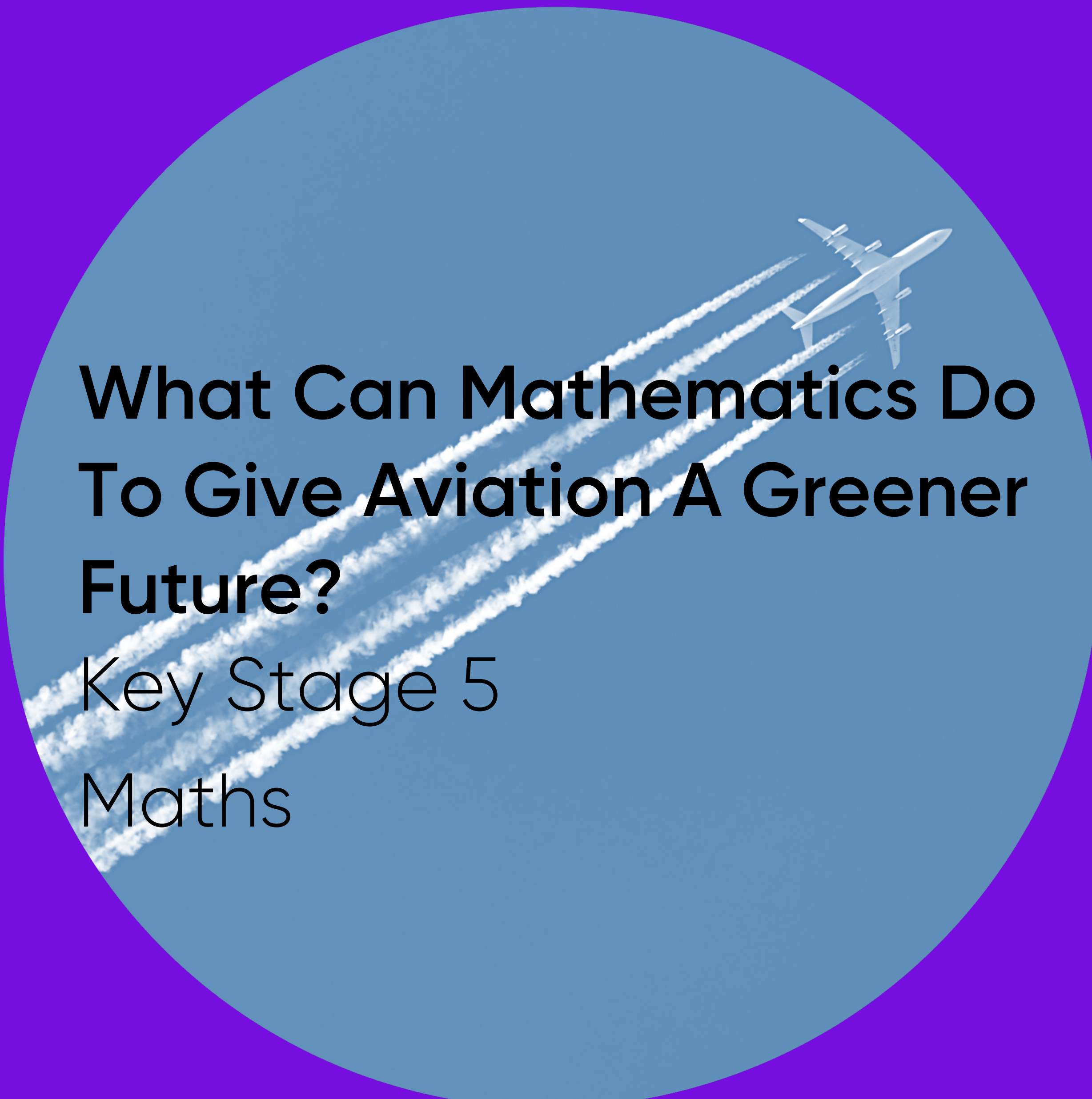
Research
Based
Curricula



# What Can Mathematics Do To Give Aviation A Greener Future?

Key Stage 5

Maths

2020

# Resource One Model Answers

Answers

1. Crossing the river:

Now try the example again, but this time you need to cross as quickly as possible. What does this mean about the direction of your velocity arrows? Try a few different routes and check your time. How can you minimise this value?

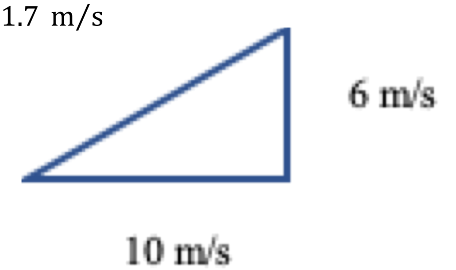
If the relative velocity vector is at right angles to the velocity of the river, then the actual velocity of the boat will be as long as possible, meaning it is travelling faster in real terms.

By Pythagoras' Theorem:

$$\sqrt{10^2 + 6^2} = 11.7 \text{ m/s}$$

Using trig: $\theta = \arctan\frac{6}{10}$

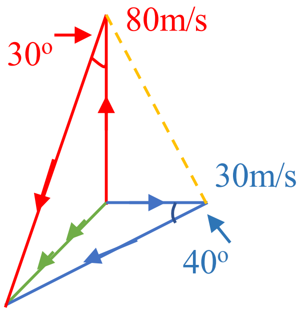
Bearing is 059°




Time to cross: $\frac{50}{10}$ = 5s

2. Which path?:

If you know what a journey looks like from two perspectives, this is often enough to pinpoint what the actual path taken and speed of travel are. If a delivery driver going due East at 30 m/s thinks that the plane she can see is going directly South West, but to a train driver traveling due North at 80 m/s it appears to be flying S30°W, on what bearing is the plane flying and how fast is it going?




We need to find the length and bearing of the green arrow which is the plane's actual path.

Joining up the right angle triangle of the speeds of the two drivers allows us to find out the length of the dotted line and the angles of the new right angled triangle.

Using the dotted line and the angles either side, all missing angles and sides can now be found using the sine and cosine rules.

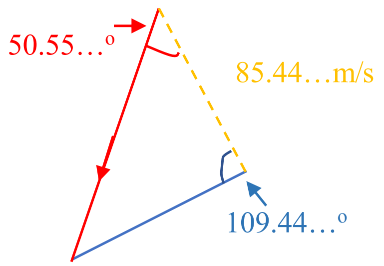
# Resource One
# Model Answers

Answers

Now the missing sides are 235.6...m/s and 192.9...m/s.

Using these lengths in one of the original triangles to get the length of the green vector: 171.0m/s

Finding one of the angles gives a bearing of 224˚.

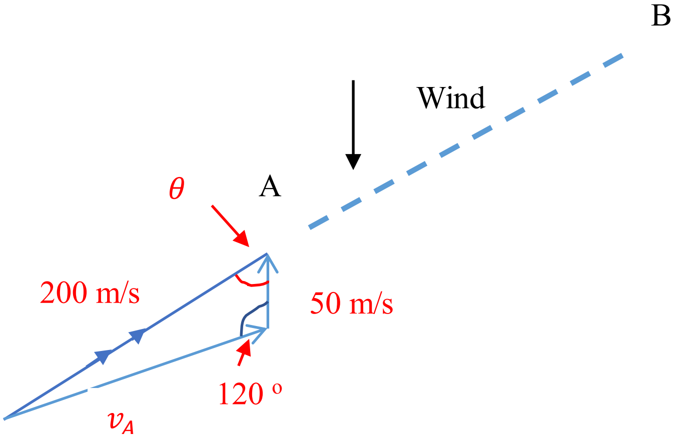



3. Windy weather:

Now imagine a plane trying to fly from Airport A to Airport B, where B is on a bearing of 060˚ from A and 300 km away. The wind is blowing from the North at 50 m/s.

Where should the pilot direct the plane? How long will the flight take if the air speed of the plane is kept to a steady 200 m/s?

NB: We are looking at this as a "flat" problem, so imagine you are seeing the plan view of the route to the airport. Here you imagine that the airports and the plane's flightpath are all at the same vertical level.



# Resource One Model Answers

Answers

The plane must actually follow the line from A to B.

