AP Calculus BC

From Simple Studies: https://simplestudies.edublogs.org and @simplestudiesinc on Instagram

Unit 3: Differentiation: Composite, Implicit, and Inverse Functions (and Important Theorems)

Chain Rule

Chain Rule

If f and g are both differentiable and $F(x)$ is the composite function defined by $F(x) = f(g(x))$ then F is differentiable and F' is given by the product

$$F'(x) = \underbrace{f'(g(x))}_{\text{Differentiate outer function}}\ \underbrace{g'(x)}_{\text{Differentiate inner function}}$$

from:https://www.onlinemathlearning.com/image-files/chain-rule.png

Implicit Differentiation

Implicit Differentiation→Differentiable in respect of x

1) Take the derivative of (Differentiate) Both sides of the equation
2) Collect all the terms involving $\frac{d}{dx}$or y’ to one side of the equation
3) Factor out $\frac{d}{dx}$or y’
4) Solve for $\frac{d}{dx}$ or y’

Explicit form: Implicit form

y= $\sqrt{25-x^2}$ $x^2+y^2=25$

y= x^2 x=y^2

$x^2+y^2-2xy=0$

$\mathbf{x^3\,y^3 - y = x}$

$$x^3\,[3y^2\,\frac{dy}{dx}] + y^3\,[3x^2] - \frac{dy}{dx} = 1$$

$$3x^3y^2\,\frac{dy}{dx} + 3\,x^2y^3 - \frac{dy}{dx} = 1$$

$$3x^3y^2\,\frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$

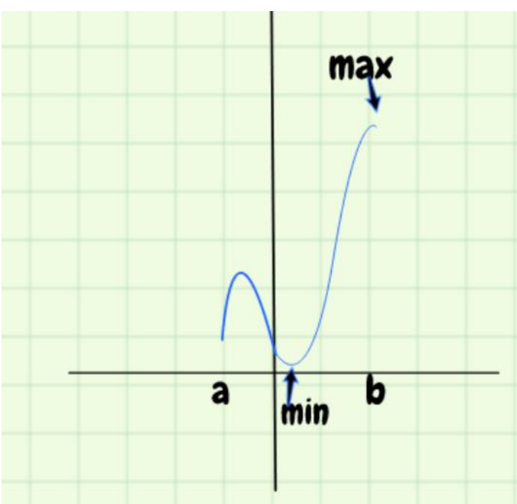
$$\frac{dy}{dx}\,(3x^3y^2 - 1) = 1 - 3\,x^2y^3$$

$$\frac{dy}{dx} = \frac{1 - 3\,x^2y^3}{3x^3y^2 - 1}$$

From: https://calcworkshop.com/wp-content/uploads/implicit-example-1.png

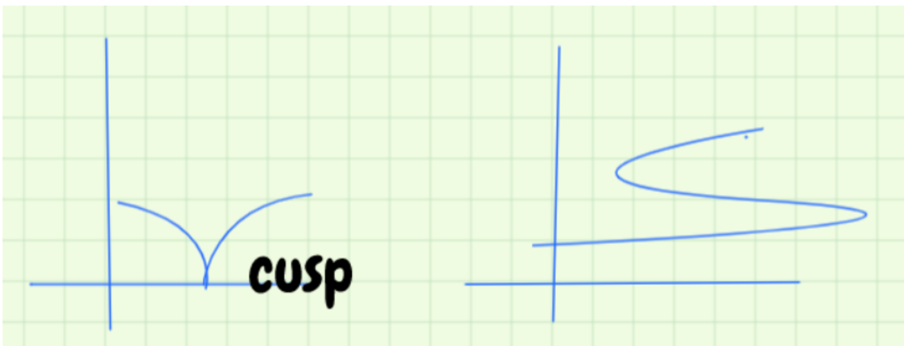
The Extreme Value Theorem

If f is continuous function on a closed interval [a,b], then f has both an absolute maximum and absolute minimum.



