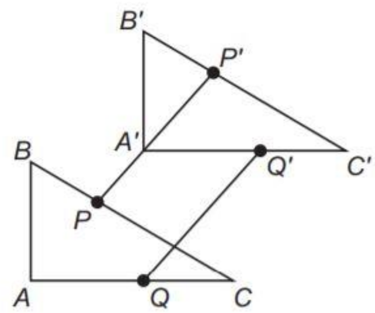
Geometry study guide

From Simple Studies: https://simplestudies.edublogs.org & @simplestudiesinc on Instagram

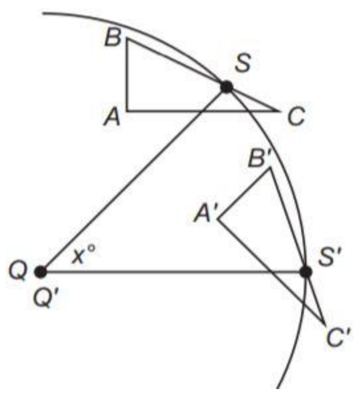
Transformations

There are two types of categories in transformations:

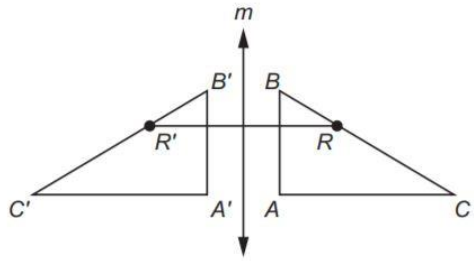
- Rigid - does not change the shape or size
    - **Translation**
        - moving an object in space without changing its size, shape or orientation



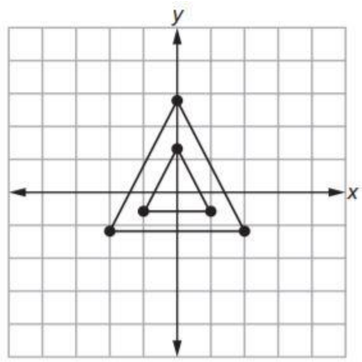
    - **Rotation**
        - rotating an object about a fixed point without changing its size or shape



    - **Reflection**
        - flipping an object across a line without changing its size or shape



- Non-rigid - change the size but not the shape
  - **Dilation**
    - expanding or contracting an object without changing its shape or orientation

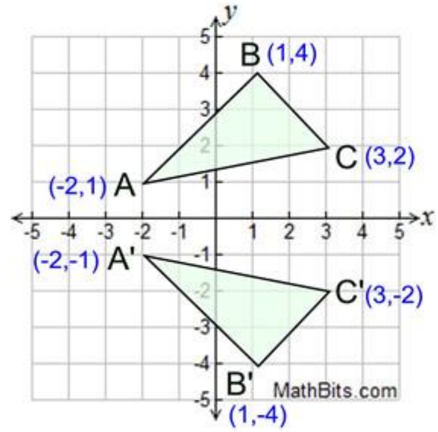


Performing Transformations

- **Translation**
  - to say the shape gets moved 30 Units in the "X" direction, and 40 Units in the "Y" direction, we can write: (x,y)→ (x+30, y+40)
- **Rotation**
  - For 90°, the rule is (x, y) → (-y, x)
  - For 180°, the rule is (x, y) → (-x, -y)
  - For 270°, the rule is (x, y) → (y, -x)
- **Reflection**

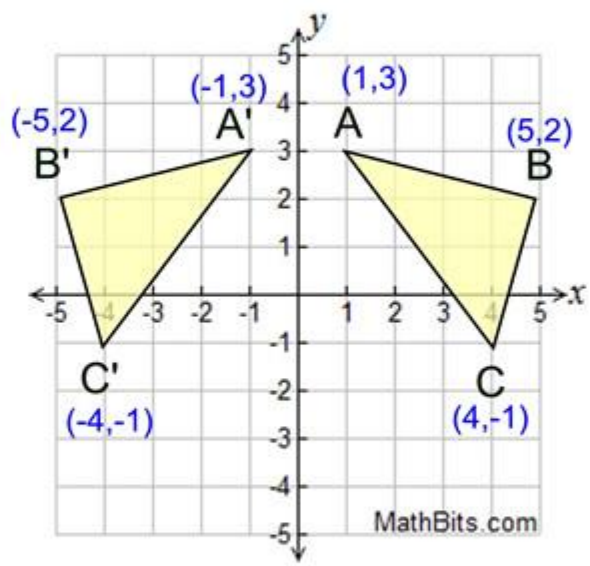
- When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite (its sign is changed).

  The reflection of the point (x,y) across the x-axis is the point (x,-y).



- When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite (its sign is changed).

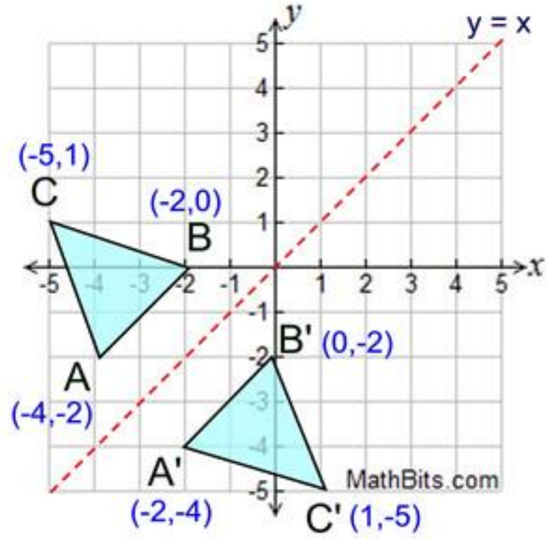
  The reflection of the point (x,y) across the y-axis is the point (-x,y).



- When you reflect a point across the line y = x, the x-coordinate and y-coordinate change places. If you reflect over the line y = -x, the x-coordinate and y-coordinate change places and are negated (the signs are changed).