AVW = VA - VW

By putting the wind vector upside down in the diagram it is the same as subtracting it.

From here two applications of the sine rule give:

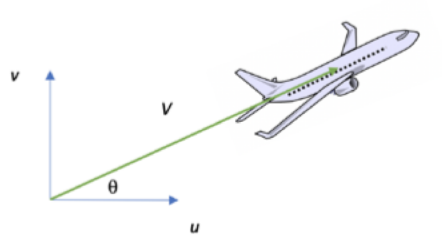
VA = 170 m/s

$\theta$ = 12.5°

Bearing: 047°

4. Taking off:

Can you find expressions for the zonal and meridional ground speeds? These will be $dx/dt$ and $dy/dt$ respectively.




$$\frac{dx}{dt} = W\,cos\,\theta + u(x,y)$$

$$\frac{dy}{dt} = W\sin\theta + v(x,y)$$

5. If we now work with the three dimensional system, how do these expressions change?

Hint: It is easier to use positions in terms of latitude and longitude in degrees for this part, rather than thinking of $x$ and $y$ as distances.

$$\frac{dx}{dt} = \frac{W\cos\theta + u(x,y)}{R_E\cos y}$$

$$\frac{dy}{dt} = \frac{W\sin\theta + v(x,y)}{R_E}$$

where $R_E$ is the Earth's radius.

# Resource Two Model Answers

$$z = x^3 - 2y^2 + 3xy - 2x^2y$$

Answers

1. Show That:

$$\frac{\partial z}{\partial x} = 3x^2 + 3y - 4xy$$

$$\frac{\partial z}{\partial y} = -4y + 3x - 2x^2$$

Hint:

When a mixed term is used then one variable is regarded as a constant coefficient.

Now there will be a more interesting set of second derivatives. Try finding these. Are there any patterns? (Hint: try researching Clairaut's Theorem.)

$$\frac{\partial^2 z}{\partial x^2} = 6x - 4y \qquad \frac{\partial^2 z}{\partial y^2} = -4 \qquad \frac{\partial^2 z}{\partial x \partial y} = 3 - 4x \qquad \frac{\partial^2 z}{\partial y \partial x} = 3 - 4x$$

So:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

2. Find and classify all of the critical points of:

i) $z = 5x^3 + 2y^2 - 60xy - 3$

$$\frac{\partial z}{\partial x} = 15x^2 - 60y$$

$$\frac{\partial z}{\partial y} = 4y - 60x$$

For critical points set these to zero and solve.

Substitute back into z function to find co-ordinates of points:

(0,0,-3) and (60,900,-540003)

# Resource Two
# Model Answers

## Answers

$$\frac{\partial^2 z}{\partial x^2} = 30x \qquad \frac{\partial^2 z}{\partial y^2} = 4 \qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = -60$$

$$D = 120x - 3600$$

$$x = 0 \Longrightarrow D < 0 \; saddle$$

$$x = 60 \Longrightarrow D > 0 \; and \; \frac{\partial^2 z}{\partial x^2} > 0 \; relative \; minimum$$

ii) $z = x^3 - 8y^3 - 2x^2y + 4xy^2 - 4x + 8y$

$$\frac{\partial z}{\partial x} = 3x^2 - 4xy + 4y^2 - 4$$

$$\frac{\partial z}{\partial y} = -24y^2 - 2x^2 + 8xy + 8$$

For critical points set these to zero and solve.

Substitute back into z function to find co-ordinates of points:

$$\left(\frac{2}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \text{ and } \left(\frac{-2}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right)$$

$$\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{16\sqrt{6}}{9}\right) \text{ and } \left(\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-16\sqrt{6}}{9}\right)$$

# Resource Two
# Model Answers

Answers $\frac{\partial^2 z}{\partial x^2} = 6x - 4y$ $\quad \frac{\partial^2 z}{\partial y^2} = -48y + 8x$ $\quad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = -4x + 8y$

$$D = (6x - 4y)(-48y + 8x) - (-4x + 8y)^2$$

$$x = \frac{2}{\sqrt{2}} \Longrightarrow D < 0 \; saddle$$

$$x = \frac{-2}{\sqrt{2}} \Longrightarrow D < 0 \; saddle$$

$$x = \frac{2}{\sqrt{6}} \Longrightarrow D > 0 \;\; \frac{\partial^2 z}{\partial x^2} > 0 \; relative\ minimum$$

$$x = \frac{-2}{\sqrt{6}} \Longrightarrow D > 0 \;\; \frac{\partial^2 z}{\partial x^2} < 0 \; relative\ maximum$$

3. Work out the Hamiltonian for any of these dynamical systems that are Hamiltonian:

i)

$$\frac{dx}{dt} = 4x^2y + 3xy^2 - 8x \qquad \frac{dx}{dt} \Longrightarrow H = 2x^2y^2 + xy^3 - 8xy + f(x)$$

$$\frac{dy}{dt} = 8y - y^3 - 4xy^2 \qquad \frac{dy}{dt} \Longrightarrow H = 2x^2y^2 + xy^3 - 8xy + g(y)$$

$$\therefore H = 2x^2y^2 + xy^3 - 8xy$$

Hamiltonian

# Resource Two
# Model Answers

Answers ii)

$$\frac{dx}{dt} = 3xy^2 + x\cos y - 2x^2 y \qquad \frac{dx}{dt} \Longrightarrow H = xy^3 - x^2y^2 + x\sin y + f(x)$$

$$\frac{dy}{dt} = 2xy^2 - y^3 - \sin y + y \qquad \frac{dy}{dt} \Longrightarrow H = xy^3 - x^2y^2 + x\sin y - xy + g(y)$$

Not Hamiltonian

iii)

$$\frac{dx}{dt} = 3x^2 + y\sin x - 4xy^2 \qquad \frac{dx}{dt} \Longrightarrow H = 3x^2y - \frac{4xy^3}{3} + \frac{y^2}{2}\sin x + f(x)$$

$$\frac{dy}{dt} = -\frac{y^2}{2}\cos x + \frac{4y^3}{3} - 6xy - \tan x$$

$$\frac{dy}{dt} \Longrightarrow H = 3x^2y - \frac{4xy^3}{3} + \frac{y^2}{2}\sin x + \sec^2 x + g(y)$$

$$\therefore H = 3x^2y - \frac{4xy^3}{3} + \frac{y^2}{2}\sin x + \sec^2 x$$

Hamiltonian

# Resource Two Model Answers

Answers

4.

i) Find the partial derivatives of $H$ with respect to $x$, $y$ and $\theta$.

$$\frac{\partial H}{\partial x} = -\lambda_x \frac{\partial u}{\partial x} - \lambda_y \frac{\partial v}{\partial x}$$

$$\frac{\partial H}{\partial y} = -\lambda_x \frac{\partial u}{\partial y} - \lambda_y \frac{\partial v}{\partial y}$$

$$\frac{\partial H}{\partial \theta} = W(-\lambda_x \sin\theta + \lambda_y \cos\theta)$$

ii) Set $\frac{\partial \lambda_x}{dt} = -\frac{\partial H}{\partial x}$ and $\frac{\partial \lambda_y}{dt} = -\frac{\partial H}{\partial y}$

$$\frac{\partial \lambda_x}{\partial t} = \lambda_x \frac{\partial u}{\partial x} + \lambda_y \frac{\partial v}{\partial x}$$

$$\frac{\partial \lambda_y}{\partial t} = \lambda_x \frac{\partial u}{\partial y} + \lambda_y \frac{\partial v}{\partial y}$$

iii) By setting $\partial H/\partial \theta$ to zero, find the relationship between $\lambda_x$ and $\lambda_y$.

$$\tan\theta = \frac{\lambda_y}{\lambda_x}$$

iv) As $H$ is not explicitly a function of time, choose to set it to zero.

$$0 = \frac{\lambda_y}{\tan\theta}(W\cos\theta + u) + \lambda_y(W\sin\theta + v) + 1$$

or

$$0 = \lambda_x(W\cos\theta + u) + \lambda_x \tan\theta\,(W\sin\theta + v) + 1$$

# Resource Two Model Answers

Answers

v) Rearrange your answer to iv) to make $\lambda_x$ the subject of the formula.

