AP PHYSICS C: ELECTRICITY + MAGNETISM

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Table of Contents:

1. **Electricity**
   1. Chapter 21 - The Electric Field I: Discrete Charge Distributions
   2. Chapter 22 - The Electric Field II: Continuous Charge Distributions
   3. Chapter 23 - Electric Potential
   4. Chapter 24 - Electrostatic Energy and Capacitance
   5. Chapter 25 - Electric Current and Direct-Current Circuits
2. **Magnetism**
   1. Chapter 26 - The Magnetic Field
   2. Chapter 27 - Sources of the Magnetic Field
   3. Chapter 28 - Magnetic Induction
   4. Chapter 29 - Alternating Current Circuits
   5. Chapter 30 - Maxwell's Equations and Electromagnetic Waves
3. **Appendix**

Electricity

Chapter 21 - The Electric Field I: Discrete Charge Distributions

- **Coulomb** (C) - SI unit of charge [elemental charge, $e = 1.6 \times 10^{-19}$, is the smallest unit of charge (i.e. - proton or electron) and it indicates that CHARGE IS QUANTIZED]. More commonly used units are the microcoulomb ($1\mu\mathrm{C} = 1 \times 10^{-6}\mathrm{C}$) or nanocoulomb ($1\mathrm{nC} = 1 \times 10^{-9}\mathrm{C}$)
- **Law of Charges**: opposite charges attract & like charges repel
- $\vec{F_e}$ = **electric force** (electromagnetic analog of gravity - force exerted from one charged particle on another)
- k = **electrostatic constant** ($k = 9.0 \times 10^9 \frac{\mathrm{Nm^2}}{\mathrm{C^2}}$). Alternate form: $k = \frac{1}{4\pi\epsilon_0}$
  - ϵ_0 = **The permittivity of vacuum**. How easily the electric force can go through a certain substance. $\epsilon_0 = 8.5 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{Nm^2}}$ in a vacuum.
- **Coulomb's Law** - $\vec{F_e} = \frac{kq_1q_2}{\vec{r}^2}$ (i.e.- the electric force is the electrostatic constant (k) times the product of charge 1 (q_1) and charge 2 (q_2) divided by the distance ($\vec{r}$) between the two charges). Same form as the Gravity formula. ($\vec{F_g} = \frac{Gm_1m_2}{\vec{r}^2}$). Expanded form: $\vec{F_e} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{\vec{r}^2}$.
  - Applies to POINT CHARGES. Uniformly charged spherical objects behave like point charges from their surface.
- **Field Strength** ($\vec{E} = \frac{\vec{F_e}}{q}$) - electromagnetic analog of $\vec{g}$ in $\vec{F_g} = m\vec{g}$. Measured in $\frac{\mathrm{N}}{\mathrm{C}}$. Thus, $\vec{F_e} = \vec{E}q$. For point masses. Thus: $\vec{E}q = m\vec{a}$

- **Electric field diagrams** - Lines representing the direction of the electric force acting on a positively charged particle are placed at that location. Near the source, the field lines are closer, meaning that the force is stronger. Since $\vec{E} = \frac{\vec{F_e}}{q} = \frac{\left(\frac{kq_1q_2}{\vec{r}^2}\right)}{q_2}$, then $\vec{E} = \frac{kq}{\vec{r}^2}$ (where q is the charge of the source of the field, not of the positively charged particle at that location). This is the electric field around a point charge with charge q. Generally, field lines point from positive to negative. Field lines can never cross; they only interact to create new, averaged, field lines.
    - **Dipoles** - A dipole is the force tangent to the curved lines. Remember, it's the direction of the force, not necessarily where it will spread out. Uneven charges will cause the arrows to lean towards the larger charge.
    - **Two like charges** - an uncharged spot exists in the center: any test charge will remain stationary if placed in the direct center as the forces exerted by the two charges are equal and opposite.
    - **Uniform Field Strength** - This occurs between two parallel and oppositely charged plates. Field lines are parallel so thus the field strength stays constant. For a sample problem, click here.

- **Electric Dipole Moment** -charges $-q$ & $+q$ separated by distance $\vec{a}$. In an $\vec{E}$ field, dipole experiences torque:
    - **Dipole Moment** - $\vec{p} = q\vec{a}$ where $\vec{a}$ points from $-q$ to $+q$. Thus torque: $\vec{\tau} = \vec{p} \times \vec{E}$

Chapter 22 - The Electric Field II: Continuous Charge Distributions

- Point charges: $\vec{E} = \frac{kq}{r^2}$
- By analogy for a charged solid, $\vec{E} = \int \frac{kdq}{r^2}$
- Consider the solid as a collection of an infinite number of point charges, dq.

- **Charge Density** - a charge distributed across an object
  - Linear Charge Density - $\lambda = \frac{q}{L}$
  - Area Charge Density - $\sigma = \frac{q}{A}$
  - Volume Charge Density - $\rho = \frac{q}{V}$
- Two major cases for use of Linear Charge Density (λ) IMPORTANT STUFF - know these two proofs!!!

CASE 1 - Electric field at the center of a charged arc	**CASE 2 - Electric field along the central axis of a charged ring**
A charged rod has charge $-Q$ and has been bent into a circular 120° arc of radius r. Find electric field $\vec{E}$ at center of curve P. Note that due to the symmetry, all of the $d\vec{E}y$ values cancel out. We only need	Given radius $\vec{a}$ and ring of charge Q, find $\vec{x}$ so that point P has maximum electric field $\vec{E}$ from the ring. In order to solve this problem, we must figure out an expression that expresses $\vec{E}$ in terms of $\vec{x}$, then find the maximum on the resulting graph. The x components

to sum up the $d\vec{E}_x$ values. $d\vec{E}_x = d\vec{E}\cos\theta$

As stated by the equation $\vec{E} = \int \frac{kdq}{\vec{r}^2}$, we must integrate over dq, or charge. It is more favorable to integrate over angle θ, so we will use linear charge density λ to put dq in terms of $d\theta$.

dQ in $d\theta$	Electric Field
$\lambda = \frac{Q}{L} = \frac{Q}{r\theta}$ (note: we must use radians for this to be true) $\lambda r\theta = Q$ $\lambda r d\theta = dQ$	$\vec{E} = \int \frac{kdQ}{\vec{r}^2}$

Only in the x direction
As we stated before: $d\vec{E}_x = d\vec{E}\cos\theta$

Mushing all of these three results together yields:

of $d\vec{E}$ will cancel out, due to the symmetry of the setup. We only want $d\vec{E}_y$. We don't want to integrate around dQ, we want to integrate around the plane of the circle. We'll name this angle φ.

As we go around the circle, φ will go from 0 to 2π. It is important to note that θ is a constant in this situation. As stated by the equation $\vec{E} = \int \frac{kdQ}{\vec{r}^2}$, we must integrate over dq, or charge. It is more favorable to integrate over angle θ, so we will use linear charge density λ to put dQ in terms of $d\theta$.

dQ in terms of $d\varphi$	Electric Field
$\lambda = \frac{Q}{L}$ $\lambda = \frac{Q}{a\varphi}$ $\lambda\varphi a = Q$ $\lambda a = \frac{dQ}{d\varphi}$ $\lambda a d\varphi = dQ$	$\vec{E} = \int \frac{kdQ}{\vec{r}^2}$
Only in the y direction	Length $\boldsymbol{r}$

$$\overrightarrow{E_x} = \int \frac{k\lambda r d\theta}{\vec{r}^2} \cdot \cos\theta$$

$$\overrightarrow{E_x} = \int \frac{k\lambda}{\vec{r}} \cdot \cos\theta d\theta$$

In order to take advantage of the symmetry, we must set the horizontal as 0° and the top end as 60° and then double the resulting force.

$$\overrightarrow{E_x} = 2\left(\int \frac{k\lambda}{\vec{r}} \cdot \cos\theta d\theta\right)$$

Because of the nature by which we derived the above equation, we must start using radians. $60^\circ = \frac{\pi}{3}$ Also, since k, λ, and r are all constants, we can move the $\frac{k\lambda}{r}$ in order to simplify the integral.

$$\overrightarrow{E_x} = 2\left(\frac{k\lambda}{\vec{r}} \int_0^{\frac{\pi}{3}} \cos\theta d\theta\right)$$

Ah...much simpler! Now we simplify, plug, and chug.

As we stated before: $d\vec{E}_y = d\vec{E} \cos\theta$	Pythagorean theorem: $\vec{a}^2 + \vec{x}^2 = \vec{r}^2$ $\vec{r} = \sqrt{\vec{a}^2 + \vec{r}^2}$

Combine everything!

$$\vec{E}_y = \int \frac{k\lambda a d\varphi}{\left(\sqrt{\vec{a}^2 + \vec{x}^2}\right)^2} \cdot \cos\theta$$

$$\vec{E}_y = \int \frac{k\lambda a d\varphi}{\vec{a}^2 + \vec{x}^2} \cdot \cos\theta$$

$$\vec{E}_y = \int \frac{k\lambda a d\varphi}{\vec{a}^2 + \vec{x}^2} \cdot \frac{\vec{x}}{\vec{r}}$$

As complicated as this integral may appear to be at first glance, one must note that every variable presented here is a constant, of course except for $d\varphi$, so we can take them out of the integral, which leaves us with:

It's a bit of a waste of calculus, but hey.

$$\vec{E}_y = \frac{k\lambda a \vec{x} 2\pi}{(\vec{a}^2 + \vec{x}^2)^{\frac{3}{2}}}$$

Remember $\lambda = \frac{Q}{a\varphi}$? Well, φ now equals 2π so we can plug in $\frac{Q}{2\pi a}$ for λ.

$$\overrightarrow{E_x} = \frac{2k\lambda}{\vec{r}} \cdot \sin\theta \Big|_{\theta=0}^{\theta=\frac{\pi}{3}}$$

$$\vec{E} = \frac{\sqrt{3}k}{\vec{r}}\lambda$$

Now we substitute in $\lambda = \frac{Q}{r\theta}$ (previously derived in table above)

$$\vec{E} = \frac{\sqrt{3}k}{\vec{r}}\left(\frac{Q}{r\theta}\right)\}$$

$$\vec{E} = \frac{\sqrt{3}k}{\vec{r}}\left(\frac{Q}{r\left(\frac{2\pi}{3}\right)}\right)$$

$$\vec{E} = \frac{\sqrt{3}k}{\vec{r}} \cdot \frac{Q}{r} \cdot \frac{3}{2\pi}$$

$\vec{E} = \frac{3\sqrt{3}kQ}{2\pi\vec{r}^2}$, right

Thus:

$\vec{E} = \frac{kQ\vec{x}}{(\vec{a}^2+\vec{x}^2)^{\frac{3}{2}}}$, up

From this, there are a few things that we can say just by looking at the nature of the equation. When $x = 0$, there is no electric field. $\lim_{x\to\infty} \frac{kQ\vec{x}}{(\vec{a}^2+\vec{x}^2)^{\frac{3}{2}}}$ also yields no electric force. When $x \gg a$, (i.e. - the hoop appears like a point mass), , which indicates that it will act like a point charge!