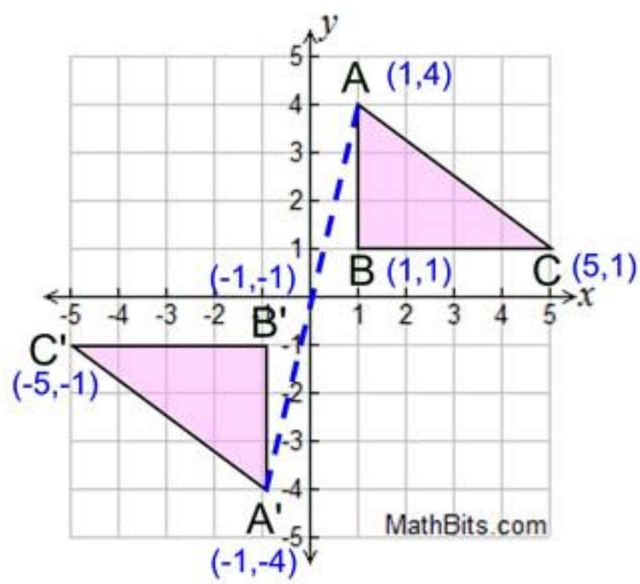
The reflection of the point (x,y) across the line y = x is the point (y, x).

The reflection of the point (x,y) across the line y = -x is the point (-y, -x).



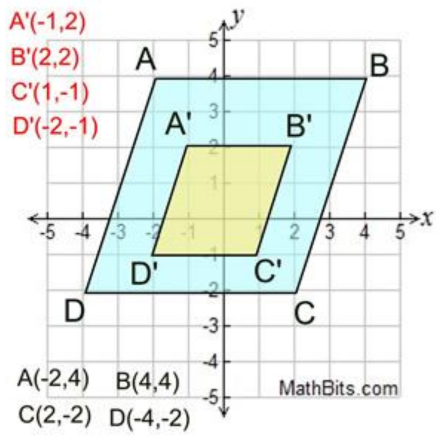
- When you reflect a point in the origin, both the x-coordinate and the y-coordinate are negated (their signs are changed).

  In a point reflection in the origin, the image of the point (x,y) is the point (-x,-y).



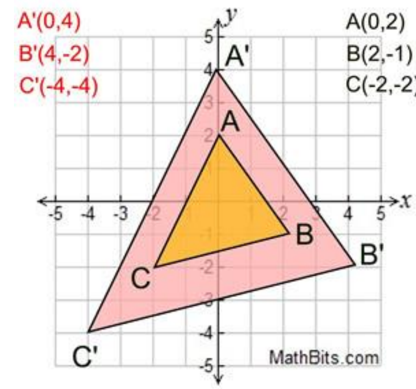
- **Dilation**
  - Dilation with scale factor ½, multiply by ½.

    (x, y) → (½x, ½y)



- Dilation with scale factor 2, multiply by 2.

  (x, y) → (2x,2y)



Postulates and Theorems:

- Defenitions:
  - **Complementary Angles:** Two angles whose measures have a sum of 90
  - **Supplementary Angles:** Two angles whose measures have a sum of 180
  - **Theorem:** A statement that can be proven
  - **Vertical Angles:** Two angles formed by intersecting lines and facing in the opposite direction
  - **Transversal:** A line that intersects two lines in the same plane at different points
  - **Corresponding angles:** Pairs of angles formed by two lines and a transversal that make an F pattern
  - **Same-side interior angles:** Pairs of angles formed by two lines and a transversal that make a C pattern
  - **Alternate interior angles:** Pairs of angles formed by two lines and a transversal that make a Z pattern
  - **Congruent triangles:** Triangles in which corresponding parts (sides and angles) are equal in measure
  - **Similar triangles:** Triangles in which corresponding angles are equal in measure and corresponding sides are in proportion (ratios equal)

- **Angle bisector:** A ray that begins at the vertex of an angle and divides the angle into two angles of equal measure
- **Segment bisector:** A ray, line or segment that divides a segment into two parts of equal measure
- **Legs of an isosceles triangle:** The sides of equal measure in an isosceles triangle
- **Base of an isosceles triangle:** The third side of an isosceles triangle
- **Equiangular:** Having angles that are all equal in measure
- **Perpendicular bisector:** A line that bisects a segment and is perpendicular to it
- **Altitude:** A segment from a vertex of a triangle perpendicular to the line containing the opposite side
- **Geometric mean:** The value of x in proportion a/x = x/b where a, b, and x are positive numbers (x is the geometric mean between a and b)
- Sine, sin For an acute angle of a right triangle, the ratio of the side opposite the angle to the measure of the hypotenuse. (opp/hyp)
- Cosine, cos For an acute angle of a right triangle the ratio of the side adjacent to the angle to the measure of the hypotenuse. (adj/hyp)
- Tangent, tan For an acute angle of a right triangle, the ratio of the side opposite to the angle to the measure of the side adjacent (opp/adj)

- **Congruence Postulates**
  - **Reflexive** Property of Congruence: A = A
  - **Symmetric** Property of Congruence: If A = B,then B = A
  - **Transitive** Property of Congruence If A = B and B = C then A = C
- **Angle postulates and theorems**
  - **Angle Addition postulate:** For any angle, the measure of the whole is equal to the sum of the measures of its non overlapping parts
  - **Linear Pair Theorem**: If two angles form a linear pair, then they are supplementary.
  - **Congruent supplements theorem:** If two angles are supplements of the same angle, then they are congruent.
  - **Congruent complements theorem:** If two angles are complements of the same angle, then they are congruent.

- **Right Angle Congruence Theorem:** All right angles are congruent.
- **Vertical Angles Theorem:** Vertical angles are equal in measure
- **Theorem:** If two congruent angles are supplementary, then each is a right angle.
- **Angle Bisector Theorem:** If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.
- **Converse of the Angle Bisector Theorem:** If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

- **Lines postulates and theorems**
  - **Segment Addition postulate:** For any segment, the measure of the whole is equal to the sum of the measures of its non-overlapping parts
  - **Postulate:** Through any two points there is exactly one line
  - **Postulate:** If two lines intersect, then they intersect at exactly one point.
  - **Common Segments Theorem:** Given collinear points A,B,C and D arranged as shown, if AB # CD then AC # BC
  - **Corresponding Angles Postulate:** If two parallel lines are intersected by a transversal, then the corresponding angles are equal in measure
  - **Converse of Corresponding Angles Postulate:** If two lines are intersected by a transversal and corresponding angles are equal in measure, then the lines are parallel
  - **Postulate:** Through a point not on a given line, there is one and only one line parallel to the given line
  - **Alternate Interior Angles Theorem:** If two parallel lines are intersected by a transversal, then alternate interior angles are equal in measure
  - **Alternate Exterior Angles Theorem:** If two parallel lines are intersected by a transversal, then alternate exterior angles are equal in measure
  - **Same-side Interior Angles Theorem:** If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary.
  - **Converse of Alternate Interior Angles Theorem:** If two lines are intersected by a transversal and alternate interior angles are equal in measure, then the lines are parallel