$$\lambda_x = \frac{-\cos\theta}{W + u\cos\theta + v\sin\theta}$$

vi) Repeat v) for $\lambda_y$.

$$\lambda_y = \frac{-\sin\theta}{W + u\cos\theta + v\sin\theta}$$

vii) Substitute your answers to v) and vi) into your $\frac{\partial\lambda_x}{dt} = -\frac{\partial H}{\partial x}$ and $\frac{\partial\lambda_y}{dt} = -\frac{\partial H}{\partial y}$ expressions.

$$\frac{\partial\lambda_x}{\partial t} = \left(\frac{-\cos\theta}{W + u\cos\theta + v\sin\theta}\right)\frac{\partial u}{\partial x} + \left(\frac{-\sin\theta}{W + u\cos\theta + v\sin\theta}\right)\frac{\partial v}{\partial x}$$

$$\frac{\partial\lambda_y}{\partial t} = \left(\frac{-\cos\theta}{W + u\cos\theta + v\sin\theta}\right)\frac{\partial u}{\partial y} + \left(\frac{-\sin\theta}{W + u\cos\theta + v\sin\theta}\right)\frac{\partial v}{\partial y}$$

viii) Now find $\frac{d\theta}{\partial\lambda_x}$.

$$\tan\theta = \frac{\lambda_y}{\lambda_x} \Longrightarrow \theta = \arctan\frac{\lambda_y}{\lambda_x}$$

$$\frac{d\theta}{\partial\lambda_x} = \frac{\lambda_x\frac{\partial\lambda_y}{\partial t} - \lambda_y\frac{\partial\lambda_x}{\partial t}}{{\lambda_x}^2 + {\lambda_y}^2} \times \frac{1}{\frac{\partial\lambda_x}{\partial t}}$$

ix) Using the Chain Rule, find $\frac{d\theta}{dt}$ as the product $\frac{d\theta}{dt} = \frac{d\theta}{\partial\lambda_x} \times \frac{\partial\lambda_x}{dt}$.

$$\frac{d\theta}{dt} = \frac{\lambda_x\frac{\partial\lambda_y}{\partial t} - \lambda_y\frac{\partial\lambda_x}{\partial t}}{{\lambda_x}^2 + {\lambda_y}^2}$$

x) Substitute in your expressions for $\frac{\partial\lambda_x}{dt}, \frac{\partial\lambda_y}{dt}$, $\lambda_x$ and $\lambda_y$ to give the final $\frac{d\theta}{dt}$ expression:

$$\frac{d\theta}{dt} = sin^2\theta\,\frac{\partial v}{\partial x} - cos^2\theta\,\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) sin\theta cos\theta$$

# Resource Three Model Answers

Answers

1.

i) Find the rainfall in a position (3.2, 5) using the nearest interpolation method: **5cm**

ii) Find the rainfall in a position (2, 3.5) using the nearest interpolation method: **5 cm**

iii) Find the rainfall in a position (1.8, 4.2) using the nearest interpolation method: **5cm**

iv) Write out the new array found by using the nearest interpolation method to split the top left original grid square into 9 squares.

$$\begin{bmatrix} 8 & 8 & 9 & 9 \\ 8 & 8 & 9 & 9 \\ 7 & 7 & 5 & 5 \\ 7 & 7 & 5 & 5 \end{bmatrix}$$

v) Write out the new array found by using the nearest interpolation method to split the bottom right original grid square into 4 squares.

$$\begin{bmatrix} 4 & 6 & 6 \\ 3 & 4 & 4 \\ 3 & 4 & 4 \end{bmatrix}$$

2.

i) Find the rainfall in a position (3.2, 5) using the linear interpolation method: **5 + 0.2 X −2 = 4.6 cm**

ii) Find the rainfall in a position (2, 4.5) using the linear interpolation method: **(6 + 5) / 2 = 5.5 cm**

iii) Find the rainfall in a position (1.8, 4.2) using the linear interpolation method. Does it matter if you interpolate vertically first and then horizontally here? **4.8 cm, Horizontally or vertically first makes no difference.**

iv) Write out the new array found by using the linear interpolation method to split the bottom right original grid square into 4 squares.

$$\begin{bmatrix} 4 & 5 & 6 \\ 3.5 & 4.25 & 5 \\ 3 & 3.5 & 4 \end{bmatrix}$$

v) Depends on choice of student.

# Resource Three Model Answers

Answers

3.

i) Find the coefficients $a$, $b$, $c$, $d$.

1. $4 = a + b + c + d$
2. $6 = 8a + 4b + 2c + d$
3. $4 = 27a + 9b + 3c + d$
4. $3 = 64a + 16b + 4c + d$

Subtracting 1 from 2, 2 from 3 and 3 from 4 gives:

5. $2 = 7a + 3b + c$
6. $-2 = 19a + 5b + c$
7. $-1 = 37a + 7b + c$

Subtracting 5 from 6 and 6 from 7 gives:

8. $-4 = 12a + 2b$
9. $1 = 18a + 2b$

Subtracting 8 from 9 gives:

10. $5 = 6a \implies a = \frac{5}{6}$

Substitute back into each successive set of equations to find:

$$a = \frac{5}{6}, b = -7, c = \frac{103}{6}, d = -7$$

# Resource Three Model Answers

Answers

3.

ii) Find the value in position 2.6 using the cubic interpolation method.

$$f(2.6) = \frac{5}{6}\times 2.6^3 + -7\times 2.6^2 + \frac{103}{6}\times 2.6 + -7 = 4.96$$

iii) Repeat the process to find the value at position 5.9 in the vector below.

$$[2 \quad 7 \quad 9 \quad 3.4 \quad 12 \quad 9 \quad 5 \quad 4 \quad 1]$$

Repeat previous tactic, but using

$$[3.4 \quad 12 \quad 9 \quad 5]$$

1. $3.4 = 64a + 16b + 4c + d$
2. $12 = 125a + 25b + 5c + d$
3. $9 = 216a + 36b + 6c + d$
4. $5 = 343a + 49b + 7c + d$

Subtracting 1 from 2, 2 from 3 and 3 from 4 gives:

5. $8.6 = 61a + 9b + c$
6. $-3 = 91a + 11b + c$
7. $-4 = 127a + 13b + c$

Subtracting 5 from 6 and 6 from 7 gives:

8. $-11.6 = 30a + 2b$
9. $-1 = 36a + 2b$

Subtracting 8 from 9 gives:

10. $10.6 = 6a \Longrightarrow a = \frac{10.6}{6}$

Substitute back into each successive set of equations to find:

$$a = \frac{53}{30}, b = -\frac{323}{10}, c = \frac{2873}{15}, d = -359$$

# Resource Three Model Answers

Answers

3.

iv) Find the value in position 5.9 using the cubic interpolation method.

$$f(5.9) = \frac{53}{30}\times 5.9^3 + -\frac{323}{10}\times 5.9^2 + \frac{2873}{15}\times 5.9 + -359 = 9.5199$$

4. Here are two arrays of wind speeds for the start of a journey across the Northern Atlantic. The wind positions cover latitudes from 42.5 to 52.5 degrees North and longitudes 30 to 40 degrees West in 2.5 degree intervals. By using the list of positions given, work out the magnitude and direction of the wind at each point.

Zonal Wind

$$\begin{bmatrix} 25.1 & 25.5 & 25.9 & 25.9 & 25.5 \\ 20.8 & 21.8 & 22.5 & 22.2 & 20.8 \\ 17.0 & 18.0 & 18.1 & 17.2 & 14.9 \\ 9.5 & 9.9 & 9.6 & 8.6 & 6.8 \\ 1.3 & 1.7 & 1.2 & 0.4 & -0.3 \end{bmatrix}$$

$\mathbf{u}$: (45N,32W) Work on the fourth row. Interpolate for columns 30W is the 5th column, 32.5W is the 4th column.

$$6.8 + \frac{2}{2.5}(8.6 - 6.8) = 8.24 \text{ m/s}$$

$\mathbf{u}$: (46N,35W) Work on the middle column. Interpolate for rows 45N is the 4th row, 47.5N is the 3rd row.

$$9.6 + \frac{1}{2.5}(18.1 - 9.6) = 13 \text{ m/s}$$

$\mathbf{u}$: (51.5N,39W) Interpolating vertically first:
37.5W is the 2nd column, 40W is the 1st column so we'll need to interpolate in both these columns to give two results.
52.5N is the 1st row, 50N is the 2nd row.