Here's a graph of what the function $E(x) = \frac{kQx}{(a^2+x^2)^{\frac{3}{2}}}$ would roughly look like:

How do we find that little maximum point? More calculus of course! We must take the derivative of $E(x)$ with respect to x, set it to zero, and then solve for x.

$$\frac{dE}{dx} = 0$$

	$$\frac{d}{dx}\left[\frac{kQx}{(a^2+x^2)^{\frac{3}{2}}}\right]=0$$ Rearrange: $$kQ\frac{d}{dx}\left[x(x^2+a^2)^{\frac{-3}{2}}\right]=0$$ Product & chain rules: Rearrange: Common denominator: $$kQ\left[\frac{-3x^2+(x^2+a^2)}{(x^2+a^2)^{\frac{5}{2}}}\right]=0$$ The kQ part doesn't really matter so we can eliminate it: $$\left[\frac{-3x^2+(x^2+a^2)}{(x^2+a^2)^{\frac{5}{2}}}\right]=0$$

	Simplify $$-2x^2+a^2=0$$ Solve for x: $$x=\left\{-\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right\}$$

- **Electric Flux** (Φ) - number of field lines intersecting a unit area. $\Phi = \vec{E} \bullet \vec{A}$ (dot product)
  - This product is a minimum at 90° and a maximum at 0°
  - $\vec{E}$ is like the density of field lines intercepting an area, so when you multiply it by the amount of area you have, you get the # of field lines in that area. $\left(\frac{\text{\# of field lines}}{\text{area}}\right)\bullet(\text{area}) = \text{\# of field lines} = \Phi$
  - The dot product accounts for the decrease in the amount of flux attained when the area is angled with respect to the electric field. Vector of $\vec{A}$ points perpendicular to the surface.
  - It's an abstract number.

- **Gauss' Law** is used to calculate $\vec{E}$ for an extended charge given certain symmetries.
  - Flux through a closed surface (box)
    - If a closed surface contains no charge, the net flux (Φ) through it is zero. The flux entering the surface equals the flux exiting the surface so the net flux is zero.

- If surface contains a dipole, the net flux (Φ) is also zero since all of the field lines eventually loop around and connect again.
- A net enclosed charge q will create a certain net flux Φ. When charge inside is positive, the flux is positive ($+\Phi$) and when the charge inside is negative, the flux is negative ($-\Phi$).

- To calculate net flux:

$$\Phi = \oint \vec{E} \bullet d\vec{A}$$

- Note that for a cube, this is very hard to integrate. Thus, for our purposes, we only actually use this when the symmetries of the problem let us get $\vec{E}$ constant all over the surface so $\Phi = \vec{E} \oint d\vec{A} = \vec{E}\vec{A}$, where $\vec{A}$ is nice and symmetric (thus, this means using the surface area of sphere or a cylinder).

<u>Example 1:</u> Taking point charge q

Consider the point charge q wrapped in the Gaussian **surface** (the surface you integrate across). $\Phi = \oint \vec{E} \bullet d\vec{A}$, the dot product goes away since vectors are always parallel.

$$\Phi = \frac{q}{\epsilon_0}$$

- This is valid for <u>any</u> enclosed charge in <u>any</u> surface. Using this, we get the definition of Gauss' law (shown below). Thus, if you know q, and the charge is arranged symmetrically, you can calculate $\vec{E}$.

GAUSS' LAW: $$\oint \vec{E} \bullet d\vec{A} = \frac{q_e}{\epsilon_0}$$

<u>However, there are 7 special cases</u>: (note: q_e signifies charge enclosed by Gaussian surface)

<u>Case 1:</u> Infinite line charge density λ	Find $\vec{E}$ as a function of distance r from wire. Consider segment wrapped in cylinder of length L and radius r. <u>Note</u>: no flux is created through ends caps as the area and electric field vectors are perpendicular, thus it is ignored. $\Phi = \oint \vec{E} d\vec{A} = \frac{q_e}{\epsilon_0}$ & $\lambda = \frac{Q}{\vec{L}} \Rightarrow q_e = \lambda \vec{L}$, thus $\oint \vec{E} d\vec{A} = \frac{\lambda \vec{L}}{\epsilon_0}$, so $\vec{E}\vec{A} = \frac{\lambda \vec{L}}{\epsilon_0}$ $\vec{E}(2\pi \vec{r} h) = \frac{\lambda \vec{L}}{\epsilon_0}$, thus $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 \vec{r}}$ Note how $\vec{E}$ is proportional to r and not r^2 since we are now dealing with the electric field that originates from a line charge and not a point charge. The strength doesn't decrease over time as quickly.
	Poke a cylinder through it! (Gaussian cylinder). We only obtain the fluxes through the end caps. The vectors of the

<u>*Case 2:</u> infinitely thin infinitely large charged sheet, charge density σ	sides are perpendicular to the vectors of the electric field, so we can ignore them. $$\sigma = \frac{Q}{\vec{A}} \Rightarrow q_e = \sigma \vec{A}$$ $$\oint \vec{E} d\vec{A} = \frac{q_e}{\epsilon_0} = \frac{\sigma \vec{A}}{\epsilon_0}$$ $$\vec{E}(2\vec{A}) = \frac{\sigma \vec{A}}{\epsilon_0}$$ $$\vec{E} = \frac{\sigma}{2\epsilon_0}$$
<u>*Case 3:</u> $\vec{E}$ above infinite conducting surface, charge density σ	It is similar to case 2, but we only get flux through top cap area $\vec{A}$. $$\oint \vec{E} d\vec{A} = \frac{q_e}{\epsilon_0}$$ $$\vec{E}\vec{A} = \frac{\sigma \vec{A}}{\epsilon_0}$$ $$\vec{E} = \frac{\sigma}{\epsilon_0}$$
<u>**Case 4:</u> Uniform spherical charge: $\vec{E}$ outside. Sphere radius R and charge Q evenly distributed across the surface.	We must find $\vec{E}$ at $r>R$. To solve, we must wrap the object in a Gaussian sphere. $$\Phi = \oint \vec{E} d\vec{A} = \frac{q_e}{\epsilon_0} = \frac{Q}{\epsilon_0}$$ $$\vec{E}(4\pi^2 r) = \frac{Q}{\epsilon_0}$$

	$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}$ $\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$ $\vec{E} = \frac{kq}{r^2}$ This should look familiar.
Case 5: If the sphere is conducting, $\vec{E}$ inside is zero.	The Gaussian sphere is placed inside the charged conducting sphere. $\Phi = \oint \vec{E} d\vec{A} = \frac{q_e}{\epsilon_0}$ The excess charge inside the object is zero! As it is a conducting surface, all of the extra charges repel each other and move to the surface of the sphere. $\vec{E} = 0$, since $q_e = 0$
**Case 6: $\vec{E}$ inside non-conducting sphere radius R, with evenly distributed charge Q	$\oint \vec{E} d\vec{A} = \frac{q_e}{\epsilon_0}$ We must use density! (charge density that is). Note that ρ (volume charge density) is the same for Q and q_e. $\rho = \frac{Q}{V} = \frac{q_e}{\frac{4}{3}\pi r^3}$ $\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$ Thus...

	$$\frac{Q}{R^3} = \frac{q_e}{r^3}$$ $$q_e = Q\frac{r^3}{R^3}$$ And plugging our findings back into the original equation... $$\vec{E}\vec{A} = \frac{q_e}{\epsilon_0} = \frac{Q\frac{r^3}{R^3}}{\epsilon_0}$$ And since $\vec{A} = 4\pi r^2$ $$\vec{E} = \frac{\left(\frac{Q\frac{r3}{R3}}{\epsilon_0}\right)}{4\pi r^2}$$ $$\vec{E} = \frac{Qr}{4\pi\epsilon_0 R^3}$$
<u>Case 7</u>: $\vec{E}$ inside non-conducting sphere radius R, non uniform charge density.	$\rho = ar^2$ (given charge density - ρ is not constant) $$\oint \vec{E}d\vec{A} = \frac{q_e}{\epsilon_0}$$ $$E(4\pi r^2) = \frac{q_e}{\epsilon_0}$$ Solve for q_e in sphere radius r. $q_e \neq \rho V$ (ρ and V are functions of r, so we must use calculus)

$$q_e = \int_0^r \rho dV$$

What this means: We take an infinite number of concentric spherical shells (all parts of one particular shell have the same q since they all have the same r value), find q for all of them, and then add all of the qs together.

Also, we want to change dV to dr since it's a more convenient variable for integration.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dr}V = \frac{d}{dr}\frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

(This is now the surface area formula since we are taking infinitely thin shell slices, which effectively have a volume equivalent to a surface area)

$$dV = 4\pi r^2 dr$$

Now...

$$q_e = \int_0^r (ar^2)(4\pi r^2 dr)$$

$$q_e = 4\pi a \int_0^r r^4 dr$$

$$q_e = \frac{4}{5}a\pi r^5 \Big|_0^r$$

	Combining with original equation $E(4\pi r^2) = \frac{q_e}{\epsilon_0}$... $$\vec{E}(4\pi r^2) = \frac{\frac{4}{5}a\pi r^5}{\epsilon_0}$$ $$\vec{E} = \frac{ar^3}{5\epsilon_0}$$

*notes on cases 2 and 3

Look at the difference: sheet - $\vec{E} = \frac{\sigma}{2\epsilon_0}$; conducting surface - $\vec{E} = \frac{\sigma}{\epsilon_0}$. If we zoom out of the sheet by a lot, it begins to look like the conducting surface (remember that it *does* have a third dimension) BUT σ is different! It is double the amount (both sides of the sheet). In reality, $\vec{E}$ is the same for both, but the way σ is obtained for both is slightly different, thus resulting in different equations.

