.015	0.089	-0.17	-0.020	0.086	-0.23	-0.016	0.082	-0.20
1932	-0.242	0.106	-2.28	-0.240	0.103	-2.33	-0.256	0.100	-2.56
1934	-0.038	0.111	-0.34	-0.053	0.108	-0.49	-0.080	0.105	-0.76
1936	-0.132	0.099	-1.33	-0.146	0.096	-1.51	-0.162	0.094	-1.72
1938	-0.092	0.108	-0.85	-0.069	0.104	-0.66	-0.016	0.102	-0.16
1940	0.201	0.104	1.94	0.204	0.099	2.05	0.208	0.097	2.15
1942	0.119	0.076	1.56	0.122	0.073	1.68	0.129	0.071	1.81
1944	0.084	0.108	0.78	0.089	0.105	0.85	0.051	0.100	0.52
1946	0.117	0.125	0.93	0.111	0.121	0.91	0.126	0.115	1.09
1948	0.213	0.105	2.02	0.218	0.101	2.15	0.231	0.098	2.37
1950	0.037	0.095	0.39	0.045	0.091	0.49	0.048	0.087	0.55
1952	0.027	0.099	0.27	0.036	0.096	0.37	0.005	0.092	0.05
1954	0.134	0.093	1.45	0.108	0.090	1.20	0.152	0.087	1.76
1956	0.285	0.086	3.32	0.296	0.083	3.59	0.262	0.079	3.30
1958	0.112	0.094	1.20	0.123	0.090	1.36	0.133	0.086	1.54
1960	0.374	0.094	3.99	0.365	0.090	4.06	0.361	0.087	4.15
1962	0.033	0.091	0.37	0.004	0.089	0.04	-0.021	0.086	-0.25
1964	0.397	0.094	4.22	0.415	0.092	4.51	0.424	0.089	4.75
1966	-0.366	0.103	-3.56	-0.364	0.100	-3.63	-0.334	0.098	-3.41
1968	0.107	0.105	1.03	0.119	0.102	1.16	0.083	0.097	0.85
1970	0.056	0.098	0.57	0.049	0.096	0.51	0.061	0.091	0.67
1972	0.185	0.098	1.88	0.206	0.096	2.16	0.255	0.094	2.71
Average std error		0.122			0.123			0.114	
R^2		0.604			0.597			0.659	
Obs		3577			3577			3577	

Appendix Table 3

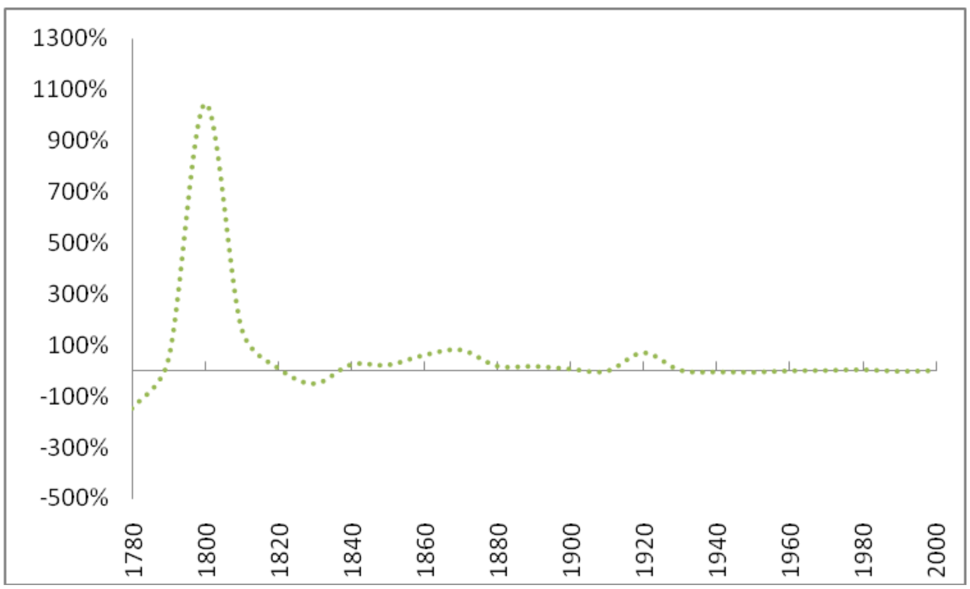
OFHEO Second Stage Regression Statistics for their QUARTERLY Housing Index : 3rd Quarter 2007

