

# Simplified analytical model for Skylark Walls (v.0.2)

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## Abstract

In the first part of this paper, a simplified analytical model to calculate the capacity and stiffness of Skylark walls is presented. The model builds upon analytical models available in literature for Cross Laminated Timber walls. In the second part, the response of a 2.9 m x 2.1 m wall is calculated by using the developed model. Then, results are compared with a numerical model based on elastic finite elements. The results show that the analytical model is in agreement with the numerical one.

## 1 Introduction

Skylark walls are designed to resist horizontal and vertical loads. While there are experimental data available on the vertical capacity of the walls, the lateral load capacity needs to be calculated by using first principle equations. In this document a simplified analytical model to calculate the capacity and stiffness of the walls is presented. The proposed model is a generalized version of the ones available for Cross Laminated Timber (CLT) walls [1, 2].

## 2 Design of walls

The shear wall has height  $h$  and width  $w$ , and it is loaded by lateral force  $F$  and vertical load  $q$ . The wall is anchored to the ground by a number  $n$  of bow ties on each face. Each bow tie is characterized by a shear capacity  $S$ , tensile capacity  $T$ , shear stiffness  $k_s$  and tensile stiffness  $k_t$ . Each horizontal  $i^{th}$  bow tie is  $d_i$  distant from the wall edge, and the distance of the furthest bow tie is labelled  $d_n$ . The panels composing the walls have thickness  $t$  and shear modulus  $G$ . A sketch of the wall is shown in Figure 1.

The design of shear walls is divided into two parts:

1. Calculation of the wall displacements (Serviceability Limit State)
2. Calculation of the wall capacity (Ultimate Limit State);

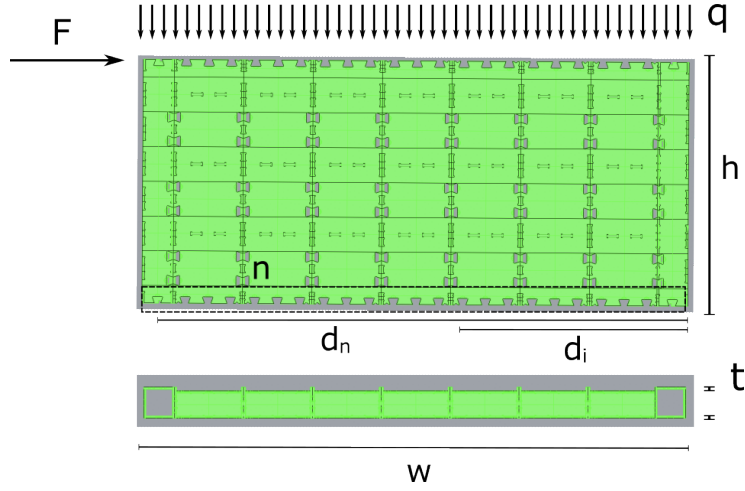


Figure 1: Geometry of the wall.

## 2.1 Serviceability Limit State (SLS)

The mechanical model considers three main sources (Figure 2) of deformation: 1) translational (or slip), 2) shear and 3) rotational (or rocking).

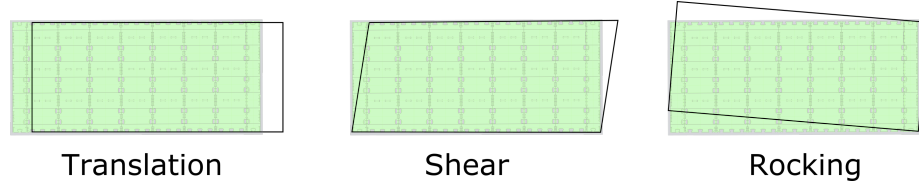


Figure 2: Deformation mechanisms.

The total displacement  $\Delta_{tot}$  is equal to:

$$\Delta_{tot} = \underbrace{\Delta_T}_{Translation} + \underbrace{\Delta_S}_{Shear} + \underbrace{\Delta_R}_{Rocking} \quad (1)$$

The term  $\Delta_T$  can be calculated as:

$$\Delta_T = \frac{F}{2nk_s} \quad (2)$$

where  $n$  is the number of bow ties and  $k_s$  the shear stiffness of the bow ties. The factor 2 appears because bow ties are placed on both faces of the wall.

The term  $\Delta_S$  can be calculated as:

$$\Delta_S = \frac{Fh}{GA_s} = \frac{Fh}{2Gwt} \quad (3)$$

where  $G$  is the shear modulus and  $A_s = 2wt$  the shear area.