σ <u>for sheet</u> - total charge on whole disk

σ <u>for conducting surface</u> - based on charge on only one surface of a larger object

**implications from the resulting equations from case 4 and case 6, $\vec{E} = \frac{kq}{r^2}$ and $\vec{E} = \frac{Qr}{4\pi\epsilon_0 R^3}$. (note: α indicates "proportional to")

<u>And now a summary...</u>

<u>**Gauss' Law 7 Cases General Problem Solving Steps**:</u> 1. <u>Find q_e through charge density (λ, σ, or ρ)*</u> <u>*way to find q_e for *case 7 only*:</u> 1. <u>Know $q_e \neq \rho V$ and thus you must use $q_e = \int_0^r \rho dV$ instead</u>

    2. Put dV in terms of dr
    3. Substitute given equation for ρ into equation from steps 1 & 2
    4. Integrate

2. Find area $\vec{A}$ of part(s) of Gaussian surface where flux is present.
3. Plug into Gauss' Law: $\oint \vec{E} \bullet d\vec{A} = \frac{q_e}{\epsilon_0}$ and solve for $\vec{E}$ (note: for all 7 cases, you can drop the $\oint$ and $\bullet$ parts and change the differential $d\vec{A}$ to $\vec{A}$ thus just making it $EA = \frac{q_e}{\epsilon_0}$)

Enclosed Charges - if you put a charge in a conducting box, you cause a separation of charge in the container. For following examples, assume the shell is neutral.

$+q$ at center	$+q$ off center
inner surface charge: $-q$ outer surface charge: $+q$ Inside the conductor, $\vec{E}=0$, everywhere else, $\vec{E}=\frac{kq}{\vec{r}^2}$ (the conducting shell effectively causes a gap in the field of the conductor) Negative charge is distributed uniformly on the inner surface and positive charge is distributed uniformly on the outer surface.	Charge density on inner surface is greater closer to enclosed q. Electric field lines are always perpendicular to the conducting surface at equilibrium (otherwise, the charges would move). Inside conducting shell, $\vec{E}=0$ On outer surface, charge $+q$ is arranged uniformly. There is no information about position of inner q. $\vec{E}=\frac{kq}{\vec{r}^2}$

In order to obtain the outside charge, just add the charge of the enclosed charge and the charge of the shell. Note that if you have a charge of value $+q$ and you have a shell of charge $-q$, $\vec{E}=0$ outside the shell.

- Conductor in an $\vec{E}$ field. - Similar to how sun rays hit the equator with higher intensity, the $\vec{E}$ field lines will hit the equator of the conducting sphere with a higher flux, causing more charge to accumulate there.

- **Conducting Sheets**
  - 1 thin conducting sheet of area $\vec{A}$, charge Q, $\sigma = \frac{Q/2}{\vec{A}} = \frac{Q}{2\vec{A}}$, so thus $\vec{E} = \frac{\sigma}{\epsilon_0} = \frac{Q/(2\vec{A})}{\epsilon_0}$
  - 2 parallel plates - It is essentially the same as adding the electric field of both plates in isolation. The field on the right is really a combination of the rigtwardly headed field lines of the left plate and the right plate, and the field on the left is really a combination of the leftwardly headed field lines of the left plate and the right plate. The field lines in the middle are heading opposite directions with the same magnitude and thus cancel out. Thus the field on the outside is $\vec{E} = \frac{\sigma}{\epsilon_0} = \frac{Q/\vec{A}}{\epsilon_0}$, which is twice as much as the field on one side of the plate if it was isolated.

Chapter 23 - Electric Potential

- Gravitational Potential
  - Gravitational Potential energy: $E_g = mgh$
    - Technically, it is only defined as a difference in energy, since h is a relative measurement.
      - $\Delta U_g = -m\vec{g} \bullet \Delta\vec{h}$ (there's a negative sign in order to make the $\vec{g}$ and $\Delta\vec{h}$ vectors point in the same direction)
    - Energy is stored in the relationship between objects and the electric field.
    - Increases when an outside force moves an object against the field (up).
    - No energy change if you move perpendicular to field lines as per dot product rules.
  - Gravitational potential (GP) - potential energy per mass $\text{gp} = \frac{U_g}{m}$ (defined at a point in the field)
    - Describes how much energy can be stored at that point in the field.
    - Objects spontaneously move to lower gp.
- Electric Potential - we will generate our equations and terms relating to Electric Potential by creating analogies to Gravitational Potential

  - Electric Potential energy: a difference between two points in a field.
    - By comparing it to ΔU_g, we can say $\Delta U_e = -q\vec{E} \bullet \Delta\vec{x}$
    - Similar to ΔU_g in the sense that charge is a replacement for mass in the formula.
    - It is a ΔU. There is no absolute point 0 for energy.
    - For a positive test charge, you increase electric potential energy by moving the charge against the $\vec{E}$ field lines.

        - For a negative charge, reverse it: increase ΔU_e by moving it *with* the field.

  - Electric Potential (V)
    - $V = \frac{U_e}{q}$ - an energy per charge (analogous to GP)
    - SI unit - Volt (V) - aka Joule per Coulomb
    - Usually, we talk about a difference in potential between two points: $\Delta V = \frac{\Delta U_e}{q}$
    - Scalar quantity, although signs must be used due to the signed nature of charges.
    - V decreases in the direction the field points (doesn't depend on the sign of q). Think of using a positive test charge as the standard.

    - Positive charges spontaneously move towards lower V.
  - Alternative definition of ΔV for uniform field

    since $\Delta U_e = -q\vec{E} \bullet \Delta\vec{x}$ and $\Delta V = \frac{\Delta U_e}{q}$, then $\Delta V = -\vec{E} \bullet \Delta\vec{x}$
  - Two charged parallel plates - $\Delta V \alpha \Delta x$ - Voltage changes proportionally to the location of a particle between the plates.
  - $|\vec{E}| = \left|\frac{\Delta V}{\Delta x}\right|$ - i.e. - Volts per meter (which is really just another way to say Newtons per Coulomb). It can be used to solve for $|\vec{E}|$ (magnitude of $\vec{E}$) between two oppositely charged plates.

  - Electric potential energy often becomes kinetic. Since $\Delta V = \frac{\Delta U_e}{q}$, then $\Delta U_e = -q\Delta V = \frac{1}{2}mv^2$ (kinetic). (must be negative because it's moving to a negative change in Voltage that is from higher energy to lower energy) (for a positive charge, that is).

- **Electronvolts** (eV) - It is a nonstandard energy unit. $1\text{eV} = 1.6\times10^{19}\text{J}$ (ELEMENTARY CHARGE! - one electron moving through one potential difference of one volt causes a transform of energy equivalent to one electron volt.
- **Equipotential Diagrams** - Essentially a topographic map - showing lines of equal potential (V) like a topographic map shows contours of equal height. When you walk on a contour line, your potential energy stays the same -- when the lines are close together and you move perpendicular, the potential energy changes the fastest. Downhill is perpendicular to the contour line. No work is done by moving along an equipotential line. Equipotential lines are ALWAYS perpendicular to the electric field.
    - Points on the same equipotential line have the same voltage. Lines closer to a positive source indicate a higher voltage. Lines further indicate a lower voltage. (positive voltage near a positive source charge & negative voltage near a negative source charge).
- **Non-Uniform Fields**
    - Electric potential energy: Between two point charges the uniform case $\Delta U_e = -q\vec{E} \bullet \Delta x$. Now taking $\vec{E}(x)$ as a function of position. Normally, we'd just use Coulomb's law to calculate $\vec{E}(x)$. Now:

$$\Delta U_e = -\int q\vec{E} \bullet d\vec{x}$$

- For two point charges...

$$\Delta U = -\int_a^b q_2\left(\frac{kq_1}{x^2}\right)dx$$

$$\Delta U = -kq_1q_2\int_a^b \frac{dx}{x^2}$$

$$\Delta U = -\frac{kq_1q_2}{x}\Big|_a^b$$

$$\Delta U = \frac{kq_1q_2}{b} - \frac{kq_1q_2}{a}$$

thus, potential energy for a pair of charges...

$$U_e = \frac{kq_1q_2}{r}$$

note how $\lim_{r\to\infty} U_e = 0$. Also, because of signums of charge, U_e is positive for like charges and negative for opposite charges.

- **Electric Potential**: **Pair of charges**. Since $V = \frac{U}{q}$, for a pair of charges, $V = \frac{kq}{r}$ (where q is the source of the field). This is like the situation above except we disregard the charge of the second particle. It gets cancelled out by division, as you're getting the potential energy per charge (i.e. - the definition of Voltage)

- Generally, V is an integral. $V = -\vec{E} \bullet \vec{x}$, so thus $\Delta V = V_b - V_a = -\int_a^b \vec{E} \bullet d\vec{x}$ (this is like the potential energy formula, except with no q).
- **WORK:** $q\Delta V = \Delta U = -W$. Work *done by the field* is negative. The work that you put on the particle to put it in a location of higher voltage is just W. Satisfies conservation of energy.

- **Conductors** - Since electric field is zero inside, and $\vec{E} = \frac{dV}{dx}$, then V is constant everywhere in it.

Special Cases for Voltage (4 cases)

A]Around a line charge $+\lambda$, radius R, infinitely long.	Since $E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$ (derived from use of Gauss' law), $\Delta V = -\int \vec{E} d\vec{x}$ $\Delta V = -\int \frac{\lambda}{2\pi\epsilon_0 r} dr$ We will simply define the surface of the curve as V_0 (frame of reference) so $V(R) = V_0$. (reference at zero (i.e. - lower limit) will make $\frac{\lambda}{2\pi\epsilon_0 r}$ undefined) $\Delta V = -\int_R^r \frac{\lambda}{2\pi\epsilon_0 r} dr$ $\Delta V = -\frac{\lambda}{2\pi\epsilon_0} \ln r \Big\vert_R^r$ (because $\int \frac{1}{x} = \ln x$) $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln R - \frac{\lambda}{2\pi\epsilon_0} \ln r$ $\Delta V = V_0 - \frac{\lambda}{2\pi\epsilon_0} \ln r$ OR $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R}{r}$
B]Above infinite thin sheet, charge density $+\sigma$ (and potential V_0)	Previously derived: $\vec{E} = \frac{\sigma}{2\epsilon_0}$ (constant) $\Delta V = -\int \vec{E} d\vec{x}$

	$$\Delta V = -\int_0^r \frac{\sigma}{2\epsilon_0} d\vec{r}$$ $$\Delta V = -\frac{\sigma}{2\epsilon_0} r \Big\|_0^r$$ $$\Delta V = -\frac{\sigma r}{2\epsilon_0} - 0$$ Recall that we incorporated V_0 in case A. This time V_0 will act like a constant of integration. $$\Delta V = -\frac{\sigma r}{2\epsilon_0} + V_0$$ $$\Delta V = V_0 - \frac{\sigma r}{2\epsilon_0}$$
*C]V at center of charged arc $+q$ radius R	Since all the charge is the same distance away and V isn't a vector, the arc acts like a point charge. All infinitesimal elements dQ are equidistant from the center of the arc. $$V = \frac{kq}{R}$$
*D]Find V along central axis of charged ring radius a, charge $+q$	All charge is distance $\sqrt{x^2+a^2}$ away from point P. Thus, $$V = \frac{kq}{r}$$ $$V = \frac{kq}{\sqrt{x^2+a^2}}$$

*D and C are relative to $V(\infty)=0$, like the point charge.

