AP Calculus BC

From Simple Studies: https://simplestudies.edublogs.org and @simplestudiesinc on Instagram

UNIT 7: Applications of Integration

(Volume, Cross Sectionals, Riemanns)

Area and Volume

Volumes of Solids with known cross sectionals

1. For cross sections of area A(x) taken perpendicular to the x-axis

volume=$\int_{\square}^{\square}$ A(x) dx

2. For cross sections of area A(y) taken perpendicular to the y-axis.

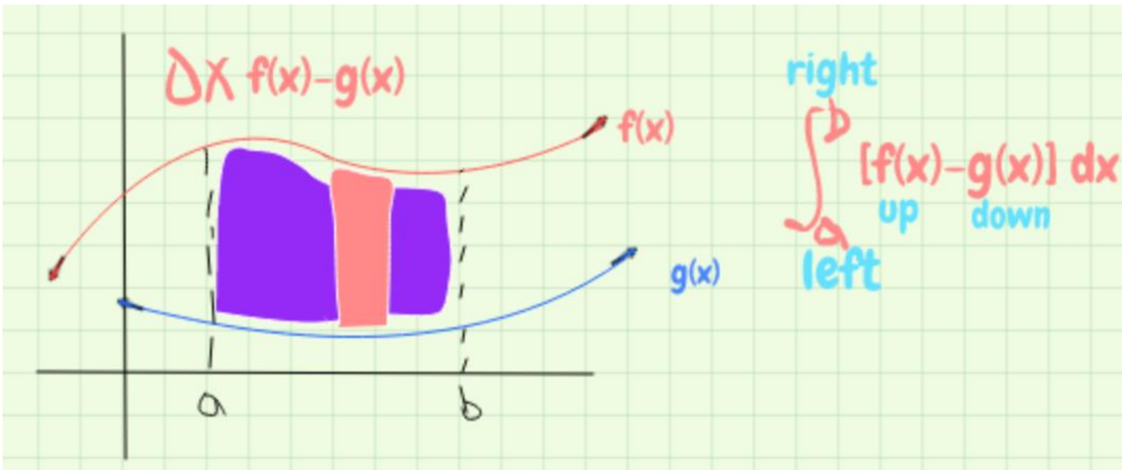
   Volume =$\int_{c}^{d}$ A(y)dy

If you wanna do a quick check:

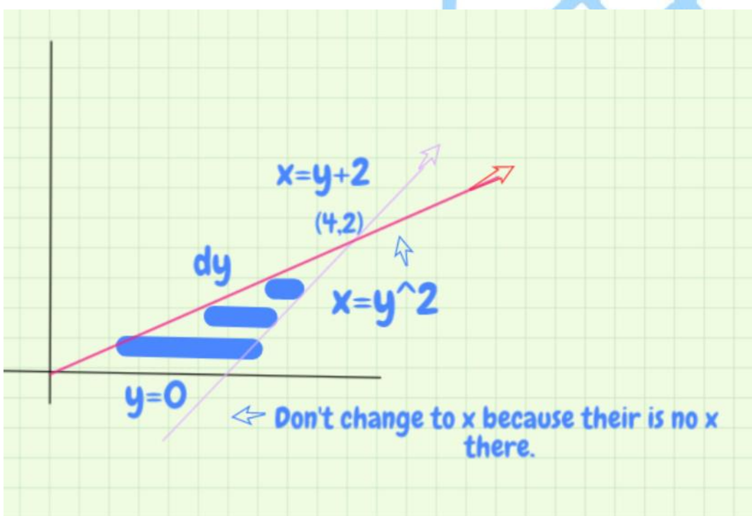
- set up, but do not evaluate the integrals that would compute the volume of the solid

Finding the Area Between Curves Expressed as Functions of x

Whenever we are talking about areas between two curves, we are talking about the TOTAL POSITIVE AREA between them.



EX: example of how it would look like in another way with y



Top-consistent(RED)

Bottom- In Consistent

Left-Consistent(Red)

Right-Consistent(purple)

The Disk Method

A similar formula can be derived if the axis of revolution is vertical.

THE DISK METHOD

To find the volume of a solid of revolution with the **disk method,** use one of the following, as shown in Figure 7.15.

Horizontal Axis of Revolution	*Vertical Axis of Revolution*
Volume $= V = \pi \int_a^b [R(x)]^2\,dx$	Volume $= V = \pi \int_c^d [R(y)]^2\,dy$

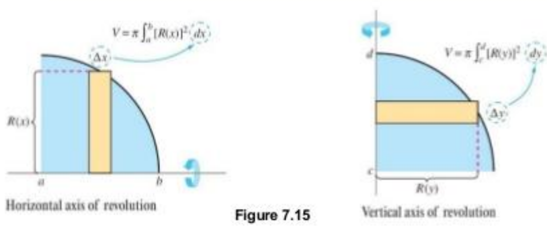


Figure 7.15

11

Example 1 – *Using the Disk Method*

Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{\sin x}$ and the x-axis ($0 \le x \le \pi$) about the x-axis.