(Constant is Restricted to be Zero)

	Linear term	Quadratic Term		Linear term	Quadratic Term
East North Central	0.001680477	-0.00000299	Maine	0.002206272	-0.00001042
East South Central	0.001413392	-0.0000018	Michigan	0.001802758	-0.00000883
Middle Atlantic	0.002074802	-0.00000055	Minnesota	0.001778828	-0.00000748
Mountain	0.002409786	-0.00001399	Missouri	0.001510312	-0.00000427
New England	0.002154035	-0.00001018	Mississippi	0.001711552	-0.00000806
Pacific	0.002429178	-0.0000141	Montana	0.001928588	-0.00000922
South Atlantic	0.002187866	-0.00000785	North Carolina	0.001565033	-0.00000252
West North Central	0.00177686	-0.00000562	North Dakota	0.001039276	-0.00000135
West South Central	0.001778648	-0.00000617	Nebraska	0.001253644	-0.00000316
Alaska	0.001670519	-0.00001364	New Hampshire	0.002027072	-0.0000166
Alabama	0.001518371	-0.00000204	New Jersey	0.002004723	-0.00001082
Arkansas	0.001386812	-0.00000118	New Mexico	0.00149599	-0.00000543
Arizona	0.001633332	-0.00000734	Nevada	0.001229675	-0.00000624
California	0.001746513	-0.0000077	New York	0.002274004	0.00000173
Colorado	0.001809468	-0.00000946	Ohio	0.001425313	-0.00000283
Connecticut	0.001763343	-0.00000839	Oklahoma	0.001735064	-0.00001022
District of Columbia	0.002680667	-0.00001472	Oregon	0.0018648	-0.00000801
Delaware	0.001384419	-0.00000705	Pennsylvania	0.001574089	0.00000112
Florida	0.001908697	-0.00000436	Rhode Island	0.001733826	-0.00001077
Georgia	0.00148583	0.00000037	South Carolina	0.001729218	-0.000002
Hawaii	0.002267208	-0.00001294	South Dakota	0.001414512	-0.00000234
Iowa	0.001448536	-0.00000564	Tennessee	0.001334526	-0.00000102
Idaho	0.001885403	-0.00001133	Texas	0.001765041	-0.00000488
Illinois	0.001269782	0.00000665	Utah	0.001433671	-0.00000553
Indiana	0.001651597	-0.00000569	Virginia	0.001594024	-0.00000623
Kansas	0.001308308	-0.00000343	Vermont	0.001695607	-0.0000107
Kentucky	0.001301519	-0.00000324	Washington	0.001739829	-0.00000457
Louisiana	0.00161069	-0.00000721	Wisconsin	0.001578844	-0.0000063
Massachusetts	0.001960005	-0.00001126	West Virginia	0.00218767	-0.00001027
Maryland	0.001480009	-0.00000671	Wyoming	0.001830342	-0.00000946