Dielectric Breakdown - in a strong enough electric field, a nonconductor (like air) can be made to conduct. For air, such a breakdown occurs when $\vec{E} \approx 3 \times 10^6 \frac{\text{V}}{\text{m}}$

When two spheres of different radii are connected to each other over a large distance, as there is no electric field at equilibrium, the voltage is constant across the surfaces of everything. We can use this data to calculate charge, charge density, and electric field outside the spheres.

Chapter 24 - Electrostatic Energy and Capacitance

- **Electric Potential Energy** - Occurs due to a collection of point charges
  - Due to work done in assembling the charges - a sum of the $q\Delta V$ terms

Q_1 - Experiences $W_1 = q_1 \Delta V_1 = q_1(0) = 0$ (no work needed - comes in for free since there are no other charges present)

Q_2 - Experiences $W_2 = q_2 \Delta V_2 = q_2\left(\frac{kq_1}{r_{12}}\right)$

Q_3 - Experiences $W_3 = q_3 \Delta V_3 = q_3\left(\frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}}\right)\}$

Q_4 - Experiences $W_4 = q_4 \Delta V_4 = q_4\left(\frac{kq_1}{r_{14}} + \frac{kq_2}{r_{24}} + \frac{kq_3}{r_{34}}\right)\}$

  - So, system potential is $U = W_1 + W_2 + W_3 + W_4 + \dots$ (sum of all works)
  - Note: Each new charge coming in experiences a larger ΔV than the previous one.
  - Now consider $2U$ - we get a nice summation. (half of all of the $k\frac{Qq}{r}$ permutations). For example above, this chart represents all of the combinations of W, which when added up make $2U$, as each combination is presented here twice:

$2U$	q_1	q_2	q_3	q_4	q_5	q_6
q_1	0	$k\frac{q_1q_2}{r_{12}}$	$k\frac{q_1q_3}{r_{13}}$	$k\frac{q_1q_4}{r_{14}}$	$k\frac{q_1q_5}{r_{15}}$	$k\frac{q_1q_6}{r_{16}}$
q_2	$k\frac{q_2q_1}{r_{21}}$	0	$k\frac{q_2q_3}{r_{23}}$	$k\frac{q_2q_4}{r_{24}}$	$k\frac{q_2q_5}{r_{25}}$	$k\frac{q_2q_6}{r_{26}}$
q_3	$k\frac{q_3q_1}{r_{31}}$	$k\frac{q_3q_2}{r_{32}}$	0	$k\frac{q_3q_4}{r_{34}}$	$k\frac{q_3q_5}{r_{35}}$	$k\frac{q_3q_6}{r_{36}}$

q_4	$k\frac{q_4 q_1}{r_{41}}$	$k\frac{q_4 q_2}{r_{42}}$	$k\frac{q_4 q_3}{r_{43}}$	0	$k\frac{q_4 q_5}{r_{45}}$	$k\frac{q_4 q_6}{r_{46}}$
q_5	$k\frac{q_5 q_1}{r_{51}}$	$k\frac{q_5 q_2}{r_{52}}$	$k\frac{q_5 q_3}{r_{53}}$	$k\frac{q_5 q_4}{r_{54}}$	0	$k\frac{q_5 q_6}{r_{56}}$
q_6	$k\frac{q_6 q_1}{r_{61}}$	$k\frac{q_6 q_2}{r_{62}}$	$k\frac{q_6 q_3}{r_{63}}$	$k\frac{q_6 q_4}{r_{64}}$	$k\frac{q_6 q_5}{r_{65}}$	0

- Thus, the resulting summation expression: $U = \frac{1}{2}\sum_{i=1}^{n} q_i V_i$
  - Since this is for an arbitrary collection of charges, for a continuous charge distribution (i.e. - charges on a conductor), then $U = \frac{1}{2}QV$ where Q is the total charge and V is the final potential relative to ∞.

- **CAPACITANCE** (C) -It is a property of a conducting object. $C = \frac{Q}{V}$ .Note capacitance is a constant for a certain surface (i.e. - whatever you do to Q (double, halve, etc.), the same happens to V as well).
  - It measures how much charge an object can hold at a given potential. Generally it's related to the size of the conductor.
  - SI Unit - Farad (F) - huge unit $1\text{F} = \frac{1\text{C}}{1\text{V}}$

MORE CASES!!!

A spherical capacitor	$$C = \frac{Q}{V}$$ $$C = \frac{Q}{\left(\frac{kQ}{r}\right)}$$ $$C = \frac{r}{k}$$ $$C = 4\pi\epsilon_0 r$$
Parallel Plates	Surface area of one side of one of the plates - A $$C = \frac{Q}{V}$$ $$C = \frac{Q}{Ed}$$ $$C = \frac{Q}{\left(\frac{\sigma}{\epsilon_0}\right)d}$$ $$C = \frac{Q}{\left(\frac{\left(\frac{Q}{A}\right)}{\epsilon_0}\right)d}$$ $$C = \frac{\epsilon_0 A}{d}$$
Cylindrical Capacitor	Radii small R_1 and large R_2 $$C = \frac{Q}{V}$$

	$C = \frac{Q}{\left(\frac{\lambda}{2\pi\epsilon_0}\ln\frac{R_2}{R_1}\right)}$ $C = \frac{Q}{\left(\frac{\left(\frac{Q}{L}\right)}{2\pi\epsilon_0}\ln\frac{R_2}{R_1}\right)}$ $C = \frac{2\pi\epsilon_0 L}{\ln\frac{R_2}{R_1}}$ note that $C \alpha L$

Specifically for AP part II problems, be able to link the cases we've done so far using the following equations in order:

Capacitors in Circuits - schematic symbol: .

- In a circuit, typically we use parallel plate capacitors.
- When connecting a capacitor to a battery: Charge will flow until potential is equal in both (almost instantly) . Hence, an electric field forms between the two plates of the capacitor
- Charge will flow until $V_{\text{capacitor}} = V_{\text{battery}}$. At this point, we can calculate Q on the plate (i.e. - amount of Q moved around on the circuit) using the definition of capacitance.

Note: red arrows indicate flow of electrons until $V_{\text{capacitor}} = V_{\text{battery}}$ condition is met.

- Note total charge on a parallel plate capacitor is always zero ($+Q$ on one plate and $-Q$ on the other plate).

- When we combine capacitors in series (i.e. - end to end), the voltage across the entire thing will be as if there were one capacitor (i.e. - the voltage of the battery).
- Note how the inside bit is isolated and separate.
- There is the same amount of charge on each capacitor, regardless of size , as when one electron gets bumped off of one capacitor, it goes on to the next one, and the charges cause it to continue along the circuit.

- Energy in a capacitor - combining $U=\frac{QV}{2}$ (we can do this because it's a continuous charge distribution) and $C=\frac{Q}{V}$ yields $U=\frac{Q^2}{2C}$
- V in various circuit setups.

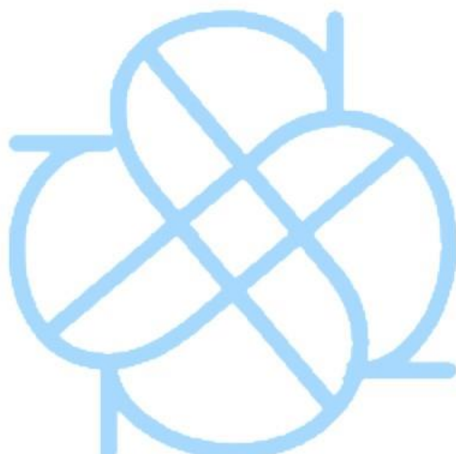
- **Capacitors in series** "see" less voltage than the battery V offers.
  - The total potential across both capacitors equals the battery V at equilibrium. $V_T=V_1+V_2$, but $Q_1=Q_2$ since the charge lost from one capacitor goes on the other.
  - Also, we can express $V=\frac{Q_T}{C_T}$, where $V_T=V_1+V_2$, thus $\frac{Q_T}{C_T}=\frac{Q_1}{C_1}+\frac{Q_2}{C_2}$. Qs cancel. Thus: $\frac{1}{C_T}=\frac{1}{C_1}+\frac{1}{C_2}$.
  - i.e. - more capacitors in series yields a smaller equivalent C_T.
  - Each capacitor sees less voltage and thus stores less charge.

- **Capacitors in parallel**
  - All capacitors see the same amount of voltage
  - So total charge Q_T moved by battery gets distributed across all capacitors. Since $Q_T=Q_1+Q_2$, and thus $C_TU_T=C_1U_1+C_2U_2$ since $C=\frac{Q}{V}$, $U_T=U_1+U_2$ and $C_T=C_1+C_2$

Dielectrics

- Dielectrics are materials that fill the gap in a capacitor.
- Generally, we use a better insulator than air (so you can apply a higher voltage without capacitor plates sparking and discharging (called **dielectric breakdown**)).
- Air breaks down at $\vec{E} = 3 \times 10^6 \frac{\mathrm{V}}{\mathrm{m}}$.
- Dielectric Constant - property of a dielectric. It's a factor by which capacitance increases (represented by the Greek letter kappa - κ)
- Replace the original ϵ_0 with $\kappa\epsilon_0$ in your expressions.
- Thus, $C_f = \kappa C_i$ and $\vec{E}_f = \frac{\vec{E}_i}{\kappa}$. Strengthens capacitance and weakens electric field.
    - Inside the dielectric, the sum of charges isn't too large, so within the volume of the capacitor, the $\vec{E}$ is less (so thus it is harder to get a spark). Note that this means that the space between the dielectric and one of the plates is going to be more. Charges on plate and dielectric (one side) are not equal and opposite.
    - Capacitor + Battery = Constant V
    - When an insert dielectric undergoes polarization and is attracted into the gap, the work done decreases potential energy of the capacitor (initially at least).
    - Battery compensates - battery puts more charges on plates
    - Final result - Goes back to original energy but there is more charge (effectively increasing the capacitance).

