# AP Statistics: Hypothesis Testing (Means & Proportions)

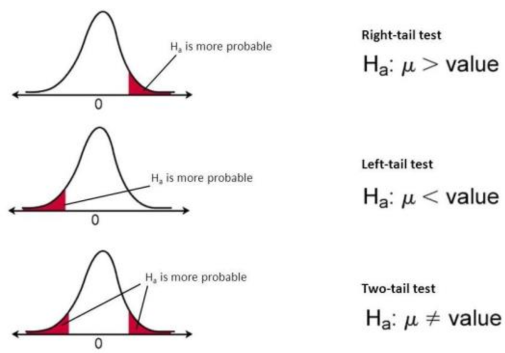
**From Simple Studies, [https://simplestudies.edublogs.org](https://simplestudies.edublogs.org) & @simplestudiesinc on Instagram**

## Hypothesis Testing

- A hypothesis test is used to see whether an assumption is statistically plausible by using sample data
- The basic formula for a hypothesis test is: $^{\text{Statistic - Parameter}}/_{\text{Standard Deviation of Statistic}}$
- The higher the Z or t score, the lower the p value, and the more evidence there is to reject the null hypothesis

## Five Steps

1. Hypothesis
2. Conditions/Assumptions
3. Formula
4. P Value
5. Conclusion




From HYPERLINK "https://towardsdatascience.com/everything-you-need-to-know-about-hypothesis-testing-part-i-

## Step 1: Hypothesis

| Hypothesis | One-Sample Mean | Two-Sample Mean | One-Sample Proportion | Two-Sample Proportion |
|---|---|---|---|---|
| $H_0$ | $\mu = x$ | $\mu_1 = \mu_2$ | $p = x$ | $p_1 = p_2$ |
| $H_a$ | $\mu$ < or > or $\neq$ x | $\mu_1$ < or > or $\neq$ $\mu_2$ | p < or > or $\neq$ x | $p_1$ < or > or $\neq$ $p_2$ |

- "Where [$\mu$, $p$, $\mu_1$, $\mu_2$, $p_1$, $p_2$] is [*context of problem*]"
  - Define ALL parameters in the context of the problem
- Whether $H_a$ is <, >, or $\neq$ depends on the problem

## Step 2: Conditions/Assumptions

- Random Sample
  - "The stem of the problem states that [*sample*] was chosen at random"
  - "The stem of the problem states that [*participants*] were randomly assigned to the groups"
- Approximate Normal Distribution
  - "The stem of the problem states the distribution is approximately normal"
  - "Since n = _ ≥ 30, by the Central Limit Theorem, we can assume the distribution is approximately normal"
    - For two samples, both $n_1$ and $n_2$ must be ≥ 30
  - "Since np = _ ≥ 10 and nq = _ ≥ 10, we can assume the distribution is approximately normal"
    - For two sample, $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, and $n_2\hat{q}_2$ must all be ≥ 10
  - "Since the [*graphical display*] shows no outliers or strong skewness, we can assume the distribution is approximately normal"
    - You must provide a graphical display (preferably a box plot) if normality cannot be assumed by the other three ways

## Step 3: Formula

- List the formula, your substitution, degrees of freedom (if using t) and your unrounded answer
  - One-Sample Means $Z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ $t = \frac{\overline{x} - \mu}{\frac{S}{\sqrt{n}}}$
  - Two-Sample Means

$$Z = \frac{(\overline{x}_1 - \overline{x}_1) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}} + \sqrt{\frac{\sigma_2^2}{n_2}}} \qquad t = \frac{(\overline{x}_1 - \overline{x}_1) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1}} + \sqrt{\frac{S_2^2}{n_2}}}$$

    - Proportions

$$Z = \frac{\widehat{p} - p}{\sqrt{\frac{pq}{n}}} \qquad Z = \frac{(\widehat{p}_1 - \widehat{p}_2) - (p_1 - p_2)}{\sqrt{(\widehat{p}_c)(\widehat{q}_c)(\frac{1}{n_1} + \frac{1}{n_2})}} \qquad \widehat{p}_c = \frac{X_1 + X_2}{n_1 + n_2}$$

## Step 4: P Value

- The probability of obtaining a test statistic (Z or t) that is this much or more extreme
- If $H_a$ is <
    - P (Z < _)
- If $H_a$ is >
    - P (Z > _)
- If $H_a$ is ≠
    - 2P (Z > _) if Z > 0
    - 2P (Z < _) if Z < 0

## Step 5: Conclusion

- "Assuming $H_0$ is true, since the p value ([*p value*]) is [*greater/less*] than α = _, we [*fail to reject/reject*] $H_0$"
    - A α will usually be given in the problem. If it is not, use α = .05
- "We [*do not/do*] have sufficient evidence to suggest $H_a$, that [*context of problem*]"

## Type I and II Errors

- Type I Error: You reject $H_0$ when you should not have
    - P (Type I Error) = α
- Type II Error: You fail to reject $H_0$ when you should have
    - P (Type II Error) = β
    - Power of the test = 1 - β
        - P (Rejecting $H_0$ when you should have)
        - Increases as α increases
- Which one is worse depends on the scenario

## Match Paired t-Testing

- Most often used in a "before and after" scenario (e.g. dexterity before and after the subjects undergo a program) and comparing two things (e.g. amount of active ingredient in a name brand and generic brand drug).
- Hypothesis
    - $H_0$: $\mu_d = 0$
    - $H_a$: $\mu_d$ > or < or ≠ 0
    - *Where $\mu_d$ is the average difference (After - Before)
- Assuming Normality
    - Draw a box plot of A - B
- Compute the Match Paired t-test as if it were a normal hypothesis test (5 steps!)

## Z vs t Distribution

- Means
    - Use a Z distribution when you have σ
    - Use a t distribution when you do not have σ (i.e. you have S)
- Proportions
    - ALWAYS use a Z distribution

## Degrees of Freedom

- Only applies to t-distributions
    - The t-distribution varies with degrees of freedom
- df = n - 1
- For a Z-distribution, df = ∞

## Calculator (TI-84 Plus)

- Stat → Test
    - Means
        - 1 = One-Sample Z-Test
        - 2 = One-Sample t-Test

    - 3 = Two-Sample Z-Test
    - 4 = Two-Sample t-Test
  - Proportions
    - 5 = One-Sample
    - 6 = Two-Sample