2nd column: $21.8 + \frac{1.5}{2.5}(25.5 - 21.8) = 24.02$

1st column: $20.8 + \frac{1.5}{2.5}(25.1 - 20.8) = 23.38$

Now interpolate horizontally:

$$24.02 + \frac{1.5}{2.5}(23.38-24.02) = 23.636 \text{ m/s}$$

# Resource Three
# Model Answers

Answers

4.

Meridional Wind

$$\begin{bmatrix} 0.8 & 2.2 & 2.6 & 1.8 & 0.4 \\ 0 & 1.6 & 2.5 & 1.8 & 0.3 \\ -0.8 & 0.9 & 1.9 & 1.1 & 0.5 \\ -1.1 & 0.6 & 1.8 & 1.2 & 0.2 \\ -1.3 & 0.5 & 1.8 & 1.9 & 1.1 \end{bmatrix}$$

Same methods as before answers are:

$v$: (45N,32W) 1 m/s

$v$: (46N,35W) 1.84 m/s

$v$: (51.5N,39W) 1.072 m/s

Wind at:

(45N,32W) $\sqrt{1^2 + 8.24^2}$=8.30 m/s to 3sf

arctan($\frac{1}{8.24}$)=6.92°

N83.1°E

(46N,35W) $\sqrt{1.84^2 + 13^2}$=13.1 m/s to 3sf

arctan($\frac{1.84}{13}$)=8.06°

N81.9°E

(51.5N,39W) $\sqrt{1.072^2 + 23.636^2}$=23.7 m/s to 3sf

arctan($\frac{1.072}{23.636}$)=2.59°

N87.4°E

# Resource Three Model Answers

Answers

5.

i) Numerical gradients for the zonal wind speeds above have been calculated for you. Can you work out how these have been obtained from the zonal wind array?

Zonal wind gradients in x direction

Column 1: $\frac{\text{Next value in row} - \text{Current value}}{2.5}$

Columns 2-4: $\frac{\text{Next value in row} - \text{Previous value}}{5}$

Column 5: $\frac{\text{Current value} - \text{Previous value}}{2.5}$

$$\begin{bmatrix} 0.16 & 0.16 & 0.08 & -0.08 & -0.16 \\ 0.4 & 0.34 & 0.08 & -0.34 & -0.56 \\ 0.4 & 0.22 & -0.16 & -0.64 & -0.92 \\ 0.16 & 0.02 & -0.26 & -0.56 & -0.72 \\ 0.16 & -0.02 & -0.26 & -0.3 & -0.28 \end{bmatrix}$$

Zonal wind gradients in $\mathbf{y}$ direction:

Row 1: $\frac{\text{Next value in column} - \text{Current value}}{2.5}$

Rows 2-4: $\frac{\text{Next value in column} - \text{Previous value}}{5}$

Row 5: $\frac{\text{Current value} - \text{Previous value}}{2.5}$

$$\begin{bmatrix} -1.72 & -1.48 & -1.36 & -1.48 & -1.88 \\ -1.62 & -1.5 & -1.56 & -1.74 & -2.12 \\ -2.26 & -2.38 & -2.58 & -2.72 & -2.8 \\ -3.14 & -3.26 & -3.38 & -3.36 & -3.04 \\ -3.28 & -3.28 & -3.36 & -3.28 & -2.84 \end{bmatrix}$$

# Resource Three Model Answers

Answers

5.

ii) Find values for $\partial u/\partial x$, $\partial u/\partial y$ at the three positions given in Exercise 4. Remember that you will need to use linear interpolation on the gradient arrays for the zonal winds.

$\frac{\partial u}{\partial x}$: (45N,32W) Work on the fourth row. Interpolate for columns 30W is the 5th column, 32.5W is the 4th column.

$-0.72+\frac{2}{2.5}(-0.56 - -0.72) = -0.592$

$\frac{\partial u}{\partial x}$: (46N,35W) Work on the middle column. Interpolate for rows 45N is the 4th row, 47.5N is the 3rd row.

$-0.26+\frac{1}{2.5}(-0.16 - -0.26) = -0.22$

$\frac{\partial u}{\partial x}$: (51.5N,39W) Interpolating vertically first:
37.5W is the 2nd column, 40W is the 1st column so we'll need to interpolate in both these columns to give two results.
52.5N is the 1st row, 50N is the 2nd row.

2nd column: $-0.34 + \frac{1.5}{2.5}(0.16 - 0.34) = 0.232$

1st column: $0.4 + \frac{1.5}{2.5}(0.16 - 0.4) = 0.256$

Now interpolate horizontally:

$0.232 + \frac{1.5}{2.5}(0.256-0.232) = 0.2464$

$\frac{\partial u}{\partial y}$: (45N,32W) Work on the fourth row. Interpolate for columns 30W is the 5th column, 32.5W is the 4th column.

$-3.04 + \frac{2}{2.5}(-3.36 - -3.04) = -3.296$

$\frac{\partial u}{\partial y}$: (46N,35W) Work on the middle column. Interpolate for rows 45N is the 4th row, 47.5N is the 3rd row.

$-3.38 + \frac{1}{2.5}(-2.58 - -3.38) = -3.06$

$\frac{\partial u}{\partial y}$: (51.5N,39W) Interpolating vertically first:
37.5W is the 2nd column, 40W is the 1st column so we'll need to interpolate in both these columns to give two results.
52.5N is the 1st row, 50N is the 2nd row.

2nd column: $-1.5 + \frac{1.5}{2.5}(-1.48 - -1.5) = -1.488$

1st column: $-1.62 + \frac{1.5}{2.5}(-1.72 - -1.62) = -1.68$

Now interpolate horizontally:

$-1.488 + \frac{1.5}{2.5}(-1.68- -1.488) = -1.6032$

# Resource Three Model Answers

Answers

5.

iii) Repeat ii) for the meridional winds this time finding $\partial v/\partial x$, $\partial v/\partial y$. You will need to calculate the gradient array first. You might like to use a spreadsheet or a computer programme to speed this up.

Same methods as before answers are:

Meridional wind gradients in $x$ direction:

$$\begin{bmatrix} 0.56 & 0.36 & -0.08 & -0.44 & -0.56 \\ 0.64 & 0.5 & 0.04 & -0.44 & -0.6 \\ 0.68 & 0.54 & 0.04 & -0.28 & -0.24 \\ 0.68 & 0.58 & 0.12 & -0.32 & -0.4 \\ 0.72 & 0.62 & 0.28 & -0.14 & -0.32 \end{bmatrix}$$

Meridional wind gradients in $y$ direction:

$$\begin{bmatrix} -0.32 & -0.24 & -0.04 & 0 & -0.04 \\ -0.32 & -0.26 & -0.14 & -0.14 & 0.02 \\ -0.22 & -0.2 & -0.14 & -0.12 & -0.02 \\ -0.1 & -0.08 & -0.02 & 0.16 & 0.12 \\ -0.08 & -0.04 & 0 & 0.28 & 0.36 \end{bmatrix}$$

$\frac{\partial v}{\partial x}$:

(45N,32W) -0.3360

(46N,35W) 0.0880

(51.5N,39W) 0.5216

$\frac{\partial v}{\partial y}$:

(45N,32W) 0.1520

(46N,35W) -0.0680

(51.5N,39W) -0.2912

# Resource Four
# Model Answers

Answers

1.

i) Water is leaking from the vertex of an inverted cone at the rate of 5 $cm^3$ per minute. What is the rate of change of the depth of the water in the cone ? If the cone is initially full of water and its radius and height are equal, what will the rate of change of the depth be when the depth has halved?