- Isolated capacitor at potential V
  - Q is constant (because of isolation)
  - If you insert a dielectric...
  - System potential energy decreases so V across the capacitor decreases.
- Some dielectrics
  - Air - $\kappa = 1$
  - Oil - $\kappa = 2.2$
  - Paper - $\kappa = 3.5$
  - Glass $\kappa = 4.7$



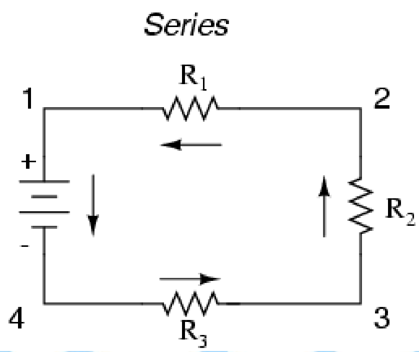
Chapter 25 - Electric Current and Direct-Current Circuits

Resistor Circuit Basics

- **Circuit** - complete path for current to flow through and consists of :
  - Energy source (battery, wall outlet, etc.)
  - Load - something to do work (light, motor, resistor, etc.)
  - Conducting path joining them.
- **Current** (I) - defined as a flow of charge. $I = \frac{\Delta Q}{\Delta t}$
- SI Unit - **Ampères**/ Amps. ($1\text{A} = \frac{1\text{C}}{1\text{s}}$)
- D.C. vs. A.C.
  - **Direct Current** - one-way flow, caused by batteries, power bricks, etc.
  - **Alternating Current** - charges oscillating in SHM (simple harmonic motion) - created by generators (wall outlets, etc.)
- **Conventional current** - fictional but conventional view that current flows from positive to negative - it's wrong when it comes to describing the direction that electrons flow in a circuit, but we use it anyway because it is a standard and all calculations yield the same results.
- **Electron current** - opposite of conventional current. The true picture of how electrons flow.

- Batteries provide a potential difference to cause current flow.
  - Sets up an electric field in the conducting path - causes free charges to move ($\vec{F} = q\vec{E}$). Charges bang into the lattice as they move, dissipating energy as heat.
  - When length of wire L experiences an $\vec{E}$ field, $\Delta V = -EL$
  - Energy is dissipated along the wire as a power: $P = \Delta VI = \frac{W}{t} = \frac{\Delta E}{\Delta t} = \frac{W}{q} \cdot \frac{q}{t}$
  - SI unit for power: Watt (W) = $1\,\mathrm{VA}$ (volt-amp) = $1\frac{\mathrm{J}}{\mathrm{s}}$
  - So energy transfer: $W = \Delta VIt$
- **Resistance** (R) - for a conductor, defined by Ohm's law: $R = \frac{V}{I}$ or $V = IR$
  - R is a property of an object.
  - SI Unit - ohm, $1\Omega = 1\frac{\mathrm{V}}{\mathrm{R}}$
  - **resistivity** (ρ) - material property related to resistance, higher ρ, lowsier conduction. $R = \rho\frac{L}{A}$, where L is the length of the wire and A is the cross sectional area of the wire.
  - Higher temperature, generally greater resistance. Temperature resistance is material dependent.
  - Power dissipation in a resistor can be represented by $P = VI = (IR)I = I^2R$ and $P = VI = V(\frac{V}{R}) = \frac{V^2}{R}$. $P = I^2R$ & $P = \frac{V^2}{R}$

- **Series Circuits** - devices connected end to end
    - Resistors connected in series: equivalent total resistance $R_T = R_1 + R_2 + R_3 + ... + R_n$
    - Current is same everywhere in a series circuit $I_T = \frac{V_T}{R_T}$.
    - Potential of the battery is the sum of the resistor potentials. The resistor potentials can be thought of voltage drops that occur as current passes through each resistor. $V_T = V_1 + V_2 + V_3 + ... + V_n$
    - More resistors causes a current decrease.



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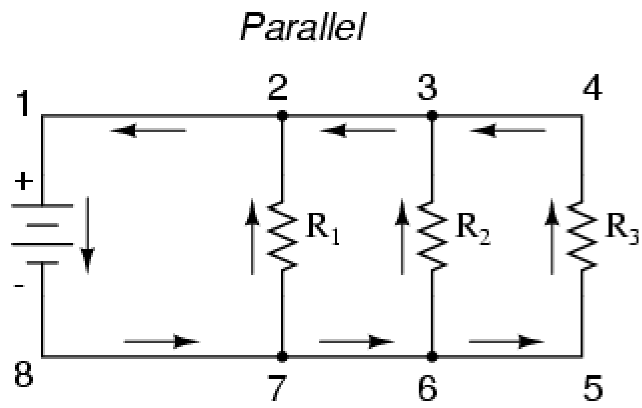
- **Parallel Circuits** - a branching circuit. All the devices in parallel to each other are really connected across the same potential.
    - Any path to and from the ends of the battery are going to "see" the same voltage. $V_T = V_1 = V_2 = V_3 = ... = V_n$ (however note that V_T may not necessarily be the voltage of the battery).
    - Battery current is the sum of currents through each branch. $I_T = I_1 + I_2 + I_3 + ... + I_n$
    - Resistance: applying Ohm's law to the currents in the equation stated above, and cancelling out for the fact that all voltages are the same, resistance is thus: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$

- More resistors in parallel decreases total resistance, but increases the total current. Causes power source to work harder (reason for circuit breakers).



https://sites.google.com/site/lamsnc2dvella/Home/grade-9e-science/unit-3---electricity/series-and-parallel-circuit

- **The Gory Details** -
  - electron motion
    - Electric effects propagate at the speed of light (3×10^8 km/s) due to the electric field.
    - Individual electron speeds are high, but random. $v \approx 10^6$ m/s (but *net* speed is zero without a battery).
    - In a circuit with current, the drift speed (average velocity of all the moving charges) is very low. $v_d \approx 10^6$ m/s.
    - Consider a wire *L* and cross sectional area *A* and charge density *n* (# of free charges/m^3). Assuming *e* is the charge on an electron and *V* is the volume of the wire, drift velocity can be estimated: thus $I = nev_d a$

- Real (as opposed to ideal) batteries:
  - Generate free charge by chemical reactions that have limited rates.
  - As more current is drawn, reactions struggle to keep up: charges have less energy and battery voltage drops. Batteries have *internal resistance*.
  - Given a battery with an EMF ($\mathcal{E}$) as its original potential (the voltage when the circuit is off), and a terminal potential V_T when the circuit is on, then: $V_T = \mathcal{E} - Ir$ (where r is the internal resistance of the battery).
  - For a real battery, EMF ($\mathcal{E}$) - the voltage drop of a battery. Batteries in series decrease the magnitude of the internal resistance as the total of the internal resistances become lower. When batteries are in series, add $\mathcal{E}$*s*.

- **KIRCHOFF'S LAWS**
    - **Voltage Law** - the sum of the potential changes around ANY loop in a circuit is zero (a.k.a. - conservation of energy).
    - **Current Law** - at any junction in a circuit, current in = current out (a.k.a. - conservation of charge).
    - Applying the law: (only necessary if you have more than one battery).
        - Write an $\Sigma V = 0$ expression and choose a direction and loop to go (using conventional current is easiest). Subtract voltages when you go through resistors in the direction of current (add them if going in reverse) and add voltages when you go through batteries (subtract them if you go in reverse).
        - Note how many currents there are and assign random directions and variables. When solved, a negative current will tell you if your assigned current is in the wrong direction.
        - Write expressions according to the current law in accordance to the arrows on your drawing.
        - If expression is complicated and will take a long time to do algebraically, use the matrix function on your graphing calculator.

- Meters -
  - Galvanometer - It is denoted by a circle with a G on it in a circuit. It's a very sensitive current meter - needle deflection is proportional to current. (delicate and can't measure big currents).
  - Ammeter - denoted by a circle with an A in it. Must be in series with the circuit. It's a galvanometer with a "shunt" resistor placed in parallel so it can handle a larger current.
  - Voltmeter - denoted by a circle with a V in it. Must be in parallel with the circuit (tests are of two points!). A galvanometer with a "shunt" resistor in series. The shunt resistor needs a high resistance so it doesn't affect the current in the device that is being measured.
- RC CIRCUITS

A] Discharging Capacitor	Start off with Kirchoff's voltage law:
Sidenote: finding $I(t)$ from $Q(t)$ $$I = -\frac{dQ}{dt}$$ negative because it's decreasing (hand-waving argument) Thus, by chain rule: $$I(t) = \frac{Q_i}{RC} e^{-\frac{t}{RC}}$$ or: $$I(t) = \frac{Q(t)}{RC}$$	$$\Sigma V = 0$$ $$V_{capacitor} - V_{resistor} = 0$$ $$\frac{Q}{C} - IR = 0$$ note, however, that Q and I have time dependencies $$\frac{Q}{C} - \frac{dQ}{dt} R = 0$$ Since current is decreasing, we need to flip the sign $$\frac{Q}{C} + \frac{dQ}{dt} R = 0$$ $$\frac{Q}{C} = -\frac{dQ}{dt} R$$ rearrange so integration with respect to Q is possible & reasonable

also, $I_i = \frac{Q_i}{RC}$ so, $I(t) = I_i e^{-\frac{t}{RC}}$ <u>Sidenote</u>: both are exponential decay to zero $\lim_{t\to\infty} Q(t) = \lim_{t\to\infty} I(t) = 0$ both start at Q_i or I_i at $t=0$	$-\frac{1}{RC}dt = \frac{dQ}{Q}$ integrate both sides $\int -\frac{1}{RC}dt = \int \frac{dQ}{Q}$ factor out constants $-\frac{1}{RC}\int dt = \int \frac{dQ}{Q}$ carry out indefinite integration $-\frac{t}{RC} + K_1 = \ln\lvert Q\rvert + K_2$ Constants of integration K_1 and K_2 may be combined $-\frac{t}{RC} + K = \ln\lvert Q\rvert$ solve for Q $Q = e^{K - \frac{t}{RC}}$ $Q = e^K \cdot e^{-\frac{t}{RC}}$ Note, how when $t=0$, $Q = e^K$, so thus $e^K = Q_i$ $Q(t) = Q_i e^{\frac{-t}{RC}}$
B] Charging Capacitor	Kirchoff's voltage law:

Sidenote: $Q(0)=0$

Since $V=\frac{Q}{C}$ and C is a constant,

$$V(t)=V_f\left(1-e^{-\frac{t}{RC}}\right)$$

Since $I=\frac{dQ}{dt}$

$$Q=CV-CVe^{-\frac{t}{RC}}$$

$$\frac{dQ}{dt}=\frac{d}{dt}\left[CV-CVe^{-\frac{t}{RC}}\right]\}$$

recall that CV is a constant

$$\frac{dQ}{dt}=\left(-CVe^{-\frac{t}{RC}}\right)\left(-\frac{1}{RC}\right)$$

$$I=\frac{V}{R}e^{-\frac{t}{RC}}$$

$$I(t)=I_ie^{-\frac{t}{RC}}$$

$$\lim_{t\to\infty} I(t)=0$$

Exponential decay!

note that the current formula is the same!