- **Converse of Alternate Exterior Angles Theorem:** If two lines are intersected by a transversal and alternate exterior angles are equal in measure, then the lines are parallel
- **Converse of Same-side Interior Angles Theorem:** If two lines are intersected by a transversal and same-side interior angles are supplementary, then the lines are parallel
- **Theorem:** If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular
- **Theorem:** If two lines are perpendicular to the same transversal, then they are parallel
- **Perpendicular Transversal Theorem:** If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other one.
- **Perpendicular Bisector Theorem:** If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment
- **Converse of the Perpendicular Bisector Theorem:** If a point is the same distance from both the endpoints of a segment, then it lies on the perpendicular bisector of the segment
- **Parallel Lines Theorem:** In a coordinate plane, two nonvertical lines are parallel if they have the same slope.
- **Perpendicular Lines Theorem:** In a coordinate plane, two nonvertical lines are perpendicular if the product of their slopes is -1.
- **Two-Transversals Proportionality Corollary:** If three or more parallel lines intersect two transversals, then they divide the transversals proportionally.

Triangle postulates and theorems

- **Angle-Angle (AA) Similarity Postulate:** If two angles of one triangle are equal in measure to two angles of another triangle, then the two triangles are similar
- **Side-Side-Side (SSS) Similarity Theorem:** If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.

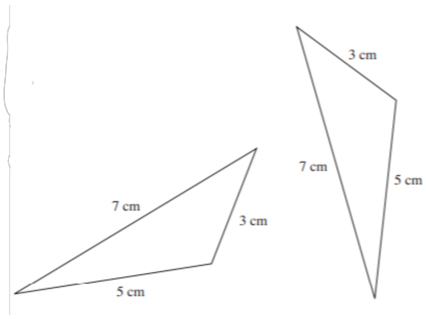
- **Side-Angle-Side (SAS) Similarity Theorem:** If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.
- **Third Angles Theorem:** If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent
- **Side-Angle-Side Congruence Postulate (SAS):** If two sides and the included angle of one triangle are equal in measure to the corresponding sides and angle of another triangle, then the triangles are congruent.
- **Side-Side-Side Congruence Postulate (SSS):** If three sides of one triangle are equal in measure to the corresponding sides of another triangle, then the triangles are congruent
- **Angle-Side-Angle Congruence Postulate (ASA):** If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
- **Triangle Sum Theorem:** The sum of the measure of the angles of a triangle is 180
- **Corollary:** The acute angles of a right triangle are complementary.
- **Exterior Angle Theorem:** An exterior angle of a triangle is equal in measure to the sum of the measures of its two remote interior angles.
- **Triangle Proportionality Theorem:** If a line parallel to a side of a triangle intersects the other two sides, then it divides those sides proportionally.
- **Converse of Triangle Proportionality Theorem:** If a line divides two sides of a triangle proportionally, then it is parallel to the third side.
- **Triangle Angle Bisector Theorem:** An angle bisector of a triangle divides the opposite sides into two segments whose lengths are proportional to the lengths of the other two sides.
- **Angle-Angle-Side Congruence Theorem (AAS):** If two angles and a non-included side of one triangle are equal in measure to the corresponding angles and side of another triangle, then the triangles are congruent.

- **Hypotenuse Leg Congruence Theorem (HL):** If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.
- **Isosceles Triangle Theorem:** If two sides of a triangle are equal in measure, then the angles opposite those sides are equal in measure
- **Converse of Isosceles Triangle Theorem:** If two angles of a triangle are equal in measure, then the sides opposite those angles are equal in measure
- **Corollary 1:** If a triangle is equilateral, then it is equiangular
- **Corollary 2**: The measure of each angle of an equiangular triangle is 60
- **Corollary 3:** If a triangle is equiangular, then it is also equilateral
- **The Inscribed Similar Triangles Theorem:** If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.
- **Pythagorean theorem:** In any right triangle, the square of the length of the hypotenuse is equal to the sum of the square of the lengths of the legs. $a^2 + b^2 = c^2$ (c is the hypotenuse and a and b are the lengths of the legs)
- **Geometric Means Corollary a:** The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments of the hypotenuse.
- **Geometric Means Corollary b:** The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse adjacent to that leg.
- **Circumcenter Theorem:** The circumcenter of a triangle is equidistant from the vertices of the triangle.
- **Incenter Theorem:** The incenter of a triangle is equidistant from the sides of the triangle.
- **Centroid Theorem:** The centroid of a triangle is located 2/3 of the distance from each vertex to the midpoint of the opposite side.
- **Triangle Midsegment Theorem:** A midsegment of a triangle is parallel to a side of triangle, and its length is half the length of that side.

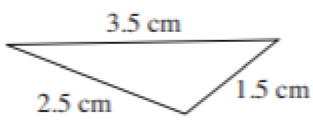
- **Theorem:** If two sides of a triangle are not congruent, then the larger angle is opposite the longer side.
- **Theorem:** If two angles of a triangle are not congruent, then the longer side is opposite the larger angle.
- **Triangle Inequality Theorem:** The sum of any two side lengths of a triangle is greater than the third side length.
- **Hinge Theorem:** If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the longer third side is across from the larger included angle.
- **Converse of Hinge Theorem:** If two sides of one triangle are congruent to two sides of another triangle and the third sides are not congruent, then the larger included angle is across from the longer third side.
- **Converse of the Pythagorean Theorem:** If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.
- **Pythagorean Inequalities Theorem:** In ¨ABC, c is the length of the longest side. If $c^2 > a^2 + b^2$, then ¨ABC is an obtuse triangle. If $c^2 < a^2 + b^2$, then ¨ABC is acute.
- 45˚-45˚-90˚ Triangle Theorem: In a 45˚-45˚-90˚ triangle, both legs are congruent, and the length of the hypotenuse is the length of a length times the square root of 2.
- **30˚-60˚-90˚ Triangle Theorem:** In a 30˚-60˚-90˚ triangle, the length of the hypotenuse is 2 times the length of the shorter leg, and the length of the longer leg is the length of the shorter leg times the square root of 3.