Hint: Remember that $dh/dV$ is just the reciprocal of $dV/dh$.

$$\frac{dV}{dt} = 5$$

$V = \frac{1}{3}\pi h^3$ as $\mathbf{height} = \mathbf{radius}$ for the whole cone, as this keeps the vertex angle the same. Take initial depth as $\mathbf{H}$.

$$\frac{dV}{dh} = \pi h^2$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{5}{\pi h^2}$$

$$When\ depth\ is\ \frac{H}{2}, radius\ is\ also\ \frac{H}{2}.$$

$$\frac{dh}{dt} = \frac{5}{\pi\left(\frac{H}{2}\right)^2} = \frac{20}{\pi H^2}$$

ii) An open cubical box of side $\mathbf{q}$ is filled with sand at a rate of $6\mathbf{q}^2$ $cm^3/s$, but the sand is also leaking out of the bottom of the box at a rate of $4\mathbf{q}$ $cm^3/s$. What is the rate at which the depth of sand in the box increases? As $\mathbf{q}$ becomes larger the rate of depth change tends to which value?

$$V = q^2 h$$

$$\frac{dV}{dh} = q^2$$

# Resource Four
# Model Answers

Answers

$$\frac{dV}{dt} = 6q^2 - 4q$$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{1}{q^2}\,(6q^2 - 4q) = 6 - \frac{4}{q}$$

As $q$ becomes larger, the rate of change of depth tends to 6.

2.

i) A population of fruit flies grows at a rate equal to 3 times the number of flies in the population at any one time, $t$, where time is measured in days. If there are originally $P_0$ flies, how long does it take for the population to double?

$$\frac{dP}{dt} = 3P \implies \int \frac{1}{P}\,dP = \int 3dt$$

$$\ln P = 3t + c$$

$$t = 0 \implies P = P_0 \implies c = \ln P_0$$

$$\therefore \ln P - \ln P_0 = 3t \implies P = P_0 e^{3t}$$

When the population doubles, P=2$P_0$ so t=$\frac{ln2}{3} = 0.231$ days

ii) A second model for the same population of flies gives population growth being equal to 3cos $3t$ times the number of flies in the population at time $t$. How long does it take the population to double now? Hint: Remember to use radians!

$$\frac{dP}{dt} = 3P\cos 3t \implies \int \frac{1}{P}\,dP = \int 3\cos 3t\,dt$$

$$\ln P = sin3t + c$$

$$t = 0 \implies P = P_0 \implies c = \ln P_0$$

$$\therefore \ln P - \ln P_0 = \sin 3t \implies P = P_0 e^{\sin 3t}$$

When the population doubles, P=2 $P_0$ so t= $\frac{\arcsin(ln2)}{3} =$ $0.255$ days

# Resource Four
# Model Answers

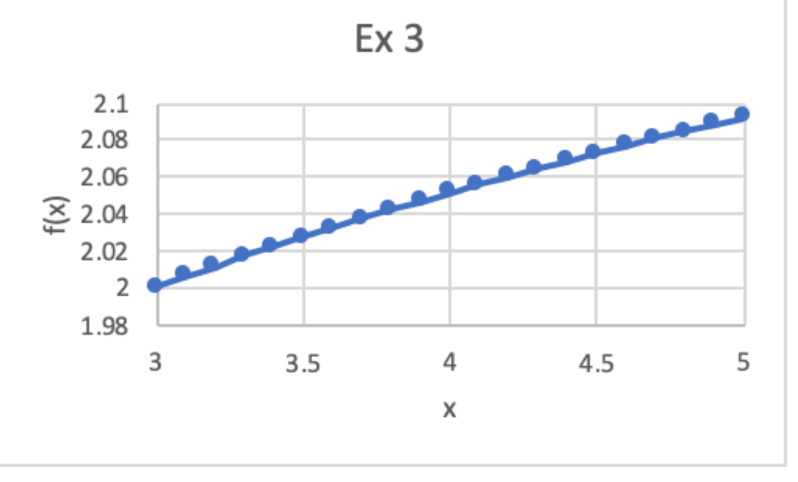
Answers

iii) Explain which model you think is better:

The second model is better as it does not lead to an infinite number of fruit flies, but a fluctuating population.

3. Continue in this way until $x = 5$ . Remember to save values on your calculator to ensure accuracy, or else use a spreadsheet. By plotting the points $(x, f(x))$ you will obtain a numerical solution to the differential equation, showing you the function $f(x)$ in the range 3 to 5.

| x | f(x) | f'(x) |
|---|---|---|
| 3 | 2 | 0.05776227 |
| 3.1 | 2.00577623 | 0.05613154 |
| 3.2 | 2.01138938 | 0.05459576 |
| 3.3 | 2.01684896 | 0.05314669 |
| 3.4 | 2.02216363 | 0.05177706 |
| 3.5 | 2.02734133 | 0.05048037 |
| 3.6 | 2.03238937 | 0.04925084 |
| 3.7 | 2.03731445 | 0.04808328 |
| 3.8 | 2.04212278 | 0.04697302 |
| 3.9 | 2.04682008 | 0.04591586 |
| 4 | 2.05141167 | 0.04490801 |
| 4.1 | 2.05590247 | 0.04394603 |
| 4.2 | 2.06029707 | 0.0430268 |
| 4.3 | 2.06459975 | 0.04214746 |
| 4.4 | 2.0688145 | 0.04130544 |
| 4.5 | 2.07294504 | 0.04049835 |
| 4.6 | 2.07699488 | 0.03972403 |
| 4.7 | 2.08096728 | 0.03898047 |
| 4.8 | 2.08486533 | 0.03826585 |
| 4.9 | 2.08869191 | 0.03757847 |
| 5 | 2.09244976 | 0.03691678 |



# Resource Four
# Model Answers

## Answers

4.

i) Try this approach for $\frac{df(x)}{dx} = \frac{(f(x))}{x}$ with a starting value of $f(1) = 4$ for $1 \leq x \leq 3.$ Use a step length of 0.1.

ii) What do you notice about the gradient?

iii) Solve the equation from i) analytically.

iv) What do you notice about your answers?

| x | f(x) | f'(x) |
|---|---|---|
| 1 | 4 | 4 |
| 1.1 | 4.4 | 4 |
| 1.2 | 4.8 | 4 |
| 1.3 | 5.2 | 4 |
| 1.4 | 5.6 | 4 |
| 1.5 | 6 | 4 |
| 1.6 | 6.4 | 4 |
| 1.7 | 6.8 | 4 |
| 1.8 | 7.2 | 4 |
| 1.9 | 7.6 | 4 |
| 2 | 8 | 4 |
| 2.1 | 8.4 | 4 |
| 2.2 | 8.8 | 4 |
| 2.3 | 9.2 | 4 |
| 2.4 | 9.6 | 4 |
| 2.5 | 10 | 4 |
| 2.6 | 10.4 | 4 |
| 2.7 | 10.8 | 4 |
| 2.8 | 11.2 | 4 |
| 2.9 | 11.6 | 4 |
| 3 | 12 | 4 |
| 3.1 | 12.4 | 4 |
| 3.2 | 12.8 | 4 |
| 3.3 | 13.2 | 4 |
| 3.4 | 13.6 | 4 |
| 3.5 | 14 | 4 |
| 3.6 | 14.4 | 4 |
| 3.7 | 14.8 | 4 |
| 3.8 | 15.2 | 4 |
| 3.9 | 15.6 | 4 |
| 4 | 16 | 4 |

The gradient is constant.

The analytical solution is y=4x.

The answer will be a straight line.

The analytical and numerical solutions are the same here.