The term  $\Delta_R$  can be calculated as:

$$\Delta_R = \frac{Fh - \frac{qw^2}{2}}{2k_t \sum_1^n d_i^2} \cdot h \quad (4)$$

where  $k_t$  is the tensile stiffness on the bow tie, and  $d_i$  the distance of each bow tie from the wall edge.

## 2.2 Ultimate Limit State (ULS)

The capacity  $C$  of the wall is the minimum between the horizontal shear capacity  $F_H$  and the overturning capacity  $F_R$ :

$$C = \min(F_R, F_H) \quad (5)$$

The term  $F_S$  can be calculated as:

$$F_S = 2nS \quad (6)$$

where  $S$  is the shear capacity of the bow tie. The factor 2 appears because there are  $n$  bow ties on each face of the wall. The term  $F_R$  can be calculated as:

$$F_R = \left( \frac{2 \sum_1^n d_i^2}{d_n} T + \frac{qw^2}{2} \right) \cdot \frac{1}{h} \quad (7)$$

where  $T$  is the tensile capacity of the bow tie.

### 3 Application

#### 3.1 Problem definition

A Skylark wall made by 4 *S columns* and 2 *S corners* is presented in Figure 3. Blocks can be downloaded from the [WikiHouse block library webpage](#).

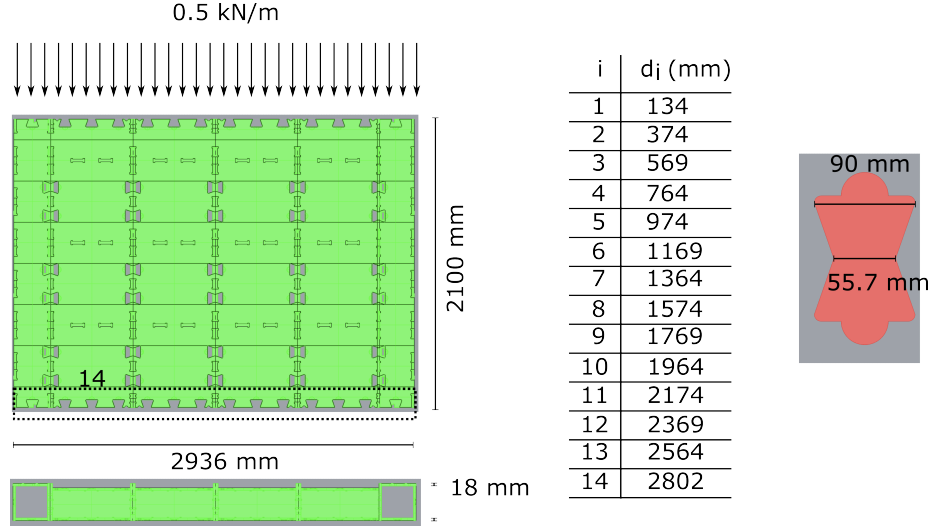


Figure 3: Deformation mechanisms.

The wall is 2936 mm wide and 2100 mm high, and subjected to a distributed vertical load equal to 0.5 kN/m. There are 14 bow ties on each side providing shear capacity and moment capacity. The shear stiffness and tensile stiffness of one bow tie are equal to  $k_s = 3 \text{ kN/mm}$  and  $k_t = 1.5 \text{ kN/mm}$ , respectively. More information on the tensile stiffness can be found in [3], and more information on the shear stiffness can be found in the [Guidelines](#).

The elements are made of 18 mm thick plywood sheets. Material properties are reported in Table 1:

Table 1: Mechanical properties of plywood.

Parameter	Value	Unit	Description
E	8000	MPa	Elastic modulus
G	350	MPa	Shear modulus
$f_{c,0}$	20	MPa	Compressive strength (main direction)
$f_{t,0}$	12	MPa	Tensile strength (main direction)
$f_{c,90}$	10	MPa	Compressive strength (weak direction)
$f_{t,90}$	6	MPa	Tensile strength (weak direction)
$f_s$	3.5	MPa	Shear strength

The walls need to be verified for a lateral force equal to  $F_{SLS} = 40kN$  at the Serviceability Limit State, and for a force equal to  $F_{ULS} = 60kN$  at the Ultimate Limit State.