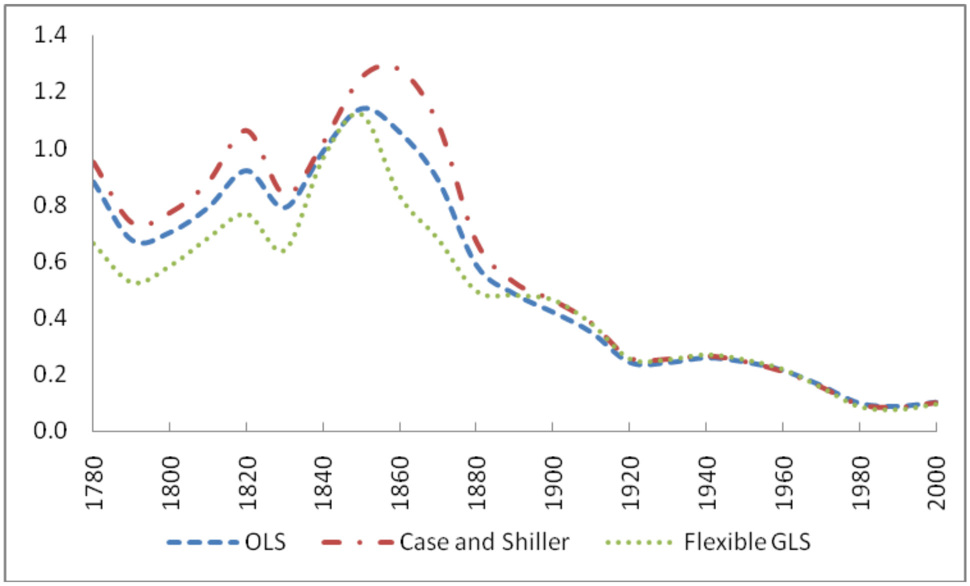


Panel A: OLS, Case and Shiller and Flexible GLS coefficients



Panel B: % Difference in Case and Shiller and Flexible GLS coefficients
% Difference = (Case and Shiller - Flexible GLS)/Case and Shiller

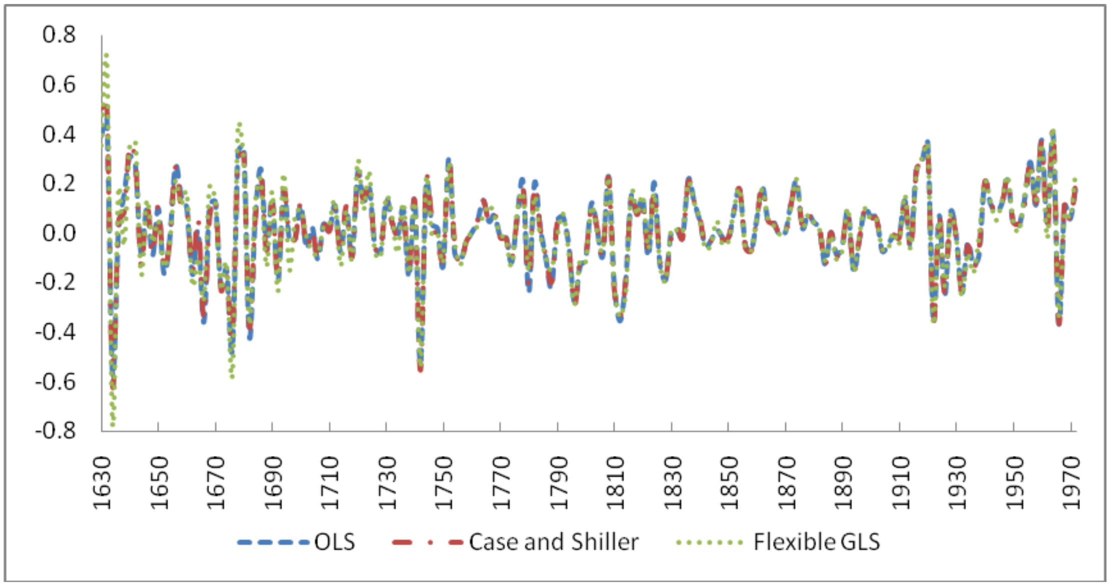
Figure 1: Amsterdam Art Estimated Cofficients



Panel A: OLS, Case and Shiller and Flexible GLS Standard Errors

Panel B: % Difference in Case and Shiller and Flexible GLS Errors
% Difference = (Case and Shiller - Flexible GLS)/Case and Shiller