# Resource Four
# Model Answers

Answers

5.

i) Try this approach for $\frac{df(x)}{dx} = \frac{f(x)}{e^x}$ with a starting value of $f(1) = 3$ for $1 \leq x \leq 3$. Use a step length of 0.1, then 0.01.

ii) Solve the equation in i) analytically and work out values for $1 \leq x \leq 3$, first in steps of 0.1, then in steps of 0.01.

iii) Plot the values from i) and ii) for the 0.1 step on one graph and for the 0.01 step on another.

iv) What do you notice about your 2 plots?

| x | f(x) | f'(x) |
|---|---|---|
| 1 | 3 | 1.10363832 |
| 1.1 | 3.11036383 | 1.03535018 |
| 1.2 | 3.21389885 | 0.96800773 |
| 1.3 | 3.31069962 | 0.9022709 |
| 1.4 | 3.40092671 | 0.8386582 |
| 1.5 | 3.48479253 | 0.77756232 |
| 1.6 | 3.56254877 | 0.71926619 |
| 1.7 | 3.63447538 | 0.66395877 |
| 1.8 | 3.70087126 | 0.61174991 |
| 1.9 | 3.76204625 | 0.56268406 |
| 2 | 3.81831466 | 0.5167527 |
| 2.1 | 3.86998993 | 0.47390514 |
| 2.2 | 3.91738044 | 0.43405813 |
| 2.3 | 3.96078626 | 0.39710385 |
| 2.4 | 4.00049664 | 0.36291687 |
| 2.5 | 4.03678833 | 0.33135976 |
| 2.6 | 4.0699243 | 0.30228784 |
| 2.7 | 4.10015309 | 0.27555289 |
| 2.8 | 4.12770838 | 0.2510062 |
| 2.9 | 4.152809 | 0.22850092 |
| 3 | 4.17565909 | 0.20789382 |