Congruence:

Two shapes are said to be congruent if they are the same shape and size: that is, the corresponding sides of both shapes are the same length and corresponding angles are the same.

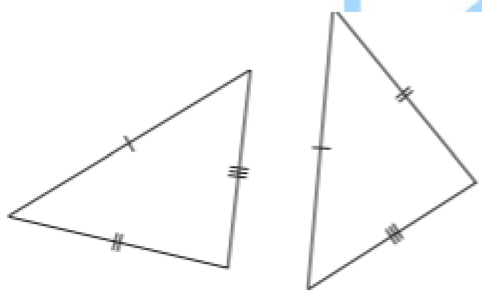


Shapes which are of different sizes but which have the same shape are said to be similar. The triangle below is similar to the triangles above but because it is a different size it is not congruent to the triangles above.



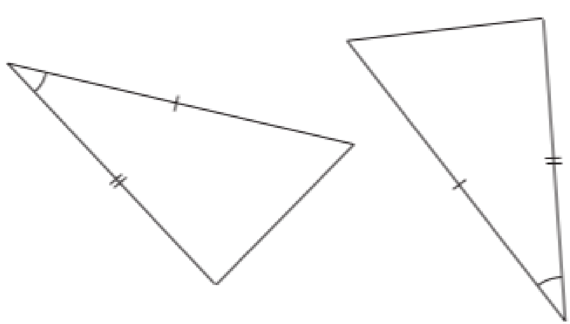
There are four tests for congruence:

- TEST 1 (Side, Side, Side): If all three sides of one triangle are the same as the lengths of the sides of the second triangle, then the two triangles are congruent. This test is referred to as SSS

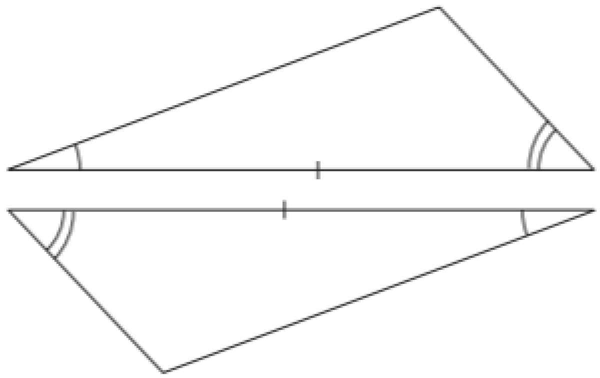


.

- TEST 2 (Side, Angle, Side) If two sides of one triangle are the same length as two sides of the other triangle and the angle between these two sides is the same in both triangles, then the triangles are congruent. This test is referred to as SAS.

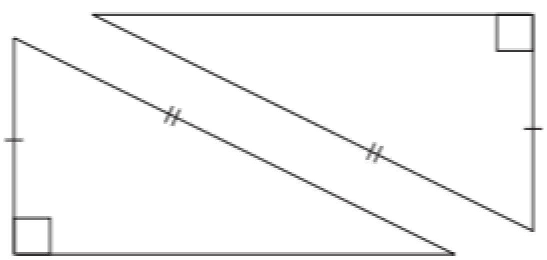


- TEST 3 (Angle, Angle, Side) If two angles and the length of one corresponding side are the same in both triangles, then they are congruent. This test is referred to as AAS



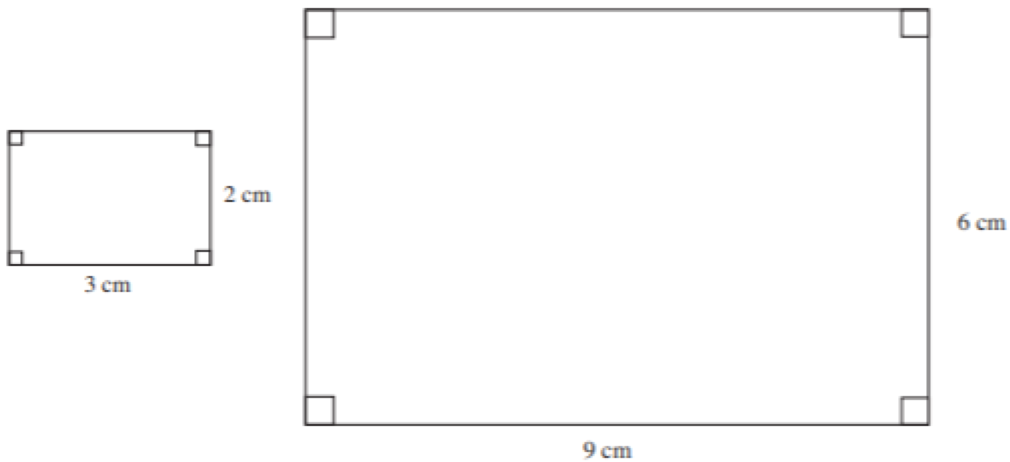
.

- TEST 4 (Right angle, Hypotenuse, Side) If both triangles contain a right angle, have hypotenuses of the same length and one other side of the same length, then they are congruent. This test is referred to as RHS.



Similarity:

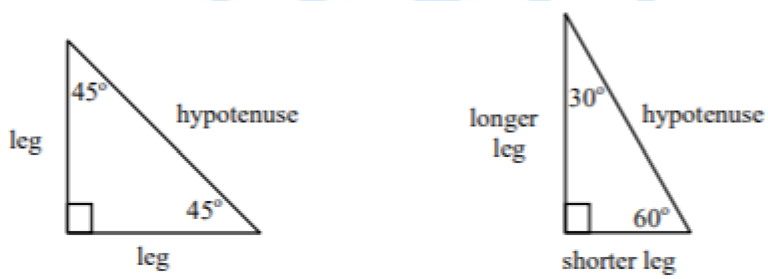
Similar shapes have the same shape but may be different sizes. The two rectangles shown below are similar – they have the same shape but one is smaller than the other. They are similar because they are both rectangles and the sides of the larger rectangle are three times longer than the sides of the smaller rectangle.



