

AP Calculus BC

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Unit 5: Analytical Applications of Differentiation

Examine $y=x^2$

Determine the absolute extreme for the function above under the following domains:

- a. $(-\infty, \infty) \rightarrow \text{Min } (x = 0, y = 0), \text{ Max} = \text{DNE}$
- b. $[0, 2] \rightarrow \text{Min } (0, 0) \quad \text{Max}(2, 4)$
- c. $(0, 2] \rightarrow \text{Min DNE} \quad \text{Max}(2, 4)$
- d. $(0, 2) \rightarrow \text{Min DNE} \quad \text{Max DNE}$

As long as your domain or interval is continuous over a closed interval- you have brackets- this implies that there MUST be a Maximum AND a Minimum value in that interval This is the Extreme Value Theorem (EVT)

Types of Extrema

Relative/local - Nearby there is no point higher or lower than this point

Absolute- Over the entire closed interval, this is no other point that can challenge the extremeness of this point

Things that are common for all types of extremes

- Your slope should change from one sign to the other
- Your derivative needed to pass through zero
- If the derivative fails to exist, the signs still manage to change

The Mean Value Theorem

If you have a continuous function over the closed interval $[a,b]$ and differentiable over the open interval (a,b) , then there must be a point $(c, f(c))$ such that c is the interval of (a,b) such that...

Formula: $f'(c) = \frac{f(b) - f(a)}{b - a} \leftarrow \text{Average Velocity}_{(\text{secant})}$

How To Use The First Derivative Test, Second Derivative Test, And Candidates Test

FIRST DERIVATIVE TEST

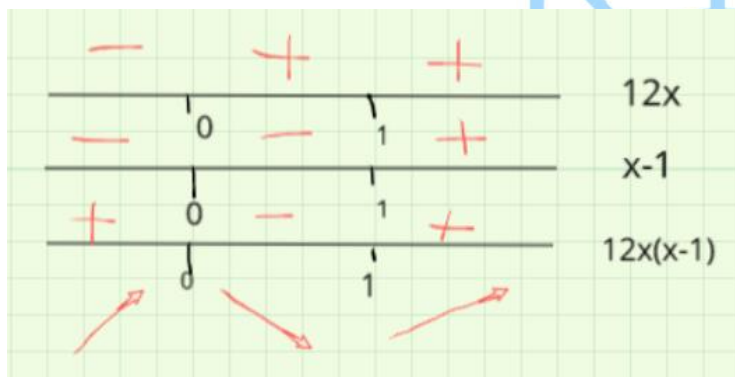
Find the extrema for $f(x) = 4x^3 - 6x^2 + 3$

$$f'(x) = 12x^2 + 12x$$

GOAL- is to determine Max & Min

$$0 = 12x(x-1)$$

$$\{x=0, x=1\}$$



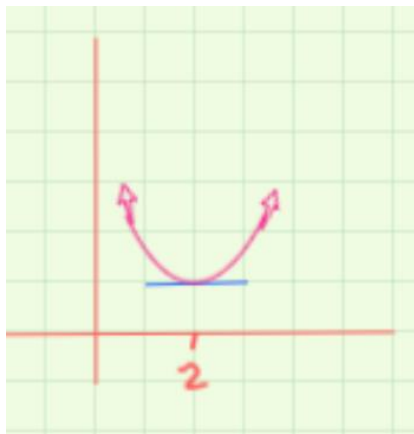
max-0 min-1

THE SECOND DERIVATIVE TEST

It serves if you have a maximum or a minimum for a critical value by using the second derivative

For example: You have a critical value of $x=2$. The concavity of the graph at $x=2$ is concave up.

By the second derivative test, $x=2$ must be a local minimum.



If you have a critical value at $x=a$, then if:

- a. $f'(a) > 0$, you have a minimum
- b. $f'(a) < 0$, you have a maximum
- c. $f'(a) = 0$, the test is inconclusive



Guide to Determine Inc. and Dec. Intervals

A] Find the Derivative

B] Find all the C.P's $\rightarrow f'(x)=0$ or $f'(x)=$ undefined

C] Graphing C.P's on the number line and writing all in interval notation

D] Choose numbers from each interval to decide the sign $[(-) \text{ or } (+)]$ for each interval

E] Using first Derivative Test. Determine the Relative extreme

F] Write final answers in interval notation

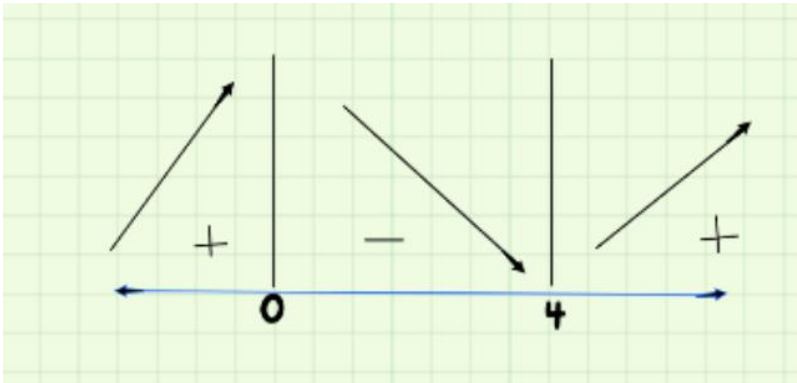
EX: Let $f(x) = x^3 - 6x^2 + 15$

- (a) Find increasing and decreasing
- (b) Find all relative extrema

$$f'(x) = 3x^2 - 12x = 0 \quad (-\infty, 0) \rightarrow x = -1 \rightarrow f'(-1) = 15 \text{ inc.}$$

$$3x(x-4) = 0 \quad (0, 4) \rightarrow x = 2 \rightarrow f'(2) = -12 \text{ dec.}$$

$$x = 0 \quad x = 4 \quad (4, +\infty) \rightarrow x = 5 \rightarrow f'(x) = 15 \text{ inc.}$$



Inc. $\rightarrow (-\infty, 0) \cup (4, +\infty)$

dec. $\rightarrow (0, 4)$



B] Relative max $\rightarrow (0, 15)$

Relative min $\rightarrow (4, -17)$

Newton's Method

Used for finding zeros of a function

- Use tangent at some value to approximate zero.
- The first number you use is a guess that gets refined.
- After repeated attempts, you will get your value

$$\int_a^b f(x) dx = F(b) - F(a)$$

For instance evaluate $\int_1^3 x^3 dx$ you can write $\int_1^3 x^3 dx = \frac{3^4}{4} - \frac{1^4}{4} = 20$

1. It is not necessary to include a constant of integration C in the antiderivative because

$$\int_a^b f(x) dx = [F(b) + c] - [F(a) + c] = F(b) - F(a)$$

FTC: if f is continuous on interval I containing a , then for every x on the interval:

$$\frac{d}{dx} \int_0^x f(t) dt$$