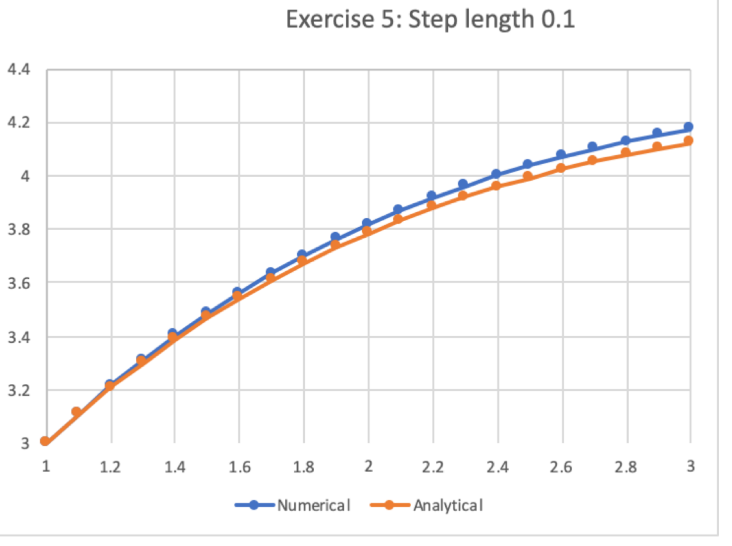
# Resource Four
# Model Answers

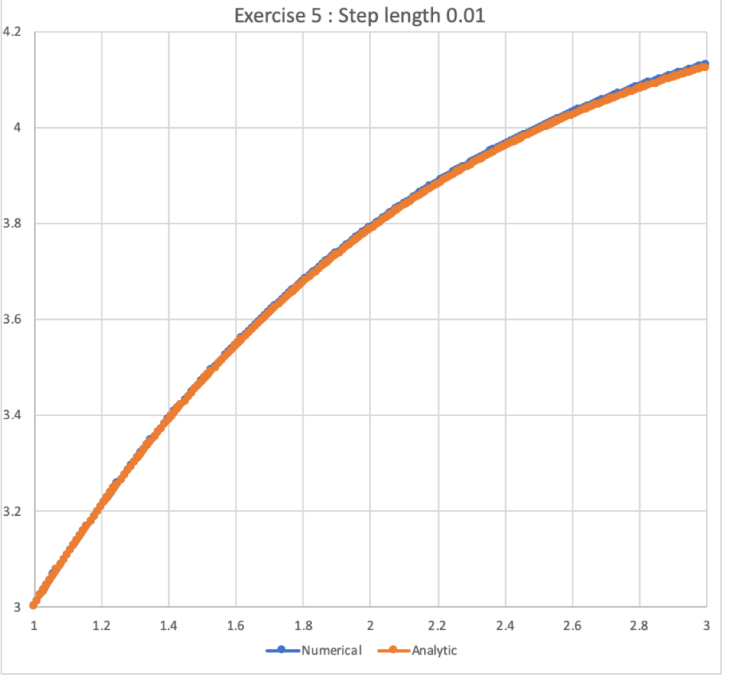
## Answers

| x | f(x) | f'(x) |
|---|---|---|
| 1 | 3 | 1.10363832 |
| 1.01 | 3.01103638 | 1.0966766 |
| 1.02 | 3.02200315 | 1.08971904 |
| 1.03 | 3.03290034 | 1.08276653 |
| 1.04 | 3.043728 | 1.07581991 |
| 1.05 | 3.0544862 | 1.06888003 |
| 1.06 | 3.065175 | 1.06194769 |
| 1.07 | 3.07579448 | 1.0550237 |
| 1.08 | 3.08634472 | 1.04810886 |
| 1.09 | 3.09682581 | 1.04120391 |
| 1.1 | 3.10723785 | 1.03430963 |
| 1.11 | 3.11758094 | 1.02742674 |
| 1.12 | 3.12785521 | 1.02055596 |
| 1.13 | 3.13806077 | 1.01369799 |
| 1.14 | 3.14819775 | 1.00685352 |
| 1.15 | 3.15826628 | 1.00002323 |
| 1.16 | 3.16826652 | 0.99320777 |
| 1.17 | 3.17819859 | 0.98640778 |
| 1.18 | 3.18806267 | 0.97962388 |
| 1.19 | 3.19785891 | 0.97285668 |
| 1.2 | 3.20758748 | 0.96610678 |
| 1.21 | 3.21724855 | 0.95937476 |
| 1.22 | 3.22684229 | 0.95266119 |
| 1.23 | 3.23636891 | 0.94596661 |
| 1.24 | 3.24582857 | 0.93929156 |
| 1.25 | 3.25522149 | 0.93263657 |
| 1.26 | 3.26454785 | 0.92600214 |
| 1.27 | 3.27380787 | 0.91938877 |
| 1.28 | 3.28300176 | 0.91279695 |
| 1.29 | 3.29212973 | 0.90622713 |
| 1.3 | 3.301192 | 0.89967978 |
| 1.31 | 3.3101888 | 0.89315533 |
| 1.32 | 3.31912035 | 0.88665422 |
| 1.33 | 3.3279869 | 0.88017686 |
| 1.34 | 3.33678866 | 0.87372366 |
| 1.35 | 3.3455259 | 0.86729501 |
| 1.36 | 3.35419885 | 0.86089128 |
| 1.37 | 3.36280776 | 0.85451286 |
| 1.93 | 3.75155731 | 0.54453178 |
| 1.94 | 3.75700262 | 0.53989612 |
| 1.95 | 3.76240158 | 0.53529219 |
| 1.96 | 3.76775451 | 0.53071995 |
| 1.97 | 3.77306171 | 0.52617932 |
| 1.98 | 3.7783235 | 0.52167024 |
| 1.99 | 3.7835402 | 0.51719264 |
| 2 | 3.78871213 | 0.51274643 |
| 2.01 | 3.79383959 | 0.50833154 |
| 2.02 | 3.79892291 | 0.50394789 |
| 2.03 | 3.80396239 | 0.49959538 |
| 2.04 | 3.80895834 | 0.49527394 |
| 2.05 | 3.81391108 | 0.49098348 |
| 2.06 | 3.81882091 | 0.48672389 |
| 2.07 | 3.82368815 | 0.48249508 |
| 2.08 | 3.8285131 | 0.47829695 |
| 2.09 | 3.83329607 | 0.47412941 |
| 2.1 | 3.83803737 | 0.46999235 |
| 2.11 | 3.84273729 | 0.46588565 |
| 2.12 | 3.84739615 | 0.46180923 |
| 2.13 | 3.85201424 | 0.45776295 |
| 2.14 | 3.85659187 | 0.45374671 |
| 2.15 | 3.86112934 | 0.4497604 |
| 2.16 | 3.86562694 | 0.44580389 |
| 2.17 | 3.87008498 | 0.44187708 |
| 2.18 | 3.87450375 | 0.43797983 |
| 2.19 | 3.87888355 | 0.43411204 |
| 2.2 | 3.88322467 | 0.43027356 |
| 2.21 | 3.8875274 | 0.42646428 |
| 2.22 | 3.89179205 | 0.42268407 |
| 2.23 | 3.89601889 | 0.41893279 |
| 2.24 | 3.90020822 | 0.41521033 |
| 2.25 | 3.90436032 | 0.41151655 |
| 2.26 | 3.90847548 | 0.40785131 |
| 2.27 | 3.912554 | 0.40421448 |
| 2.28 | 3.91659614 | 0.40060593 |
| 2.29 | 3.9206022 | 0.39702551 |
| 2.3 | 3.92457246 | 0.3934731 |
| 2.31 | 3.92850719 | 0.38994854 |
| 2.35 | 3.94389642 | 0.3761261 |
| 2.36 | 3.94765768 | 0.37273872 |
| 2.37 | 3.95138507 | 0.36937835 |
| 2.38 | 3.95507885 | 0.36604483 |
| 2.39 | 3.9587393 | 0.36273803 |
| 2.4 | 3.96236668 | 0.3594578 |
| 2.41 | 3.96596126 | 0.35620398 |
| 2.42 | 3.9695233 | 0.35297643 |
| 2.43 | 3.97305306 | 0.34977501 |
| 2.44 | 3.97655081 | 0.34659955 |
| 2.45 | 3.98001681 | 0.34344992 |
| 2.46 | 3.98345131 | 0.34032597 |
| 2.47 | 3.98685457 | 0.33722753 |
| 2.48 | 3.99022684 | 0.33415447 |
| 2.49 | 3.99356839 | 0.33110662 |
| 2.5 | 3.99687945 | 0.32808384 |
| 2.51 | 4.00016029 | 0.32508598 |
| 2.52 | 4.00341115 | 0.32211289 |
| 2.53 | 4.00663228 | 0.3191644 |
| 2.54 | 4.00982393 | 0.31624038 |
| 2.55 | 4.01298633 | 0.31334066 |
| 2.56 | 4.01611974 | 0.31046509 |
| 2.57 | 4.01922439 | 0.30761353 |
| 2.58 | 4.02230052 | 0.30478582 |
| 2.59 | 4.02534838 | 0.3019818 |
| 2.6 | 4.0283682 | 0.29920132 |
| 2.61 | 4.03136021 | 0.29644423 |
| 2.62 | 4.03432465 | 0.29371038 |
| 2.63 | 4.03726176 | 0.29099962 |
| 2.64 | 4.04017175 | 0.28831179 |
| 2.65 | 4.04305487 | 0.28564673 |
| 2.66 | 4.04591134 | 0.2830043 |
| 2.67 | 4.04874138 | 0.28038435 |
| 2.68 | 4.05154523 | 0.27778672 |
| 2.69 | 4.05432309 | 0.27521126 |
| 2.7 | 4.05707521 | 0.27265782 |
| 2.71 | 4.05980178 | 0.27012625 |
| 2.72 | 4.06250305 | 0.26761639 |
| 2.73 | 4.06517921 | 0.2651281 |
| 2.77 | 4.07563717 | 0.2553876 |
| 2.78 | 4.07819105 | 0.25300489 |
| 2.79 | 4.0807211 | 0.25064284 |
| 2.8 | 4.08322752 | 0.24830132 |
| 2.81 | 4.08571054 | 0.24598017 |
| 2.82 | 4.08817034 | 0.24367925 |
| 2.83 | 4.09060713 | 0.2413984 |
| 2.84 | 4.09302111 | 0.23913748 |
| 2.85 | 4.09541249 | 0.23689635 |
| 2.86 | 4.09778145 | 0.23467486 |
| 2.87 | 4.1001282 | 0.23247287 |
| 2.88 | 4.10245293 | 0.23029022 |
| 2.89 | 4.10475583 | 0.22812678 |
| 2.9 | 4.1070371 | 0.22598241 |
| 2.91 | 4.10929692 | 0.22385695 |
| 2.92 | 4.11153549 | 0.22175027 |
| 2.93 | 4.113753 | 0.21966223 |
| 2.94 | 4.11594962 | 0.21759268 |
| 2.95 | 4.11812555 | 0.21554148 |
| 2.96 | 4.12028096 | 0.2135085 |
| 2.97 | 4.12241605 | 0.21149359 |
| 2.98 | 4.12453098 | 0.20949662 |
| 2.99 | 4.12662595 | 0.20751744 |
| 3 | 4.12870112 | 0.20555593 |

# Resource Four
# Model Answers

## Answers




Where more steps have been taken, the numerical result more closely approaches the analytically result.

# Resource Four
# Model Answers

Answers

6.

i) What effect will changing the time step have?

A smaller time step will normally give a more accurate result. As planes travel at around 240 m/s in cruise phase, the length of the time step will limit how closely a trajectory can approach the destination airport.

ii) How can the time of flight be recovered from this method?

Just count the time steps to get as close as possible to the destination airport and multiply by how long they are.

iii) Given that Heathrow has co-ordinates (51.5°N, 0.454°W), that the initial heading angle is 187° , that the zonal wind speed is 20 m/s and the meridional wind speed is 5 m/s, using a 10 second time step and an air speed of 230 m/s, find the first position reached.

Hint: Remember you'll need to convert to radians and then back to degrees.

$$lon_2 = -0.0079\ldots + \frac{230\times \cos\left(\frac{187\times\pi}{180}\right) + 20}{111000}\times 10 = -0.0108\ldots$$

So the next point will have a longitude of 0.621°W.

Using the latitude equation in the same way, $lat_2 = 0.8967\ldots^{c}$=51.4 °N.

iv) The gradient of the zonal wind at the starting airport is -0.4 in the direction of longitude and 0.3 in the direction of latitude. For the meridional wind these values are -2.7 and -3.2 respectively. What is the next heading angle?

Substituting all of these values into the equation gives:

$\theta_2$=186.99970…≈187°

So the new heading remains almost the same as the initial heading.

v) How many steps of 10s is the journey likely to take if the plane is due to land in New York?

The journey takes about 8 hours, so with a 10s time step this gives 2880 steps. As journey times differ, just ensure you have divided your chosen journey time in hours by the number of time steps in an hour.

# Resource Five
# Model Answers

Answers

1.

On the next page are some results for looking at fuel used in simulations versus actual flights. The plane being used is a Boeing 787-9 and the real world data comes from a low cost airline.

i) How would you start to analyse the data?

Try sweeping through by eye to check for obvious inconsistencies.

Now work out some summary statistics and check these for anything unusual.

Next analyse mathematically for outliers.

ii) Are there any values of data that need "cleaning"?

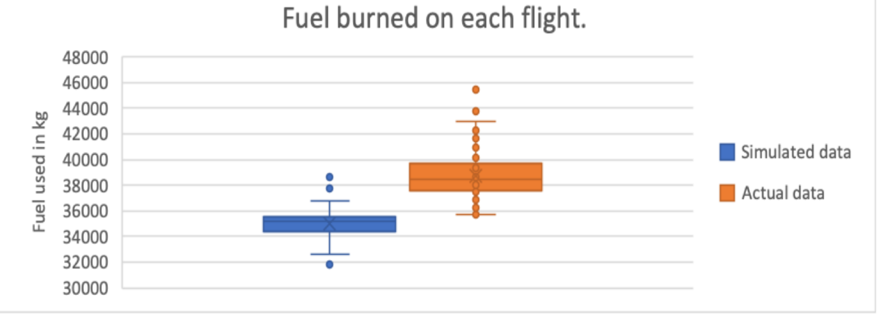
No values look incorrect although there may be outliers.

iii) What further calculations would help you to argue your case if you wanted to prove airlines could save fuel by flying your new routes?

Look at differences between the two data sets and percentage differences too. It is often useful to find the average percentage difference. Try a hypothesis test to see if fuel reductions really are significant.

iv) How could you present your findings graphically to make your point?

Box and whisker plots show comparisons of data well.