And from Ohm's law:

$$\Sigma V=0$$

$$V-V_c-V_r=0$$

$$V-\frac{Q}{C}-IR=0$$

$$V-\frac{Q}{C}-\frac{dQ}{dt}R=0$$

$$V-\frac{Q}{C}=\frac{dQ}{dt}R$$

$$CV-Q=\frac{dQ}{dt}RC$$

$$\frac{dt}{RC}=\frac{dQ}{CV-Q}$$

u-substitution! let $u=CV-Q$, thus $dQ=-du$

$$\frac{dt}{RC}=-\frac{du}{u}$$

$$\int\frac{dt}{RC}=-\int\frac{du}{u}$$

$$\frac{t}{RC}+K_1=-\ln|u|+K_2$$

$$-\frac{t}{RC}+K=\ln|CV-Q|$$

$$e^{-\frac{t}{RC}+K}=CV-Q$$

$$Q=CV-e^{-\frac{t}{RC}+K}$$

recall that K is our constant of integration

$$Q=CV-e^{-\frac{t}{RC}}\cdot e^K$$

note that when $t=0$, $Q=CV-e^K=0$, so thus $e^K=CV$

$I_i = \frac{V}{R}$, where V is battery voltage	$Q = CV - CVe^{-\frac{t}{RC}}$ $Q(t) = CV\left(1 - e^{-\frac{t}{RC}}\right)\}$ note that $Q_f = \lim_{t \to \infty} Q(t) = CV$ $Q(t) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$

RC Time Constant (τ) - a measure of how quickly the capacitor discharges. $\tau = RC$, $\frac{\tau}{RC} = 1$

-discharging capacitor: $Q(\tau) = Q_i e^{-\frac{\tau}{RC}} = Q_i e^{-1} = \frac{Q_i}{e} \approx 37\% Q_i$

-charging capacitor: $Q(\tau) = Q_f\left(1 - \frac{1}{e}\right) \approx 67\% Q_f$

Graphs

Discharging Capacitor	Charging Capacitor
$Q(t)=Q_i e^{\frac{-t}{RC}}$ & $Q_i=CV$	$Q(t)=Q_f\left(1-e^{-\frac{t}{RC}}\right)$ & $Q_f=CV$
$V(t)=V_i e^{\frac{-t}{RC}}$ & $V_i=\frac{Q_i}{C}$	$V(t)=V_f\left(1-e^{-\frac{t}{RC}}\right)$ & $V_f=\frac{Q_f}{C}$
$I(t)=I_i e^{-\frac{t}{RC}}$ & $I_i=\frac{Q_i}{RC}$	$I(t)=I_i e^{-\frac{t}{RC}}$ & $I_i=\frac{Q_i}{RC}$

Behavior of capacitors over extended periods of time:

- For a capacitor in a battery circuit, (uncharged) when you first turn the circuit on, the capacitor **acts like a wire** (ignore it in the circuit) -- a long time later, it **acts like a break** (take the corresponding branch out of the circuit).

A preview of magnetism: Inductors!

Inductor - wire coils in a circuit - schematic symbol is a bunch of connected loops

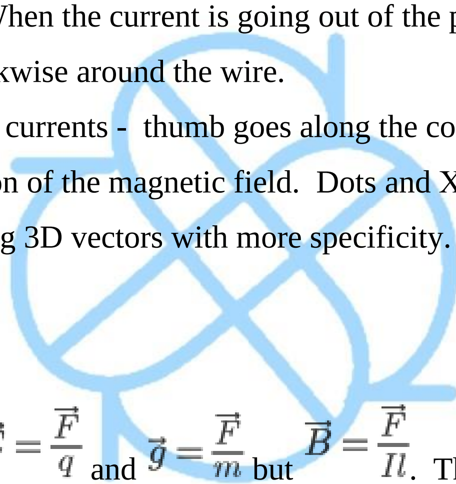
- Magnetic analog of a capacitor but they act the opposite
- Stores energy in a $\vec{B}$ field (magnetic field).
- Exerts a "back EMF" when first turned on - stopping the current as the magnetic field builds up.
- A long time later, once the magnetic field is full strength, it just looks like a wire.
- Inductance (L) - a measure of strength of magnetic field set up - SI Unit - Henry (H)

- Potential drop across inductor: $V = -L\frac{dI}{dt}$
  - Maximum reverse voltage when $\frac{dI}{dt}$ reaches max.
- In a circuit containing a battery, resistor, and inductor connected in series, Kirchoff's voltage law yields the differential equation $V - IR - L\frac{dI}{dt} = 0$
- NON AP: **oscillator** - Capacitor and inductor connected in a series loop. Energy goes from capacitor to inductor and back (electric potential to magnetic potential and back). Emits radio waves (a radio transmitter).

Magnetism

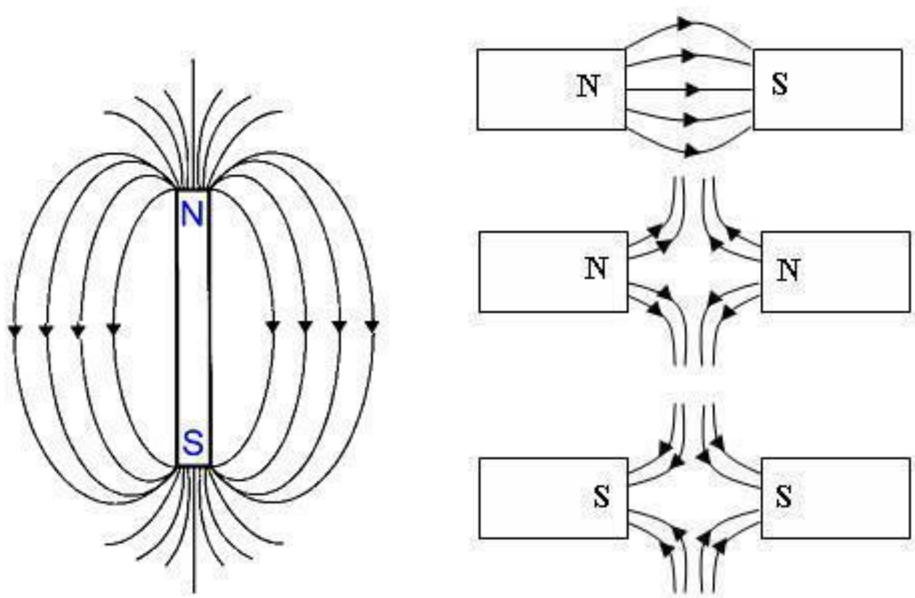
Chapter 26 - The Magnetic Field

- Magnetic Field - caused by moving charges and exerts a force on other moving charges. A vector field, given by direction of the north pole of a compass at a given point in space.
- 3D Vector notation - dots indicate vectors coming out of the page and crosses indicate vectors going into the page (perpendicular to the surface).
- Magnetic Field ($\vec{B}$) around a wire circles the current. As viewed from above, when the current is going into the page, the magnetic field circles clockwise around the wire. When the current is going out of the page, the magnetic field circles counterclockwise around the wire.
- Right hand rule for currents - thumb goes along the conventional current; fingers wrap in the direction of the magnetic field. Dots and X's make such diagrams clearer by indicating 3D vectors with more specificity.



- Field Strengths - $\vec{E} = \frac{\vec{F}}{q}$ and $\vec{g} = \frac{\vec{F}}{m}$ but $\vec{B} = \frac{\vec{F}}{Il}$. The current length ($I \cdot l$) is the source of the magnetic field. In the case of a wire: $|\vec{B}| = \frac{\mu_0 I}{2\pi r}$, where $\mu_0 = 4\pi \times 10^{-7} \frac{\mathrm{N}}{\mathrm{A}^2}$ (called the permeability of free space -- note that ϵ_0 is the permittivity of free space).
- SI Unit for magnetic field strength - the Tesla (T) --> $1\mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{A \cdot m}}$
- Bar magnets - certain solids can be magnetic in the absence of an overall current: ferromagnetic materials (iron, nickel, cobalt, neodymium)
- Electrons appear to be little magnets (property of magnetic spin). "spin up" or "spin down"

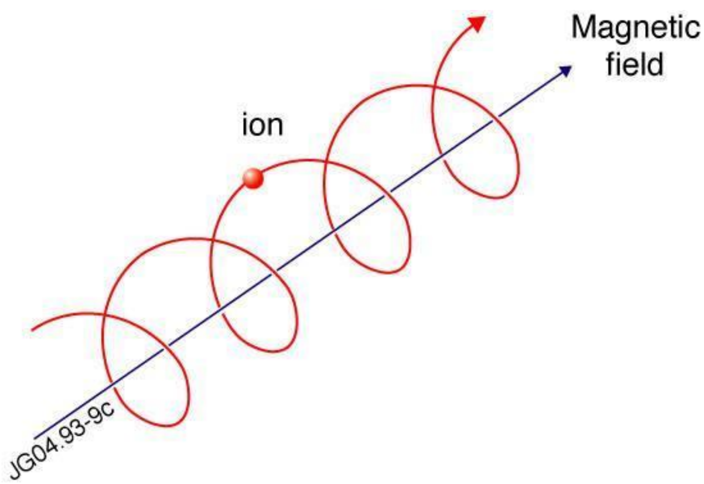
- Domain theory: 1. In some atoms, the electrons have an overall magnetic field, so the atom itself "looks" like a magnet. 2. Domain - a group of atoms magnetically aligned 3. Typically, a lump of iron has many small domains randomly aligned - weakly magnetized overall. 4. If you put an object in a strong magnetic field, the domains get bigger and tend to line up, increasing its magnetic field strength.
- What do magnetic fields look like?



http://www.coolmagnetman.com/maggallery.htm

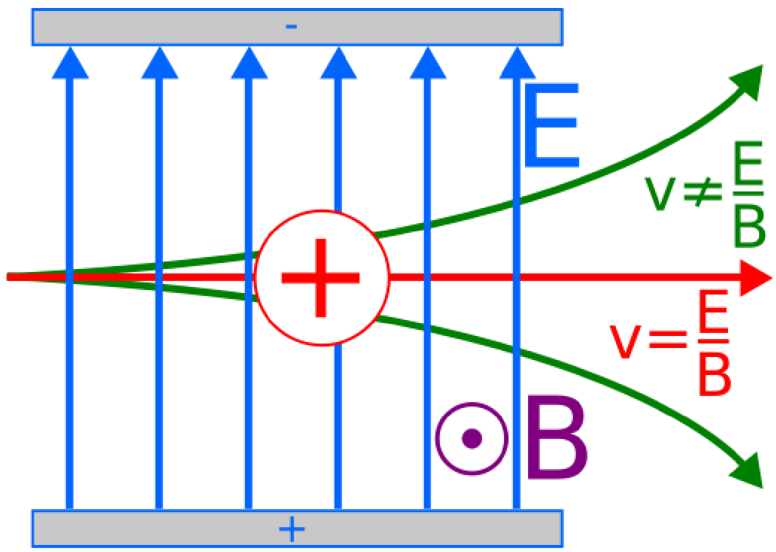
- Breaking a magnet creates two smaller ones (no magnetic monopoles).
- Magnetic field lines are always closed loops. They do not begin or end anywhere (unlike electric field lines). There are no magnetic "monopoles" like there are for an electric charge.
- Magnetic force on a current. Since $|\vec{B}| = \frac{F}{Il}$, we get the direction of the magnetic field from $\vec{F} = I\vec{l} \times \vec{B}$ (cross product between the length vector and the magnetic field vector).
- To determine the direction of the force, use the right hand slap rule. Fingers align up with the magnetic field and the thumb aligns along the current-length. Perpendicular from your palm is the direction of the force.