### 3.2 SLS check

The SLS is verified if:

$$\Delta_{tot} = \Delta_T + \Delta_S + \Delta_R \leq \frac{h}{300} = 7 \text{ mm}$$

The first term is equal to:

$$\Delta_T = \frac{\overbrace{F}^{40}}{2 \underbrace{n}_{14} \underbrace{k_s}_3} = 0.48 \text{ mm}$$

The second term is equal to:

$$\Delta_S = \frac{\overbrace{Fh}^{40000 \cdot 2100}}{2 \underbrace{Gwt}_{350 \cdot 2936 \cdot 18}} = 2.27 \text{ mm}$$

The third term is equal to:

$$\Delta_R = \frac{\overbrace{Fh}^{40000 \cdot 2100} - \frac{\overbrace{qw^2}^{0.5 \cdot 2936^2}}{2}}{2 \underbrace{k_t}_{1.5 \cdot 10^3} \underbrace{\sum_{i=1}^{n_h} d_i^2}_{39.5 \cdot 10^6}} \cdot \overbrace{h}^{2100} = 1.45 \text{ mm}$$

Therefore:

$$\Delta_{tot} = 4.2 \text{ mm} \leq 7 \text{ mm} \checkmark$$

### 3.3 ULS check

The tensile capacity of a bow tie  $T$  is calculated as the minimum force between a tension failure  $T_t$  and a compression/shear failure  $T_c$  (Figure 4).

$T_t$  can be calculated as:

$$T_t = \underbrace{f_{t,0} A_t}_{12 \cdot 55.7 \cdot 18} = 12 \text{ kN}$$

The calculation of  $T_c$  requires the following iterative process:

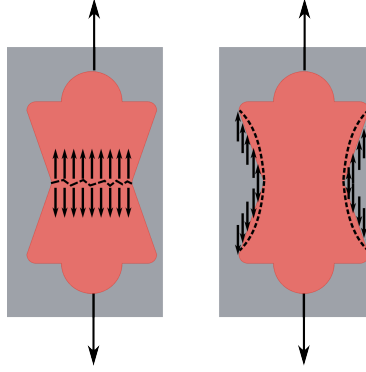


Figure 4: Bow tie failure mechanisms.

1. Select an arbitrary  $T_c$ , for example 1 kN.
2. Calculate the bow tie slip  $s = \frac{F_2}{k}$  with  $k$  the bow tie stiffness in tension.
3. Calculate the contact zone between the bow tie and the panel considering the bow tie slip.
4. Decompose  $T_c$  into  $T_{c,comp}$  (compression force) and  $T_{c,shear}$  (shear) on the contact zone.
5. Calculate the compression stress  $\sigma_C$  and  $\tau_S$  by considering the contact area.
6. Check if the element has failed according to  $\frac{\sigma_c}{f_{c,d}} + \frac{\tau_a}{f_s} = 1$ . If not, increased  $T_c$  until failure.

The details of the calculations are reported in the spreadsheet attached. The "goal seek" function was used. This leads to  $T = 5.2kN$ . By considering a  $k_{mod} = 1.1$  (load duration factor for instantaneous events), this leads to  $T = 5.7kN$ .

The shear capacity  $S$  of a bow tie is equal to:

$$S = \underbrace{A_s}_{55.7 \cdot 18} \overbrace{f_s}^{3.5} = 3.5 \text{ kN}$$

The shear capacity of the wall can be calculated as:

$$F_S = 2nS = 98 \text{ kN}$$

Considering a safety coefficient equal to 1.2 leads to  $F_S = 81.6kN$ .

The rocking capacity  $F_R$  is equal to:

$$F_R = \left( \frac{\overbrace{2 \sum_{i=1}^n d_i^2}^{39.5 \cdot 10^6}}{\underbrace{d_n}_{2802}} \underbrace{T}_{5700} + \frac{\overbrace{qw^2}^{0.5 \cdot 2936^2}}{2} \right) \cdot \underbrace{\frac{1}{h}}_{2100} = 77.5 \text{ kN}$$

Considering a safety coefficient equal to 1.2 leads to  $F_R = 64.6 \text{ kN}$ . Therefore:

$$C = \min(F_S, F_R) = \min(81.6, 64.6) = 64.6 \text{ kN} \geq 60 \text{ kN} \checkmark$$

### 3.4 Finite Element model

An elastic finite element model was developed in Karamba3D [4] to check the results. The wall was modelled by using a shell element with thickness equal to 36 mm. Bow tie couples (one each side) were modelled with a single elastic springs having  $k_t = 3 \text{ kN/mm}$  and  $k_s = 6 \text{ kN/mm}$ , i.e., double the value of a single bow tie.