Right Angle

Trigonometry:

In Trigonometry, we frequently deal with angle measures that are multiples of 30 , 45 , and 60 . Because of this fact, there are two special right triangles which are useful to us as we begin our study of Trigonometry. These triangles are named by the measures of their angles, and are known as 45 -45 -90 triangles and 30 -60 -90 triangles. A diagram of each triangle is shown below:



In a 45 -45 -90 triangle, the legs are congruent, and the length of the hypotenuse is sqrt2 times the length of either leg.

In a 30 -60 -90 triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is sqrt3 times the length of the shorter leg.

Trigonometric Ratios:

The three basic trigonometric ratios are defined in the table below. The symbol θ , pronounced “theta”, is a Greek letter which is commonly used in Trigonometry to represent an angle, and is used in the following definitions. Treat it as you would any other variable.

Trigonometric Function		Abbreviation		Ratio of the Following Lengths
The sine of θ	=	$\sin(\theta)$	=	$\frac{\text{The leg opposite angle } \theta}{\text{The hypotenuse}}$
The cosine of θ	=	$\cos(\theta)$	=	$\frac{\text{The leg adjacent to angle } \theta}{\text{The hypotenuse}}$
The tangent of θ	=	$\tan(\theta)$	=	$\frac{\text{The leg opposite angle } \theta}{\text{The leg adjacent to angle } \theta}$

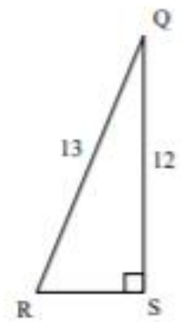
SOH-CAH-TOA

SOH stands for **s**in(θ), **O**pposite, **H**ypotenuse: $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH stands for **c**os(θ), **A**djacent, **H**ypotenuse: $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA stands for **t**an(θ), **O**pposite, **A**djacent: $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

Example
Find the sine, cosine, and tangent ratios for each of the acute angles in the following triangle.



Solution:

We first find the missing length of side *RS*.
Solving the equation $(RS)^2 + 12^2 = 13^2$, we obtain $RS = 5$.

We then find the three basic trigonometric ratios for angle R:

$$\sin R = \frac{\text{The leg opposite angle R}}{\text{The hypotenuse}} = \frac{12}{13}$$

$$\cos R = \frac{\text{The leg adjacent to angle R}}{\text{The hypotenuse}} = \frac{5}{13}$$

$$\tan R = \frac{\text{The leg opposite angle R}}{\text{The leg adjacent to angle R}} = \frac{12}{5}$$

We then find the three basic trigonometric ratios for angle Q:

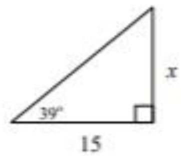
$$\sin Q = \frac{\text{The leg opposite angle Q}}{\text{The hypotenuse}} = \frac{5}{13}$$

$$\cos Q = \frac{\text{The leg adjacent to angle Q}}{\text{The hypotenuse}} = \frac{12}{13}$$

$$\tan Q = \frac{\text{The leg opposite angle Q}}{\text{The leg adjacent to angle Q}} = \frac{5}{12}$$

Finding missing sides of right triangles:

In Trigonometry, there are two basic types of angle measure known as degrees and radians. In this text, we will be using only degree measure, so you should make sure that your calculator is in degree mode.

1. 

<u>Solution:</u>
Using the 39° angle as our reference angle, x is the length of the opposite leg and 15 is the length of the adjacent leg. Therefore, we will use the tangent ratio:

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan(39^\circ) = \frac{x}{15}$$

$$x = 15 \cdot \tan(39^\circ)$$ *(Enter this into the calculator; make sure first that you are in degree mode.)*

$$x \approx 12.1468$$

Reciprocal trig ratios:

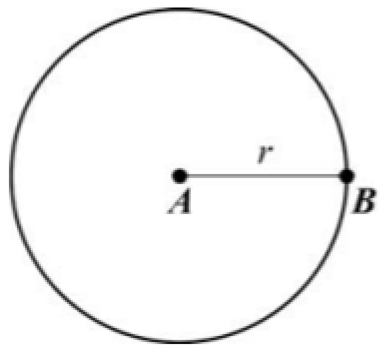
There are three other trigonometric ratios called the reciprocal trigonometric ratios, and they are defined as follows:

Trigonometric Function		Abbreviation		Reciprocal Relationship		Ratio of the Following Lengths
The cosecant of θ	=	$\csc(\theta)$	=	$\frac{1}{\sin(\theta)}$	=	$\frac{\text{The hypotenuse}}{\text{The leg opposite angle } \theta}$
The secant of θ	=	$\sec(\theta)$	=	$\frac{1}{\cos(\theta)}$	=	$\frac{\text{The hypotenuse}}{\text{The leg adjacent to angle } \theta}$
The cotangent of θ	=	$\cot(\theta)$	=	$\frac{1}{\tan(\theta)}$	=	$\frac{\text{The leg adjacent to angle } \theta}{\text{The leg opposite angle } \theta}$

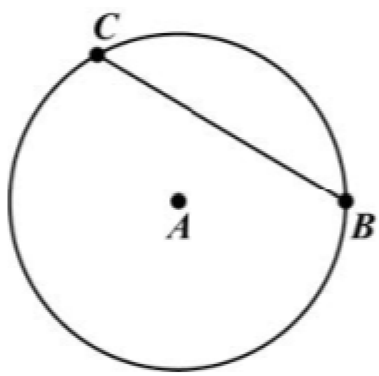
Circles:

- A circle is the set of points in a plane equidistant from a given point, which is the center of the circle. All circles are similar.

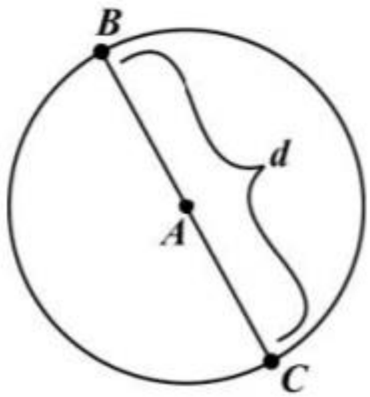
- A **radius** is a line segment from the center of a circle to any point on the circle. The word radius is also used to describe the length, r, of the segment. AB is a radius of circle A.