Definition of Critical numbers

- A critical number C is a number, such that f’(c)=0 or f’(c)= Undefined.
- Relative extrema only occurs at critical numbers



Derivative is not defined

Guide lines fore solving absolute extrema

1] Find all critical numbers: f’(c)=0, f’(c)=DNE

2] Compare the values of f at the critical numbers and at the endpoints

3] The largest value of f is the absolute maximum and the smallest value is absolute maximum

Rolle’s Theorem: If f is continuous function on [a,b] and differentiable on (a,b), and f(a)=f(b), then there exists at least one number “c” such that f’(c)=0

EX: Let f(x)= $x^4 - 2x^2$

Verify that Rolle’s Theorem can be applied in the interval [-2,2]

- ❏ (check)Continuous
- ❏ (check)Differentiable

f(-2)=$(-2)^4 - 2(-2)^2 = 8$

f(2)= $(2)^4 - 2(2)^2 = 8$

f(-2)=f(2) →Rolle’s Theorem can be applied

Since f(-2)=f(2) and f(x) is continuous on [-2,2] and diff. On (-2,2), then Rolle's Theorem can be applied.

Connecting Physics and Calculus

Math Physics

Original function= Position Graph

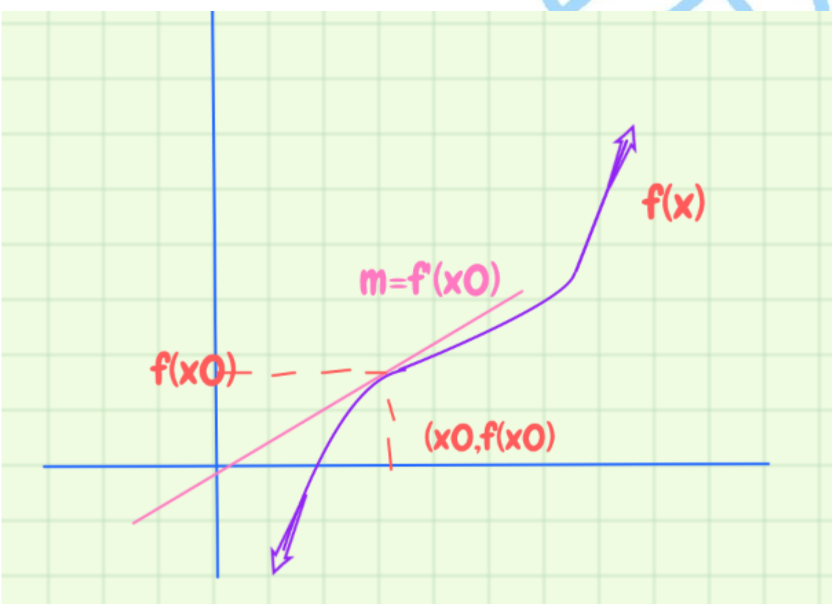
First Derivative= Velocity Function (slope of your position)

Second derivative= Acceleration(slope of your velocity)

Third Derivative= Jerk (snap you feel when a vehicle changes acceleration rapidly)

Linear Approximation

x0= x_0



$y - y_1$=m(x$-x_1$)

y-f(x_0)= f'(x_0) (x-x_0)

0-f(x_0)= f'(x_0) (x-x_0)

$$\frac{-f(x_0)+ -x_0 * f'(x_0)}{f'(x_0)} = \frac{f'(x_0) * x}{f'(x_0)}$$

$$x_0 - \frac{f(x_0)}{f'(x_0)} = x$$

The fundamental theorem of Calculus

FTC- if a function continuous on a closed interval [a,b] and F is an antiderivative of f on the interval [a,b] , then $\int_{\square}^{\square}$ f(x) dx= F(b)-F(a)

(integral represents the area) $\int_{\square}^{\square}$ f(x) dx=F(x)←answer

Guidelines for using the Fundamental Theorem of Calculus

1. Provided you can find an antiderivative of , you now have a way to evaluate a definite integral without having to use the limit of a sum.
2. When applying the FTC, the following nation us convenient