

AP Statistics: Confidence Intervals (Means & Proportions)

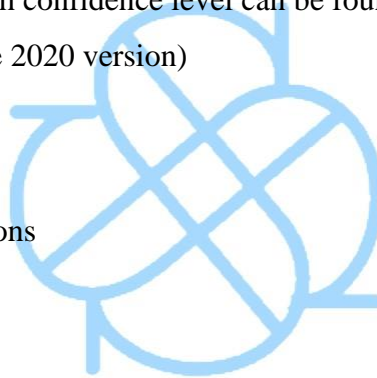
From Simple Studies, <https://simplestudies.edublogs.org> & @simplestudiesinc on Instagram

Confidence Interval Definition

- A confidence interval is a range of values obtained from a sample that is likely to contain the parameter
- The basic formula for a confidence interval is: $Statistic \pm (Critical Value)(Standard Deviation of Statistic)$
- The margin of error is defined by the Critical Value (Standard Deviation of Statistic)
- The critical values for each confidence level can be found on Table B of the AP Statistics formula sheets (p. 5 of the 2020 version)

Three Steps

1. Conditions/Assumptions
2. Formula
3. Interpretation



Step 1: Conditions/Assumptions

- Random Sample
 - “The stem of the problem states that [*sample*] was chosen at random”
 - “The stem of the problem states that [*participants*] were randomly assigned to the groups”
- Approximately Normal Distribution
 - “The stem of the problem states that the distribution is approximately normal”
 - “Since $n = _ \geq 30$, by the Central Limit Theorem, we can assume the distribution is approximately normal”
- For two samples, both n_1 and n_2 must be ≥ 30

- “Since $np = _ \geq 10$ and $nq = _ \geq 10$, we can assume the distribution is approximately normal”
- For two samples, $n_1\hat{p}_1$, $n_1\hat{q}_1$, $n_2\hat{p}_2$, and $n_2\hat{q}_2$ must all be ≥ 10
 - “Since the [graphical display] shows no outliers or strong skewness, we can assume the distribution is approximately normal”
- You must provide a graphical display (preferably a box plot) if normality cannot be assumed by the other three ways

Step 2: Formula

- List the formula, your substitution, degrees of freedom (if using t^*) and your unrounded answer

- One-Sample Means $\bar{x} \pm \frac{Z_c\sigma}{\sqrt{n}}$ $\bar{x} \pm \frac{t^*S}{\sqrt{n}}$
- Two-Sample Means

$$\bar{x} \pm \frac{Z_c\sigma}{\sqrt{n}} \quad \bar{x} \pm \frac{t^*S}{\sqrt{n}}$$

- Proportions

$$(\bar{x}_1 - \bar{x}_2) \pm Z_c\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \quad (\bar{x}_1 - \bar{x}_2) \pm t^*\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$\hat{p} \pm Z_c\sqrt{\frac{\hat{p}\hat{q}}{n}} \quad (\hat{p}_1 - \hat{p}_2) \pm Z_c\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Step 3: Interpretation

- “Based on these samples, we are $_ \%$ confident that the true [context of problem] is between [lower value] and [upper value]”
- “Since zero [is/is not] in the $_ \%$ confidence interval [interval], we [do not/do] have sufficient evidence to suggest there is a difference at the $\alpha = [1 - _ \%]$ level”
- Always put the $\%$ as a decimal (e.g. $95\% = .95$)
- Use this second statement for two-sample confidence intervals only

Z vs t Distribution

- Means
 - Use a Z distribution when you have σ
 - Use a t distribution when you do not have σ (i.e. you have S)
- Proportions
 - ALWAYS use a Z distribution

Degrees of Freedom

- Only applies to t-distributions
 - The t-distribution varies with degrees of freedom
- $df = n - 1$
- For a Z-distribution, $df = \infty$

Finding Critical Values

- AP Statistics formula sheet table B
- TI-84 Plus
 - 2nd → Vars
 - 3 = Z-distribution
 - 4 = t-distribution
 - Area = $\frac{1}{2}(1 - \%)$
 - Always put the % as a decimal (e.g. 95% = .95)

Calculator (TI-84 Plus)

- Stat → Test
 - Means
 - 7 = 1-sample Z-Interval
 - 8 = 1-sample t-Interval
 - 9 = 2-sample Z-Interval
 - 10 = 2-sample t-Interval

- Proportions
 - A = 1-Sample
 - B = 2-Sample