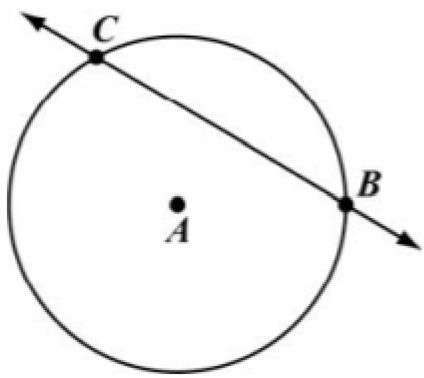
- A **chord** is a line segment whose endpoints are on a circle. BC is a chord of circle A.



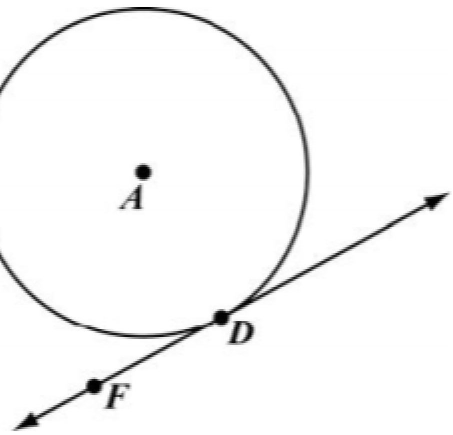
- A **diameter** is a chord that passes through the center of a circle. The word diameter is also used to describe the length, d, of the segment. BC is a diameter of circle A.



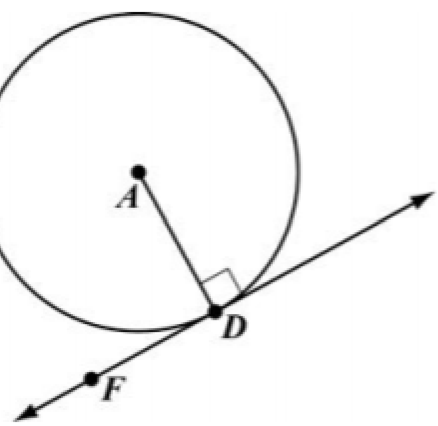
- A **secant** line is a line that is in the plane of a circle and intersects the circle at exactly two points. Every chord lies on a secant line. BC is a secant line of circle A



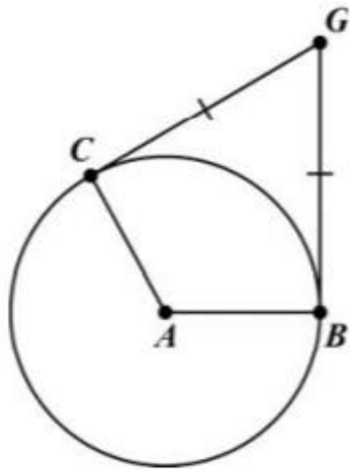
- A **tangent** line is a line that is in the plane of a circle and intersects the circle at only one point, the point of tangency. DF is tangent to circle A at the point of tangency, point D.



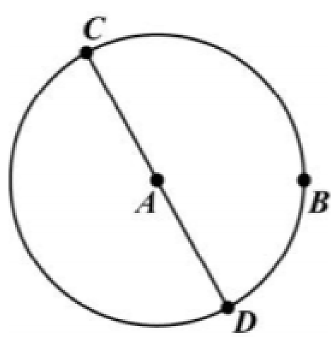
- If a line is **tangent to a circle**, the line is perpendicular to the radius drawn to the point of tangency. DF is tangent to circle A at point D, so AD D ⊥ F.



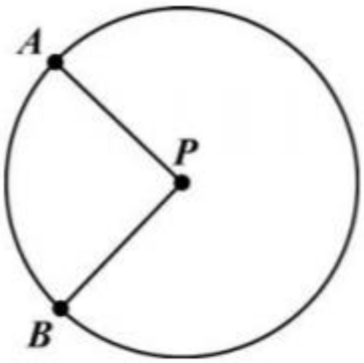
- **Tangent segments** drawn from the same point are congruent. In circle A, CG ≅ BG .



- **Circumference** is the distance around a circle. The formula for circumference C of a circle is C = πd, where d is the diameter of the circle. The formula is also written as C = 2πr, where r is the length of the radius of the circle. π is the ratio of circumference to diameter of any circle.
- An **arc** is a part of the circumference of a circle. A **minor arc** has a measure less than 180°. Minor arcs are written using two points on a circle. A semicircle is an arc that measures exactly 180°. Semicircles are written using three points on a circle. This is done to show which half of the circle is being described. A **major arc** has a measure greater than 180°. Major arcs are written with three points to distinguish them from the corresponding minor arc. In circle A, CB is a minor arc, CBD is a semicircle, and CDB is a major arc.

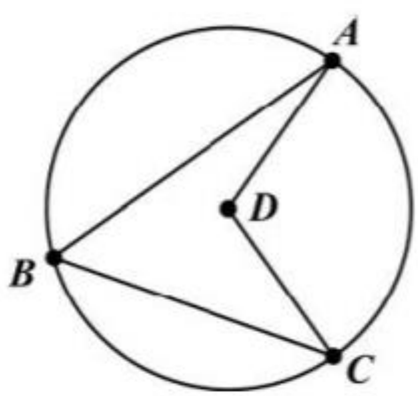


- A **central angle** is an angle whose vertex is at the center of a circle and whose sides are radii of the circle. The measure of a central angle of a circle is equal to the measure of the intercepted arc. ∠APB is a central angle for circle P, and AB is the intercepted arc.



$$m\angle APB = m\widehat{AB}$$

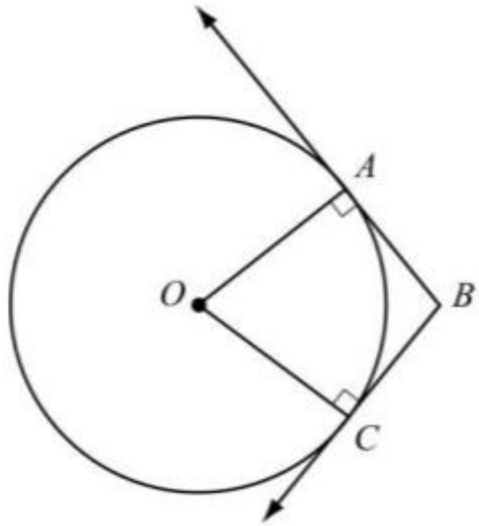
- An **inscribed angle** is an angle whose vertex is on a circle and whose sides are chords of the circle. The measure of an angle inscribed in a circle is half the measure of the intercepted arc. For circle D, ∠ABC is an inscribed angle, and AC is the intercepted arc.