- Since $I=\frac{q}{t}$, we can substitute and get $\vec{F}=q(\vec{v}\times\vec{B})$. This is the force on a moving charge in a magnetic field.
- In all cases, if the current or charge is negative, reverse the vector effect (direction of the magnetic field or force).
- **Cyclotron effect** - magnetic force is a centripetal force because it is always perpendicular to the velocity. Magnetic forces don't do work on moving charges as they only affect the direction of a particle's motion, not speed. If a particle moves at an angle to the magnetic field, it will spiral along the magnetic field line. Overall direction of motion is the component of the original velocity that was parallel to the magnetic field.



http://new.math.uiuc.edu/math198/MA198-2015/lhansel2/index.html

- Recall the centripetal force $\Sigma F=\frac{mv2}{r}$ (sum of radial forces only). For a charged particle in a magnetic field: $q(\vec{v}\times\vec{B})=\frac{mv2}{r}$, thus $r=\frac{mv}{qB}$.
- **Velocity Selector** - crossed electric and magnetic fields. In the image below, the magnetic field deflects a positive charge down. The electric field deflects a positive charge up (with parallel plates). For the charge to remain undeflected, $F_B=F_E$, and thus $qE=qvB$, and thus the velocity that will pass through undeflected is $v=\frac{E}{B}$.



https://en.wikipedia.org/wiki/Wien_filter

- You can either calculate or select out particles of a certain velocity by adjusting the electric and magnetic fields.
- The electric field, velocity, and magnetic fields must all be mutually perpendicular.
- **Mass Spectrometer** - used to calculate mass of atoms or molecules.
- Particles are often singly or doubly ionized (charge of +e or +2e). Acceleration across some voltage will produce some final velocity. Since $U = qV = \frac{1}{2}mv^2$, then $v = \sqrt{\frac{2qV}{m}}$. If you can't calculate this, use $v = \frac{E}{B}$ on the velocity selector. Fire into the magnetic field, so cyclotron effect yields mass $m = \frac{qBr}{v}$, and thus solving for v and combining with $v = \sqrt{\frac{2qV}{m}}$ and then solving for m, once can find the the mass from $m = \frac{mB^2r^2}{2V}$.

Current Loops and torques - motor

A current loop behaves like a bar magnet. It thus tends to line up with an external field. We can wrap our right hand fingers around the loop to get the magnetic field generated by the loop (i.e. - up). OR, you can look at it as having an upward force on the left and a downward force on the right. In a stable state:

Close and far wires go opposite now so everything is balanced (note how they are not parallel to the magnetic field now). There is maximum torque when the area vector is perpendicular to the magnetic field.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Here, theta is the angle between the area vector and the magnetic field.

$$\vec{\tau} = \frac{c}{2} IlB \sin\theta$$

but there are two forces, and length here is d

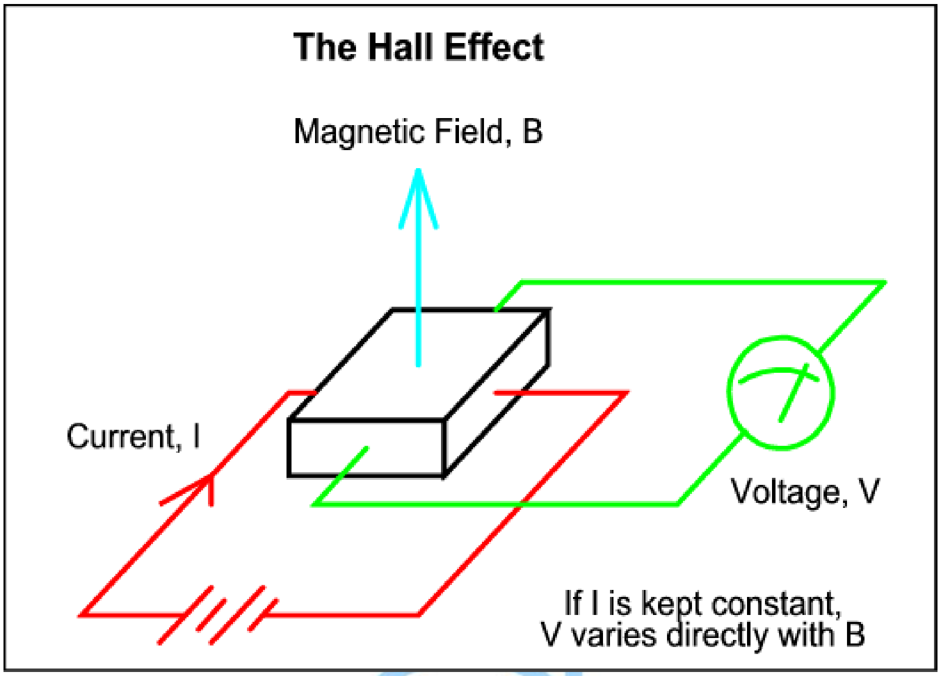
$$\vec{\tau} = (cd)IB \sin\theta$$

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

and generally, for a coil of n number of loops.

$$\tau = nI\vec{A} \times \vec{B}$$

- **Hall Effect -** if a current bearing conductor is placed in a magnetic field, moving charges will be deflected to one side setting up a voltage, V_H, across the width of the conductor. Consider a conducting strip width w, and thickness t. Electrons (assuming that they are moving) set an electric field across the width. Quickly, the electric and magnetic forces balance out.



http://www.coolmagnetman.com/magflux.htm

- We know from velocity selectors that in perpendicular magnetic and electric fields, our drift velocity will be defined by $v_d = \frac{E}{B}$. We also know the Hall Voltage can be defined as follows: $V_H = Ew$. Combining gives $V_H = v_d Bw$. This is a simple way to find drift velocity if you know the hall voltage, the magnetic field, and the width.
- If your material is a particular sort of semiconductor, the hall voltage reverses across the width. These are called p-type materials. Implication: some materials have "positive charge carriers", not electrons, transferring current.
- The Hall Effect is also used to measure weak magnetic fields. Recall for charge-carrier density in a material: $I = nev_d A$. Combining with the previous equation, $n = \frac{I}{e\left(\frac{V_H}{B}\right)t}$, which solved for magnetic field is $B = \frac{neV_H t}{I}$.

Chapter 27 - Sources of the Magnetic Field

All our force formulas are inverse square laws ($\propto \frac{1}{r^2}$). With magnetic fields, additional things need to be taken into account.

Gravitational	Electric	Magnetic
$d\vec{g} \propto \frac{dm}{r^2}\widehat{r}$ For a point mass $\vec{g} = -G\frac{m}{r^2}\widehat{r}$	$d\overrightarrow{E} \propto \frac{dq}{r^2}\widehat{r}$ For a point charge $\overrightarrow{E} = -k\frac{q}{r^2}\widehat{r}$	$d\overrightarrow{B} \propto \frac{Id\vec{l}}{r^2} \times \widehat{r}$ Or as an equality (for an element of current) $d\overrightarrow{B} = \frac{\mu_0}{4\pi} \cdot \frac{Id\overrightarrow{L}}{r^2} \times \widehat{r}$ $d\overrightarrow{B} = \frac{\mu_0}{4\pi} \cdot \frac{dq(\vec{v} \times \widehat{r})}{r^2}$ For a line of current (integration of above) $\overrightarrow{B} = \left(\frac{\mu_0 I}{2\pi r}\right)(d\widehat{L} \times \widehat{r})$

$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{L}}{r^2} \times \hat{r}$ is called the **Biot-Savart Law.** It is the most general expression for calculating the electric field but is very limited in its practicality. There are only two cases to memorize.

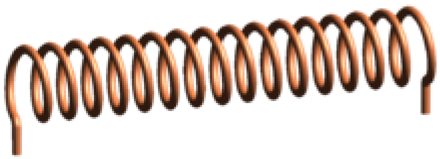
Biot-Savart Case 1: Magnetic field from a curved wire	**Biot-Savart Case 2: Magnetic field from a current ring**
Because of the geometry, the cross product goes away. Start with the Biot-Savart law. $\lvert\vec{B}\rvert = \frac{\mu_0}{4\pi}\int \frac{IdL}{r^2}$ Substitute $dL = Rd\varphi$ $B = \frac{\mu_o I}{4\pi}\int_0^{\varphi} \frac{Rd\varphi}{R^2}$ $B = \frac{\mu_0 I\varphi}{4\pi R}$ Resulting magnetic field is into the page.	by pythagorean theorem: $r^2 = y^2 + R^2$ $dB_y = dB\cos\theta$ Because of the geometry, θ appears twice, thus $\cos\theta = \frac{R}{\sqrt{R^2+y^2}}$ Since we know the resultant magnetic field is up, we can drop the cross product (since we are accounting for components already). $\vec{B} = \frac{\mu_0 I R^2}{2(R^2+y^2)^{3/2}}$ upwards

- **Gauss' Law for Magnetism**: It's silly!!! Since all magnetic field lines are loops, the flux in will equal the flux out, so thus $\oint \vec{B} \cdot d\vec{A} = 0$. This can't really be used to calculate anything. It just serves a reminder of the nature of magnetic flux (Φ_M).
- **Ampère's Law -** a useful way to calculate magnetic fields, but for a current!

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_e$$

- Instead of integrating around a Gaussian surface, we are now integrating around a Ampèrian loop. We integrate the magnetic field around a closed loop surrounding a current in a plane perpendicular to the current so the magnetic field is always parallel to the Ampèrian loop. For this equation, we have cases.

