AP Calculus BC

From Simple Studies: https://simplestudies.edublogs.org and @simplestudiesinc on Instagram

Unit 4: Contextual Applications of Differentiation

Related Rates

4.4 Intro to related rates

In related rates, we are often using implicit differentiation with respect to dt. In these types of problems, one thing changes but we are interested in seeing how something else changes.

EX: In a special balloon, the radius will change at a rate different than the rate in which air is being pushed into the balloon.

V = $\frac{4}{3}\pi r^3$ ←static equation

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \leftarrow \mathit{Dynamic\ Equation}$(analyze change)

Five cubic inches of air per second are blown into the balloon exactly when the radius is 3 inches. At what rate does the radius change?

(Hint: plug into the above Dynamic Equation)

$5=4\pi(3)^2\frac{dr}{dt}$

$\frac{5}{36\pi}=\frac{dr}{dt}$ $\frac{5}{36\pi}$in/sec of expansion

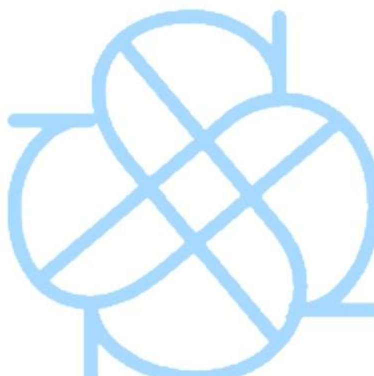
4.5 solving related rates problems

Helpful tips for related rates

1.) Label your variables
2.) Draw your problem out if you don't understand it
3.) Using your diagram, create a relevant equation
4.) Organize your variables even if you labeled them correct

EX:

Lapis is chasing Ruby in a spectacular unicycle race. RUby is ahead by three blocks when Lapis is still lagging behind. Ruby had turned going north and had traveled two blocks north already. Lapis is going west bound and is one block away from the turn. RUby can travel at the first 2 blocks per minute while lapis travels at 3 blocks per minute. What is the rate at which the two girls are being separated?



4.6 approximating values of a function using linearity and linearization

Approximate $\sqrt{0.02}$without a calculator

f'(x)=$\sqrt{x}$ 0.02 is 0.02 away from 0

Use linearization @ x=0 to approximate $\sqrt{0.02}$

f'(x)=$\frac{1}{2\sqrt{x}}$ →f'(0)=....UNDEFINED

Since x=0 gives us undefined slope, use x=1

f'(x)=$\frac{1}{2\sqrt{x}}$ → $\frac{1}{2}$slope   f(1)=$\sqrt{1}$ → $(1,1)$

y-1=$\frac{1}{2}$(x-1)

y=$\frac{1}{2}$(x-1)

y=$\frac{1}{2}$(0.02 -1) +1

y=$\frac{1}{2}$(-0.98) + 1 →-.49 +1=0.51

$(0.51)^2 = 0.2601$ →HORRIBLY off from 0.02

L'Hopital rules

L'Hopital's rule is a method of dealing with limits that are in indeterminate form. The limit must be written so they think it will display.

List of 7 indeterminate forms for BC

1]0/0

2]$\infty * 0$

3]∞/∞

4]$\infty - \infty$

5] 1^{∞}

6] 0^{0}

7] ∞^{0}

4.6 Partial Fraction

Partial Fractions

Partial fractions says “What if we go backward ?’

$\int \frac{1}{x^2-4}dx = ???$

$\int \frac{\dots\dots}{x+2}dx + \int \frac{\dots\dots}{x-2}dx$

Doing partial sums is like this however it is a bit more complex

EX:

$$\frac{6}{x^4-5x^2+4}$$

$\frac{A}{(x-2)}+\frac{B}{(x+2)}+\frac{C}{(x-1)}+\frac{D}{(x+1)}$

6= A(x+2)(x-1)(x+1) + B(x-2)(x-1)(x+1) + C(x-2)(x+2)(x+1) + D(x-2)(x+2)(x-1)

x= {-2,-1,1,2}

x=1 6= C (-1)(3)(2)→C=-1

x=2 6= A (4)(1)(3)→A=1/2

x=-2 6= B(-4)(-3)(-1)→B=-1/2

x=-1 6= P(-3)(1)(-2) →D=1

$\frac{1}{2(x-2)}-\frac{1}{2(x+2)}-\frac{1}{x-1}+\frac{1}{x+1}$