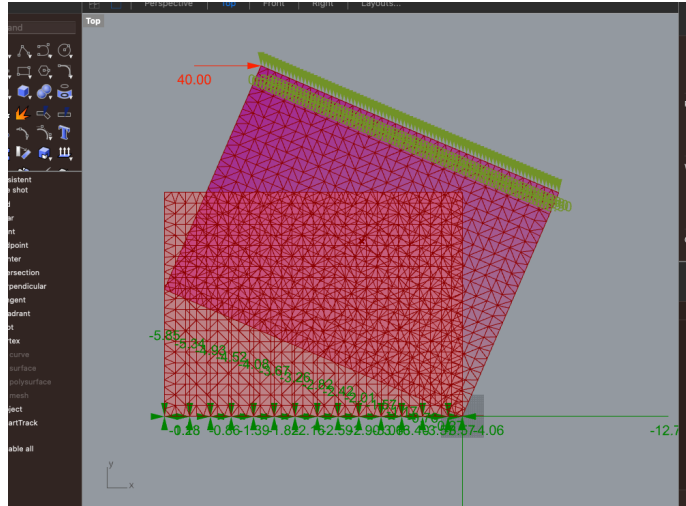


Figure 5: Finite element model of the wall subjected to a lateral load equal to 40 kN.

The wall was subjected to a lateral load equal to 40 kN (Figure 5). It can be seen that the force  $T_n$  in the  $n^{th}$  bow tie couple is equal to  $T_n = 5.85 \text{ kN}$ . This leads to a vertical uplift equal to:

$$\Delta_n = \frac{\overbrace{T_n}^{5.85}}{\underbrace{k_t}_3} = 1.95 \text{ mm}$$

Hence:

$$\Delta_R = \frac{\overbrace{\Delta_n}^{1.95}}{\underbrace{d_n}_{2802}} \cdot \overbrace{h}^{2100} = 1.46 \text{ mm}$$

which is very close (0.01 mm) to the value reported by the analytical model.

For a lateral load equal to  $77.5 \text{ kN}$ , it can be seen (Figure 6) that the force in the  $n^{\text{th}}$  bow tie couple is equal to  $2T = 11.5 \text{ kN}$ . This leads to a force  $T = 5.75$  which is only 0.05 kN different from the values calculated by using the analytical model.

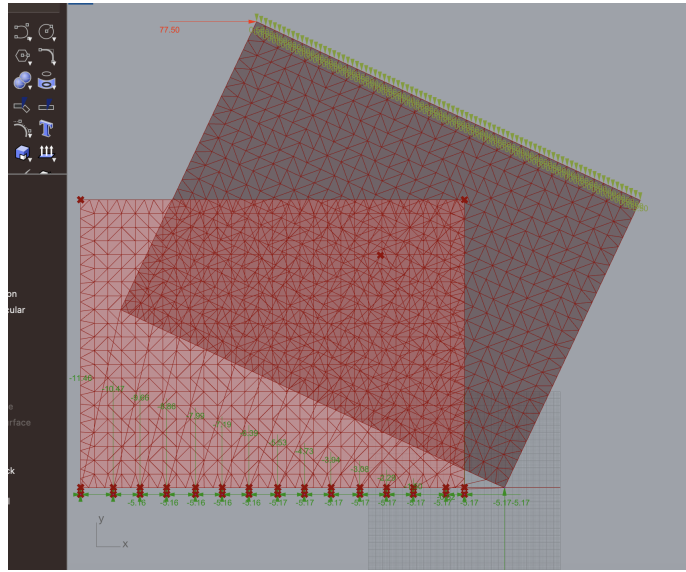


Figure 6: Finite element model of the wall subjected to a lateral load equal to 77.5 kN.

## Appendix A: Derivation of the rotation terms

While the translational terms and the shear terms in the above equations are more intuitive, a more detailed explanation is reported for the rotational terms. The rotation mechanism of the wall is shown in Figure 7.