Case 1 - for a straight wire	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_e$ Because of geometry, we can drop the $\oint$, dot product, and differentials. $B(2\pi r) = \mu_0 I$ $B = \frac{\mu_0 I}{2\pi r}$
Case 2 - inside a wire radius R and distance r from the center. Cross section:	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_e$ In order to find the portion of the current that is enclosed by our Ampèrian loop, we must use a proportion. $I_e = I\left(\frac{\pi r^2}{\pi R^2}\right) = \frac{r^2}{R^2} I$

	$$B(2\pi r) = \mu_0\left(\frac{r^2}{R^2}I\right)$$ $$B = \frac{\mu_0 r I}{2\pi R^2}$$
Case 3 - Inside a solenoid  https://en.wikipedia.org/wiki/Solenoid Putting a rectangular loop cross sectioning the solenoid...	$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_e$$ Let's assume that our solenoid has a current I and our Ampèrian loop is enclosing N number of wires. $\oint_a^b$ doesn't matter because it's outside the coil $\oint_b^c$ and $\oint_d^a$ don't matter because $\vec{B}$ is perpendicular to $d\vec{s}$ $$\oint_d^c \vec{B} \cdot d\vec{s} = \mu_0 I_e$$ $$BL = \mu_0 N I$$ $$B = \mu_0 I \frac{N}{L}$$ note how $\frac{N}{L}$ is a density of wire coils if we call this density n, we can simplify this expression to say $$B = n\mu_0 I$$

Case 4 - Inside a toroid Toroid = a wire-wrapped doughnut Current I and number of loops N Inner radius a and outer radius b	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_e$ Integrate across radius where $a<r<b$ $\oint B(2\pi r) = \mu_0 N I$ $B = \frac{\mu_0 N I}{2\pi r}$, where $a<r<b$

Extra note. An old unit for magnetic field: The *gauss* (G). Conversion factor: $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$

Forces due to parallel currents. Parallel currents attract each other. Each wire lies in the electric field caused by the other current. This creates a magnetic force that attracts the two if they're running in the same direction and a magnetic force that repels them if they're running in the opposite direction.

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$\frac{F}{L} = IB$$

this is a force per length, which is practical if the length is unknown or is ideally infinite

$$\frac{F}{L} = I_1\left(\frac{\mu_0 I_2}{2\pi r}\right)$$

and thus

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

which looks a ton like coulomb's law and newton's law of gravitation!

Use your knowledge of the directions of the currents to determine the direction of the forces.

Chapter 28 - Magnetic Induction

Inducing EMF. Here, instead of V, we will use $\mathcal{E}$. Recall that EMF is not really a force, but is rather a voltage. Recall that magnetic fields are created by one of two things: 1. a current or 2. a changing electric field. When a capacitor charges, initially, an imaginary current is created between the two plates since it acts like there is no break. Really, a magnetic field is caused by the changing electric field between the plates.

Faraday's Law - a changing magnetic current creates an electric field (or an EMF). Recall $\Phi_b = \vec{B} \cdot \vec{A}$.

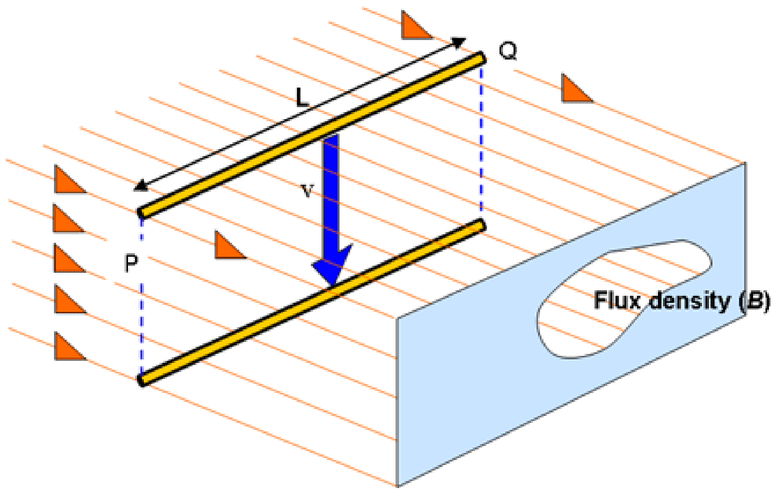
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

the negative is due to lenz's law (below)

Take note that Φ_B can change one of two ways:

1. Change in the magnetic field: $\mathcal{E} = -\frac{d\vec{B}}{dt} \cdot \vec{A}$. Examples: pushing a magnet into a wire coil, or turning an electromagnet on or off.
2. Change in the area: $\mathcal{E} = -\vec{B} \cdot \frac{d\vec{A}}{dt}$. Examples: spin a wire coil in a magnetic field, so angle between the area vector and the magnetic field vector keeps changing.

Special case: moving a wire so that it cuts a magnetic field. This also generates an EMF.



https://www.slideshare.net/mrmeredith/electromagnetic-induction-2

Here, we have a wire of length $\vec{\ell}$ traveling with velocity $\vec{v}$ through a magnetic field $\vec{B}$. Note that $\vec{v}t$ is the distance traveled, and that $\vec{\ell}\times\vec{v}t$ is the area swept out by the wire.

$\mathcal{E} = -\vec{B}\cdot\frac{d\vec{A}}{dt} = -\vec{B}\cdot\frac{d}{dt}(\vec{\ell}\times\vec{v}t)$

$\mathcal{E} = -B\ell v$ (if everything is at right angles)

If not: $\mathcal{E} = \ell\vec{v}\times\vec{B}$

Lenz' Law - an induced current flows in such a way as to oppose the change that caused it. If the flux is increasing, the induced current will create a magnetic field that goes against the increasing magnetic field. If the flux is decreasing, the induced current will create a magnetic field that goes with the increasing magnetic field. Examples: Pushing a magnet into a loop will cause repulsion. Pulling it out will cause attraction. (It's pretty much a specific application of the conservation of energy).

Eddy Currents - (bulk conductor and changing magnetic field). When inserting a conducting block into a region with a magnetic field, charges are separated in the section intersecting the magnetic field. The voltage causes the charges to flow around the other part of the block (the part that doesn't have any magnetic field lines piercing it). When the block is at a place where the magnetic field covers the entire block, no eddy current occurs (only separation of charge occurs). These eddy currents dissipate energy as heat and flow as to oppose the motion.

Induced Electric Field - since $\mathcal{E} = -\frac{d\Phi_B}{dt}$ and $\Delta V = -\int \vec{E} \cdot d\vec{\ell}$, we get an alternate form of Faraday's Law:

$$-\frac{d\Phi_B}{dt} = \oint \vec{E} \cdot d\vec{\ell}$$

There is a negative because of Lenz' Law (applied to the equation after the fact) . Integrate the electric field across a closed loop. Enclose the magnetic flux that you're talking about. If the path of integration is a conduction loop, the electric field will do the work to move the charges for an induced current, not the magnetic field. The electric field is parallel to the EMF. This energy transfer is NOT conservative. The work done by the electric field is typically dissipated as heat or transferred as kinetic energy in a way that it is not reversible.

Inductance (L) - How much a coil resists changes in current, due to magnetic effects. SI Unit: Henry (H). It's an effect of Lenz' law. Causes a reverse EMF ($\mathcal{E}$) when you turn the circuit on (when the current changes). Definition:

$$\mathcal{E} = L\frac{dI}{dt}$$

Magnetic Energy

1. Electromagnets store energy. When you turn a circuit on with an inductor, the magnetic field starts expanding around it. Field lines expanding cut the coil, inducing a reverse emf: this acts like a temporary resistor in the coil. Once a magnetic field is stable, reverse emf disappears. The coil looks like a wire again. When you switch the current off, the collapsing field induces an emf that tries to keep the current flowing, opposes the collapse of the field.
2. Inductor - a wire coil used in a circuit. The creation/destruction of the field acts like a break on the charges to the circuit current. It acts the opposite of a capacitor. Inductors: first turned on/off, acts like a break in the circuit - a long time later, it looks like a wire. (high resistance to low resistance). Capacitors: first turned on/off, looks like a wire. Later, it becomes a break in the circuit.
3. Inductance (L) - property of a particular inductor or coil.
4. emf induced in an inductor can be obtained by the definition of inductance.
5. Power consumed by an inductor can be found by $P = IL\frac{dI}{dt}$. Not very useful...only gives power if current changes.
6. Energy stored by an inductor can be found by $U = -\frac{1}{2}LI^2$
7. RL Circuits - by Kirchhoff's voltage law, the differential equation $\mathcal{E} = IR + L\frac{dI}{dt}$. The solution is $I(t) = \frac{\mathcal{E}}{R}(1 - e^{-\frac{Rt}{L}})$. Note that this indicates that the time constant is $\tau = \frac{L}{R}$.

Chapter 29 - Maxwell's Equations and Electromagnetic Waves

This is not a full chapter. You should know these equations already anyway:

Maxwell's Equations

1. $\oint_S \vec{E} \cdot d\vec{A} = \frac{q_e}{\epsilon_0}$ (gauss' law)
2. $\oint_C \vec{B} \cdot d\vec{A} = 0$ (gauss' law for magnetism)
3. $\oint_S \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$ (variant of Faraday's law)
4. $\oint_S \vec{B} \cdot d\vec{l} = \mu_0 I_e (+\mu_0 I_d)$ (Ampère's Law)

Note: displacement current (I_d) - imaginary current from a capacitor when initially turned on. Creates a magnetic field just as if it was a wire. (the $(+\mu_0 I_d)$ serves somewhat as an addendum to Ampère's law)

Appendix

Store the following values in your calculator using the "➔ sto" function

E	Elementary Charge	1.6×10^{-19}
K	Coulomb's Constant	9.0×10^{9}
U	Permeability of free space	$4\pi\times10^{-7}$

Remember also that you can get the permittivity of free space by coulomb's constant ($\epsilon_0 = \frac{1}{4\pi k}$)

Reminder for **Rebecca Karger**:

$$\int \cos\theta d\theta \neq \cos\theta + C$$

E+M topic breakdown for the test:

1. Electrostatics (ch. 21-23) 30% - charge, coulomb's law, gauss' law, electric field and potential
2. Capacitance (ch. 24) 14% - electric potential energy, capacitance, dielectrics
3. Circuits (ch. 26-27) 20% - current, resistance, power, steady-stable DC Currents, transients with capacitors (changing current).
4. Magnetic fields (ch. 26-27) 20% - force on moving charges and circuits, Biot-Savert law, Ampère's law
5. Electromagnetism (ch. 28-30) 16% - Faraday's law, Lenz's Law, induction, Maxwell's Equations