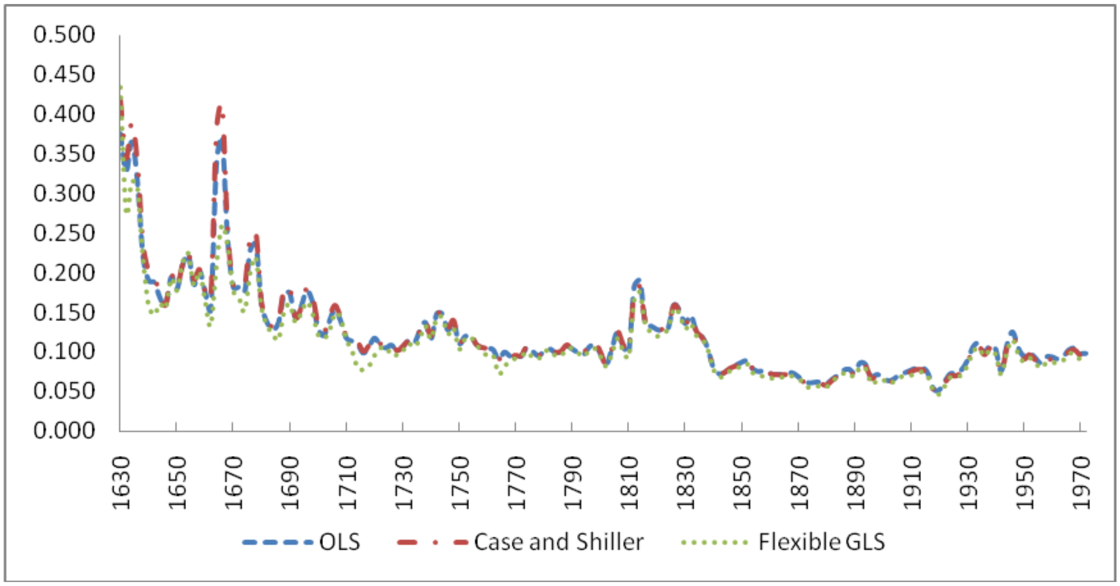
Figure 2: Amsterdam Art Estimated Standard Errors



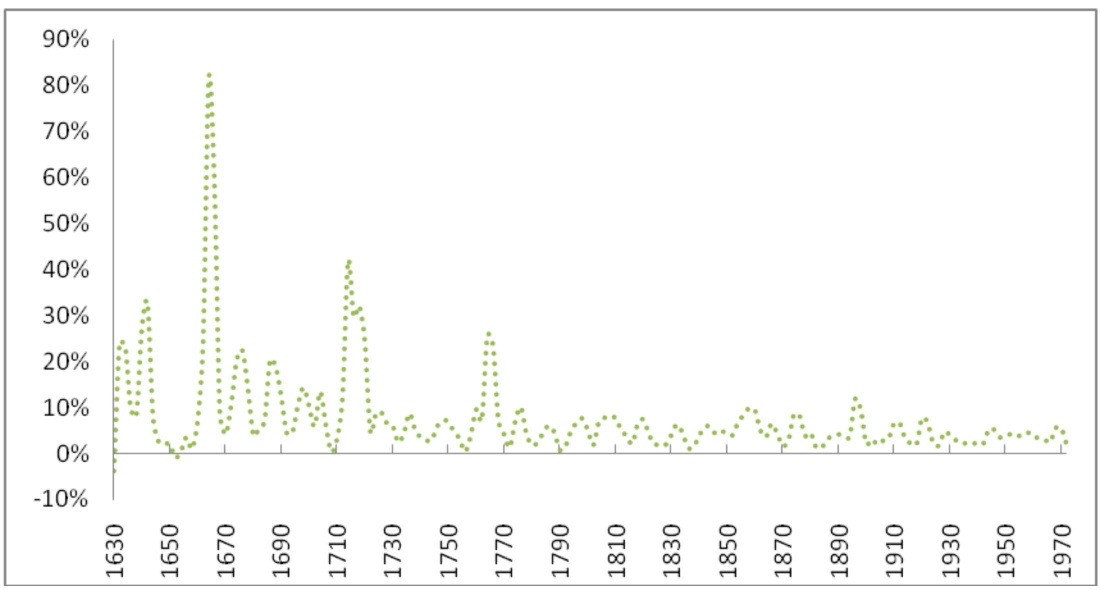
Panel A: OLS, Case and Shiller and Flexible GLS coefficients

Panel B: % Difference in Case and Shiller and Flexible GLS coefficients
% Difference = (Case and Shiller - Flexible GLS)/Case and Shiller

Figure 3: Herengracht Estimated Cofficients



Panel A: OLS, Case and Shiller and Flexible GLS Estimated Standard Errors



Panel B: % Difference in Case-Shiller and Flexible GLS Standard Errors
% Difference = (Case and Shiller - Flexible GLS)/Case and Shiller

Figure 4: Herengracht Estimated Standard Errors