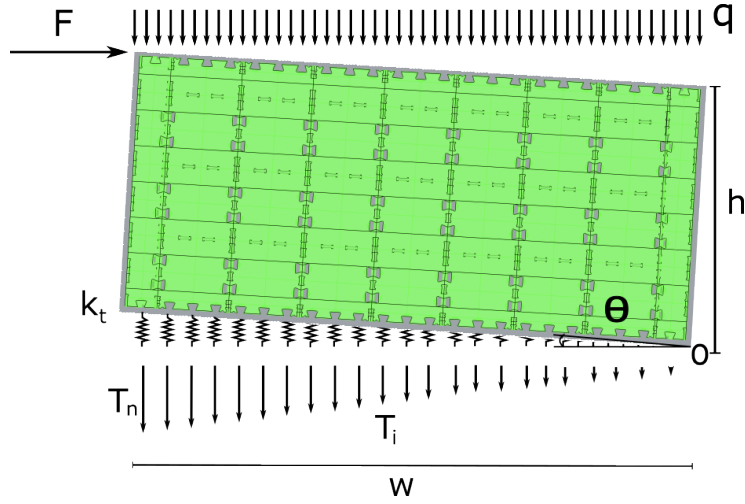


Figure 7: Rocking of the wall over the point O.

The overturning moment is resisted by  $2n$  bow ties. The  $i^{th}$  bow tie is providing a force  $T_i$  at a distance  $d_i$  from the wall edge. Imposing the equilibrium of overturning moments over  $O$  leads to:

$$Fh - \frac{qw^2}{2} = 2 \sum_{i=1}^{n_{bt}} d_i T_i \quad (8)$$

### 3.5 Derivation of the displacement terms

Assuming a rigid rotation, the  $T_i$  contribution of the  $i^{th}$  bow tie can be expressed as a function of the furthest one  $T_n$ :

$$T_i = \frac{d_i}{d_n} T_n \quad (9)$$

Substituting Equation 9 into Equation 8 leads to:

$$Fh - \frac{qw^2}{2} = 2 \sum_{i=1}^n \frac{d_i^2}{d_n} T_n \quad (10)$$

Solving for  $T_n$  this leads to:

$$T_n = \frac{Fh - \frac{qw^2}{2}}{2 \sum_{i=1}^n \frac{d_i^2}{d_n}} \quad (11)$$

The vertical uplift  $\Delta_n$  of the  $n^{th}$  bow tie can be calculated as:

$$\Delta_n = \frac{T_n}{k_t} = \frac{Fh - \frac{qw^2}{2}}{2k_t \sum_1^n \frac{d_i^2}{d_n}} \quad (12)$$

Finally, the rotation of the panel  $\Theta$  and the horizontal displacement  $\Delta_R$  can also be calculated:

$$\Theta = \frac{\Delta_n}{d_n} = \frac{T_n}{k_t} = \frac{Fh - \frac{qw^2}{2}}{2k_t \sum_1^n d_i^2} \quad (13)$$

$$\Delta_R = \Theta \cdot h = \frac{\Delta_n}{d_n} = \frac{T_n}{k_t} = \frac{Fh - \frac{qw^2}{2}}{2k_t \sum_1^n d_i^2} \cdot h \quad (14)$$

### 3.6 Derivation of the capacity terms

Equation 10 can be re-arranged to retrieve  $F$  as a function of  $T_n$  as:

$$F = \left( \frac{2 \sum_1^n d_i^2}{d_n} T_n + \frac{qw^2}{2} \right) \cdot \frac{1}{h} \quad (15)$$

The capacity  $F_R$  can be found by imposing  $T_n = T$ , leading to:

$$F_R = \left( \frac{2 \sum_1^n d_i^2}{d_n} T + \frac{qw^2}{2} \right) \cdot \frac{1}{h} \quad (16)$$

## References

- [1] Daniele Casagrande et al. “Proposal of an analytical procedure and a simplified numerical model for elastic response of single-storey timber shear-walls”. In: *Construction and Building Materials* 102 (2016), pp. 1101–1112.
- [2] Ildiko Lukacs, Anders Björnfot, and Roberto Tomasi. “Strength and stiffness of cross-laminated timber (CLT) shear walls: State-of-the-art of analytical approaches”. In: *Engineering Structures* 178 (2019), pp. 136–147.
- [3] Aidan Napier. “Experimental testing of plywood integral mechanical attachment joints to define tensile and stiffness characteristics”. Master thesis. Department of Civil and Environmental Engineering, University of Strathclyde, 2022.
- [4] Clemens Preisinger. “Linking structure and parametric geometry”. In: *Architectural Design* 83.2 (2013), pp. 110–113.