# Resource Five
# Model Answers

Answers

2.

i) Analyse the data for the air incidents to find any outliers using both of these methods

| For data 1 outliers: | | For data 2 outliers: | |
|---|---|---|---|
| | | | |
| IQR method: | | IQR method: | |
| Lower | Upper | Lower | Upper |
| -10.5 | 25.5 | -5.5 | 14.5 |
| | | | |
| For data 1 outliers: | | For data 2 outliers: | |
| >2? | | | |
| stdev method: | | stdev method: | |
| 25 is an outlier | | No outliers | |

So the IQR method finds no outliers, but the standard deviation method gives 25 as an outlier.

ii) Try removing any you find of these and recalculating the $t$ value for the data.

Running the t-test without the outlier:

| Data 1 | Data 2 | Difference | Square |
|---|---|---|---|
| 2 | 2 | 0 | 0 |
| 3 | 7 | -4 | 16 |
| 12 | 2 | 10 | 100 |
| 3 | 0 | 3 | 9 |
| 3 | 3 | 0 | 0 |
| 7 | 11 | -4 | 16 |
| 3 | 3 | 0 | 0 |
| 16 | 11 | 5 | 25 |
| | Sums: | 10 | 166 |

| | |
|---|---|
| Mean of differences | 1.25 |
| variance P | 153.5 |
| 7*8 | 56 |
| t | 0.75500502 |

iii) Is there any difference between your results?

This is smaller than the critical value

so the alternative hypothesis must be rejected

at this level of significance

You can see that without the outlying value any reductions in incident numbers are even more likely to be by chance.

# Resource Five
# Model Answers

Answers

2.

iv) Try the spreadsheet ttest function on both the complete data set and the reduced data set to find the probabilities. Do the results agree with your previous conclusions?

Computer t test probability

0.11323999

This is more than 5% so our evidence is not significant at this level.

Computer t test probability

0.23743832

This is even further above the 5% level so our evidence is not significant at this level.

3. Now try to apply this theory to the set of data values given for fuel use for both the simulated and flown routes. What conclusions can you draw from your test?

| | Fuel (kg) | |
|---|---|---|
| Outliers | Simulated flights | Real world data |
| | 35724.29748 | 37514.15575 |
| Higher | 38619.31587 | 40103.31898 |
| Higher | 37757.64777 | 38565.19376 |
| Higher | 37970.5371 | 38049.60359 |
| | 36738.28202 | 38049.60359 |
| | 34313.22523 | 42242.0353 |
| | 36140.70586 | 38132.3135 |
| | 35080.28877 | 39630.94163 |
| | 34879.91332 | 36892.37578 |
| | 34748.1323 | 37049.43017 |
| | 34529.43188 | 36505.10712 |
| | 35480.14216 | 37549.89289 |
| | 35292.10673 | 38021.70951 |
| | 35505.54561 | 39666.91764 |
| | 36450.47778 | 38278.44987 |
| | 35414.61454 | 36249.41467 |
| | 36254.82761 | 37776.70125 |
| | 35860.63307 | 37605.20125 |
| | 35758.78842 | 41250.19213 |
| | 35448.40138 | 39753.21559 |
| | 35158.89655 | 38143.50724 |
| | 35979.27698 | 39064.08465 |
| | 33689.32298 | 37037.25002 |
| | 34087.82526 | 36992.09935 |
| | 33466.90042 | 38003.73812 |
| | 33013.66201 | 36730.46029 |
| | 35192.12909 | 36954.93138 |
| | 34389.02387 | 40199.41951 |
| | 33731.2895 | 40449.1355 |
| | 34581.51024 | 35669.8451 |
| | 36157.67997 | 36253.95761 |
| | 35549.79557 | 36716.36344 |
| | 35344.20775 | 38426.21959 |
| | 34994.99008 | 39374.95196 |
| | 35139.29919 | 37844.72297 |
| | 34995.98818 | 38642.75529 |

| | | |
|---|---|---|
| | 33859.31136 | 37631.49226 |
| | 33445.91716 | 37342.4977 |
| | 33678.46293 | 38232.62485 |
| | 35276.91462 | 37262.03919 |
| | 35519.65863 | 38920.51302 |
| | 35130.17112 | 38519.36692 |
| | 34362.32868 | 38875.36032 |
| | 34672.53 | 39695.6386 |
| Lower | 31789.4501 | 40971.22678 |
| Lower | 32157.19635 | 38179.0248 |
| Lower | 32637.6823 | 38476.5452 |
| | 33152.15153 | 38479.08547 |
| | 33103.42105 | 39364.48693 |
| | 35297.65848 | 45443.19536 |
| | 35583.16724 | 41093.31062 |
| Higher | 35471.64309 | 42964.54788 |
| Higher | 36182.52211 | 43748.95096 |
| | 35266.76028 | 41320.07522 |
| | 33148.96295 | 41408.43726 |
| | 34768.28022 | 37864.31618 |
| | 36110.23 | 39460.92335 |
| | 35352.36231 | 38876.10781 |
| | 35472.35 | 38577.68885 |
| | 34658.40905 | 37694.49053 |
| | 34438.19922 | 39811.5962 |
| | 34997.39319 | 41256.76904 |
| | 35938.81544 | 39817.78605 |
| | 35933.17023 | 38683.55027 |
| | 34939.61335 | 40242.17401 |
| | 33938.16048 | 38549.52992 |
| | 33922.9366 | 38105.87335 |
| | 34917.30762 | 39818.45448 |
| | 35830.01458 | 41597.34998 |
| | 34626.16476 | 36685.58097 |
| | 35248.88997 | 36376.83267 |
| | 33922.39802 | 37279.04837 |
| | 35216.0674 | 38990.67573 |
| | 35230.14875 | 38194.70842 |
| | 35417.43081 | 35709.89772 |

# Resource Five
# Model Answers

## Answers

| | | |
|---|---|---|
| Mean | 34992.67811 | 38724.70653 |
| St Dev | 1182.680884 | 1836.475557 |
| Quartile 1 | 34389.02387 | 37549.89289 |
| Quartile 2 | 35158.89655 | 38426.21959 |
| Quartile3 | 35519.65863 | 39695.6386 |
| IQR | 1130.634758 | 2145.745702 |
| | | |
| Outlier lower | 32693.07174 | 34331.27434 |
| Outlier higher | 37215.61077 | 42914.25715 |

There are some outliers, but nothing is immediately obvious without testing.

| | |
|---|---|
| ttest= | 2.21506E-25 |
| ttest without outliers= | 1.2698E-24 |

Significant regardless, as these probabilities are tiny.

Using the t-value method, t without outliers is 15.711626 and t with outliers is 15.4025053.

From the table the value of the t-statistic for 68 degrees of freedom and probability 0.05: **1.6676**

From the table the value of the t-statistic for 76 degrees of freedom and probability 0.05: **1.6649**

So these reductions are definitely significant. Looking at the table, we can see they are significant even beyond the 0.05% level, which agrees with the spreadsheet probabilities.

# Resource Six Model Answers

Answers

1. Choose a topic from the list below and try finding some relevant papers. Read through as discussed and take notes. You can then refer to these in university interviews and academics will be delighted that you have used your own initiative to explore Mathematics beyond the curriculum. Results will depend on student choices.

2.

i) Prepare a short presentation based on a paper you read in Exercise 1. Try recording it and watching it back. This is a really good way to make sure that you sound knowledgeable and convincing. Never put too much writing on slides and do always assume that your audience can read, so talk around your slides, highlighting your point, rather than reading them out.

ii) Look online for some relevant conferences. You should see details of the "call for abstracts". This is when academics are required to send in their 300 word summaries to pitch for a place presenting at the conference. Write an abstract for your presentation that would be suitable to send to the most relevant conference you find. You can be inspired by reading abstracts from other conferences. Results will depend on student choices.

3. **Good points**:

- It gives results clearly
- The diagram in the bottom left corner is interesting
- The method is explained briefly
- The aim and motivation are clear

**Improvements**:

- Too crowded with small font
- Diagram backgrounds are too pale; from a little way off you don't really notice them.
- The references could have been placed on the back to leave more space.
- Graphs in figure 2 are a little too fiddly to see clearly on a poster.

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@_AccessEd

hello@access-ed.ngo

100 Black Prince Road
London, SE1 7SJ