Solution:

From the representative rectangle in the upper graph in Figure 7.16, you can see that the radius of this solid is

$$R(x) = f(x) = \sqrt{\sin x}.$$

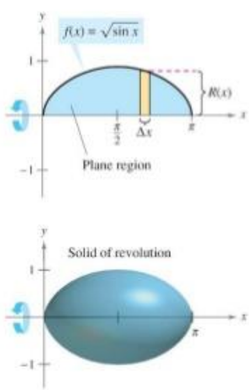


Figure 7.16

12

The volume Equation you need to memorize is:

V=A*H

A stands for the area of your main cross section square→ $A = x^2$

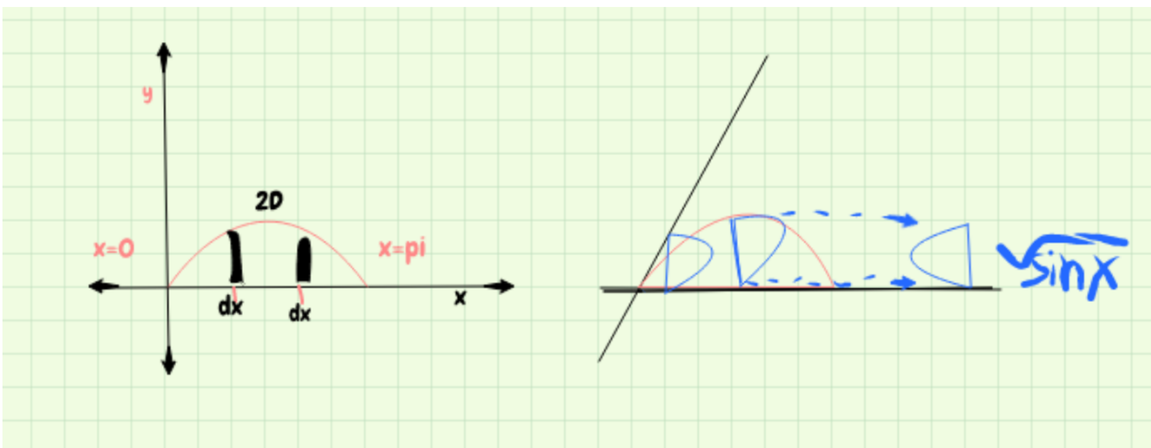
rectangular→ $A = l * w$

triangle→ $A = \frac{1}{2}$b*h

circle→ $A = \pi r^2$

trapezoid→ $A = \frac{1}{2}(b_1 \ + b_2 \ )h$

Ex: Cross sectional of semicircle perpendicular to the x-axis are laid on the area bounded by y=$\sqrt{sin\ x}$,the x-axis, and between x=0 and x=π.



$V = \int_a^b \quad A * dx$ r=$\frac{1}{2}\sqrt{sin\ x}$

=$\int_0^\pi \quad \frac{\pi}{8}$sin dx A=$\frac{1}{2}(\pi r^2)$

=$\frac{\pi}{8}\int_0^\pi$ sin dx =$\frac{1}{2}\pi(\frac{1}{2}\sqrt{sin\ x}\ )^2$

= -$\frac{\pi}{8}$(cos$\pi - cos\ 0) \ \rightarrow -\frac{\pi}{8}(-1-1)$ =$\frac{\pi}{8}$sin x

=$\frac{2\pi}{8}$=$\frac{\pi}{4}$

7

Terms to use when taking about Riemann Sums

- Subinterval- Each part of the area you are calculating (each of the smaller parts)
- Partition- the area for which you are approximating the area for which you are approximating the area for the whole thing
- Norm - the length of each subinterval→ $\Delta x ORP$

  The norm is probably the most controllable and beneficial thing to manipulate for a riemann sum.

Trapezoid Sum = Average of your left and right riemann sums

If you don't have either sum, you can still candidate the trapezoid sum via the old standard formula

T$T_{sum} = \frac{\Delta x}{2}(h_0 + 2h_1 + 2h_2 + 2h_3 + ... + 2h_{n-1} + h_n)$

$\Delta x = (\frac{b-a}{n})$

Use the tables to determine the areas

x	1	3	4	7	8	10	13
f(x)	2	4	4	1	7	9	10

1) Find the area using 6 subintervals

A] LRAM →2(2) + 1(4) + 3(4) + 1(1) + 2(7) + 3(9) = 62

B] RRAM →2(4) + 1(4) + 3(1) + 1(7) + 2(9) + 3(10)= 70

C] TRAP→ $\frac{L+R}{2} = \frac{62-70}{2} = \frac{132}{2}$ = 66

D] SIMP → N/A Requires a constant ΔX