$$m\angle ABC = \frac{1}{2}m\widehat{AC} = \frac{1}{2}\,m\angle ADC$$

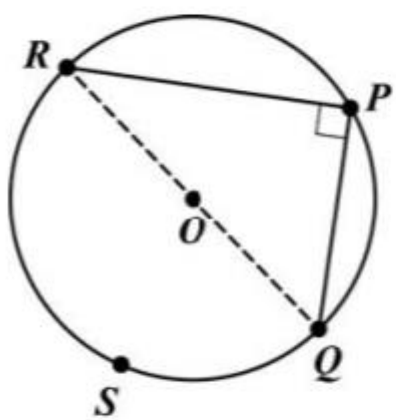
$$m\angle ADC = m\widehat{AC} = 2(m\angle ABC)$$

- A **circumscribed angle** is an angle formed by two rays that are each tangent to a circle. These rays are perpendicular to radii of the circle. In circle O, the measure of the circumscribed angle is equal to 180° minus the measure of the central angle that forms the intercepted arc. The measure of the circumscribed angle can also be found by using the measures of two intercepted arcs.



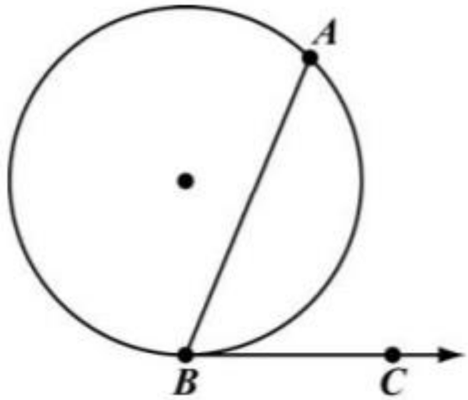
$$m\angle ABC = 180° - m\angle AOC$$

- When an inscribed angle intercepts a semicircle, the inscribed angle has a measure of 90°. For circle O, $\angle RPQ$ intercepts semicircle RSQ as shown.



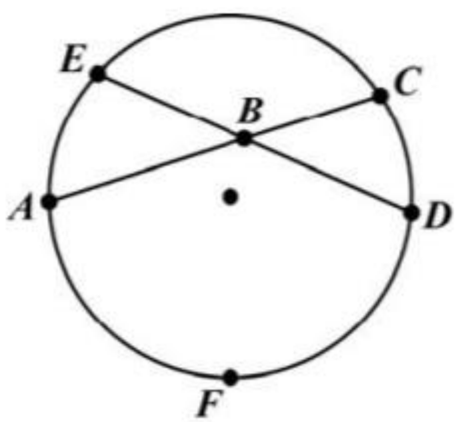
$$m\angle RPQ = \frac{1}{2}\left(m\widehat{RSQ}\right) = \frac{1}{2}(180°) = 90°$$

- The measure of an angle formed by a tangent and a chord with its vertex on the circle is half the measure of the intercepted arc. AB is a chord for the circle, and BC is tangent to the circle at point B. So, ∠ABC is formed by a tangent and a chord.



$$m\angle ABC = \frac{1}{2}\left(m\widehat{AB}\right)$$

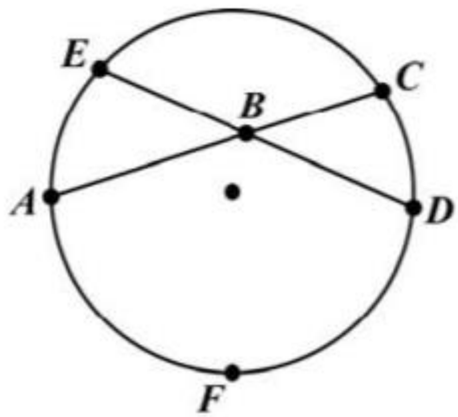
- When two chords intersect inside a circle, two pairs of vertical angles are formed. The measure of any one of the angles is half the sum of the measures of the arcs intercepted by the pair of vertical angles.



$$m\angle ABE = \frac{1}{2}\left(m\widehat{AE} + m\widehat{CD}\right)$$

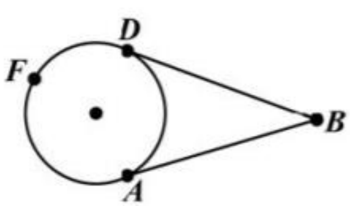
$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} + m\widehat{EC}\right)$$

- When two chords intersect inside a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

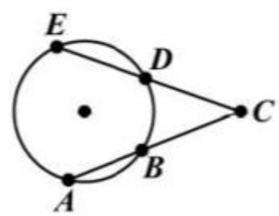


$$AB \cdot BC = EB \cdot BD$$

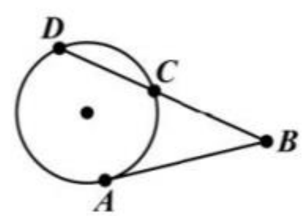
- Angles outside a circle can be formed by the intersection of two tangents (circumscribed angle), two secants, or a secant and a tangent. For all three situations, the measure of the angle is half the difference of the measure of the larger intercepted arc and the measure of the smaller intercepted arc.



$$m\angle ABD = \frac{1}{2}\left(m\widehat{AFD} - m\widehat{AD}\right)$$

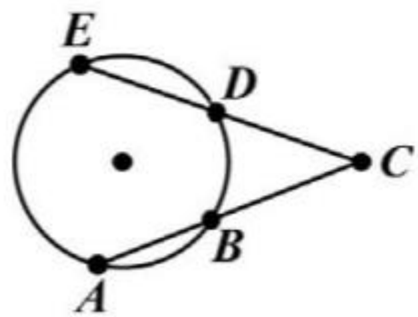


$$m\angle ACE = \frac{1}{2}\left(m\widehat{AE} - m\widehat{BD}\right)$$



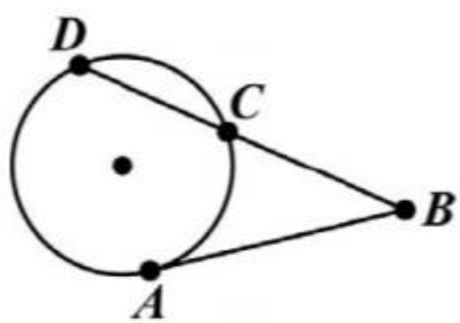
$$m\angle ABD = \frac{1}{2}\left(m\widehat{AD} - m\widehat{AC}\right)$$

- When two secant segments intersect outside a circle, part of each secant segment is a segment formed outside the circle. The product of the length of one secant segment and the length of the segment formed outside the circle is equal to the product of the length of the other secant segment and the length of the segment formed outside the circle.



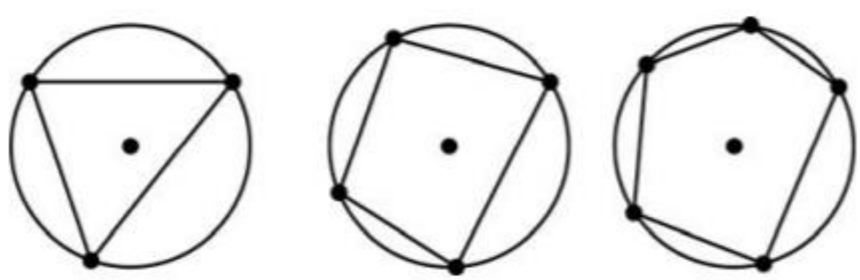
$$EC \cdot DC = AC \cdot BC$$

- When a secant segment and a tangent segment intersect outside a circle, the product of the length of the secant segment and the length of the segment formed outside the circle is equal to the square of the length of the tangent segment.

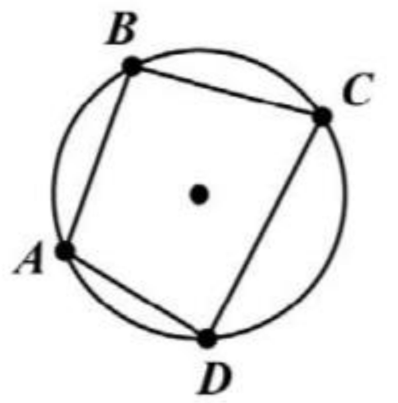


$$DB \cdot CB = AB^2$$

- An inscribed polygon is a polygon whose vertices all lie on a circle. This diagram shows a triangle, a quadrilateral, and a pentagon each inscribed in a circle.



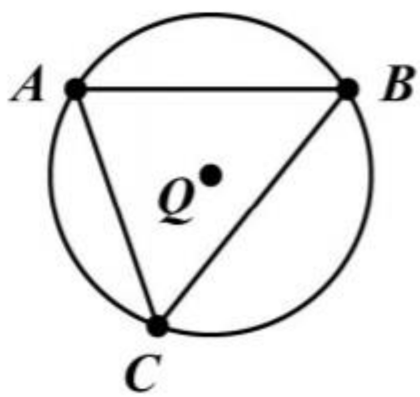
- In a quadrilateral inscribed in a circle, the opposite angles are supplementary.



$m\angle ABC + m\angle ADC = 180°$

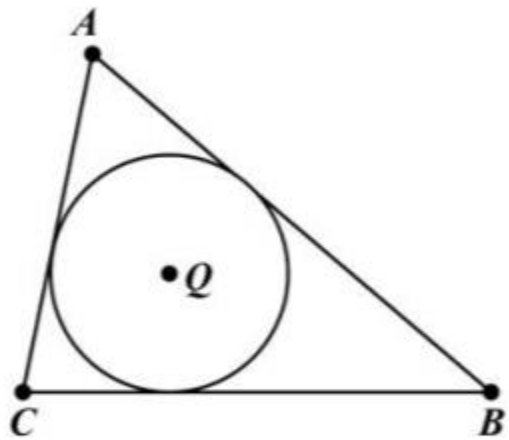
$m\angle BCD + m\angle BAD = 180°$

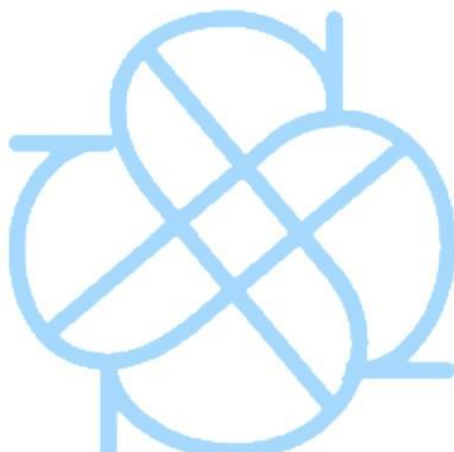
- When a triangle is inscribed in a circle, the center of the circle is the circumcenter of the triangle. The circumcenter is equidistant from the vertices of the triangle. Triangle ABC is inscribed in circle Q, and point Q is the circumcenter of the triangle.



$AQ = BQ = CQ$

- An inscribed circle is a circle enclosed in a polygon, where every side of the polygon is tangent to the circle. Specifically, when a circle is inscribed in a triangle, the center of the circle is the incenter of the triangle. The incenter is equidistant from the sides of the triangle. Circle Q is inscribed in triangle ABC, and point Q is the incenter of the triangle. Notice also that the sides of the triangle form circumscribed angles with the circle.





Citations:

https://www.lcps.org/cms/lib/VA01000195/Centricity/Domain/8538/Geometry%20Study%20Guide.pdf

https://ccps.ss10.sharpschool.com/UserFiles/Servers/Server_54431/File/Resources/GMASGuides/10th%20Grade%20Geometry/Geometry%20Study%20Guide%20and%20Rubrics.pdf
http://www.cimt.org.uk/mepjamaica/unit33/StudentText.pdf
https://online.math.uh.edu/MiddleSchool/Modules/Module_4_Geometry_Spatial/Content/RightTriangleTrigonometry-TEXT.pdf
https://cpb-us-e1.wpmucdn.com/cobblearning.net/dist/0/2000/files/2016/08/U4-Circles-and-Volume-GADOE-23baw1p.pdf