

AP Calculus BC- Unit 1

From Simple Studies: <https://simplestudies.edublogs.org> and
@simplestudiesinc on Instagram

1.2 Intro to limits/Finding Limits using graphs, table and numerically

$$\lim_{x \rightarrow c} f(x) = L \quad \text{*the arrow means that it is approaching}$$

We read "Limit of $f(x)$ = to L as X approaches c "

EX: Evaluate using a table

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = 3 \quad f(1) = \text{undefined} \quad \frac{1^3 - 1}{1 - 1} = 0/0$$

x	0.9	0.99	0.999	0.9999	1.0001	1.001	1.01	1.1
y	2.71	2.9701	2.99700	2.99970	3.00030	3.00300	3.0301	3.31
			1	001	001	1		

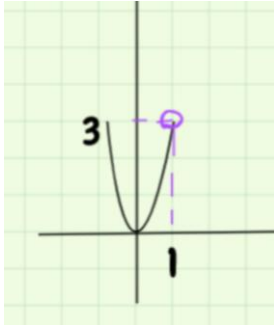
Table: it is clear that as x approaches closer and closer to one, $f(x)$ approaches 3.

$$a^3 - b^3 \rightarrow (a - b)(a^2 + ba + b^2)$$

$$(x-1)(x^2 + x + 1) \rightarrow \text{factored numerator}$$

The $(x-1)$ will cancel out, ensuring the presence of a hole at $x=1$.

Plug $x=1$ into the leftover equation: $(x^2 + x + 1)$, and the hole is found to be at $(1,3)$, validating the results of the table.



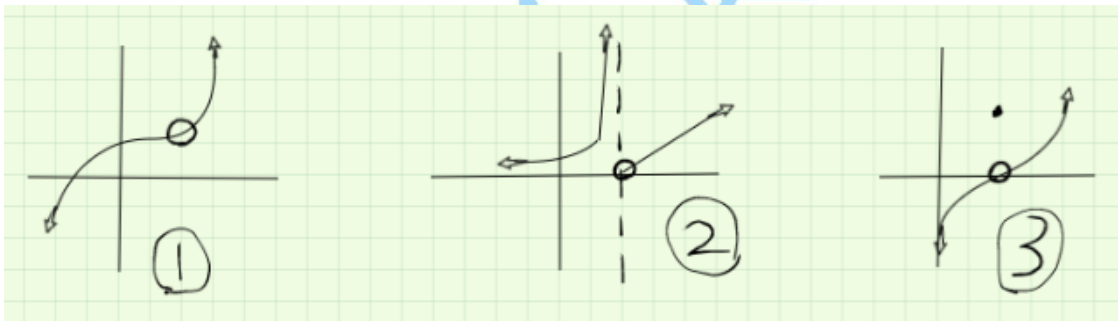
1.3 Evaluating Limits Algebraically (see above example)

1.4 Continuity

A function is continuous at point $x=c$ if and only if:

- 1.) $f(c)$ exists
- 2.) Limit at $x=c$ must exist
- 3.) The value of $f(c)$ and the limit of $x=c$ must be the same

The following below shows the function not continuous because the rules above are not being applied



When we zoom out on a graph to look at the whole graph, we can discuss continuity of the whole function.

We can't test every single point for a graph... that's impossible So we take a short cut

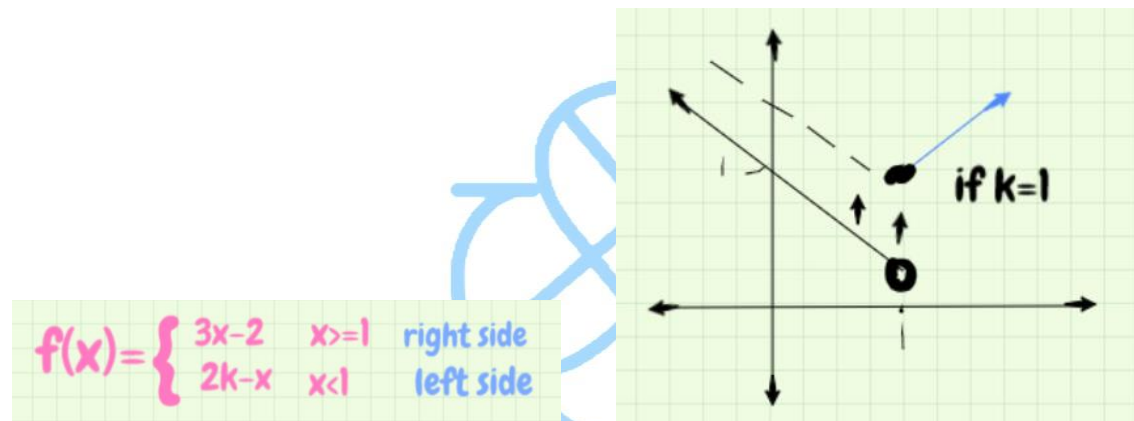
We can say: A function is continuous over (a,b) if it is continuous at every single point in between ... this still has an infinite amount of points, so let's try something else.

Examine the domain. If your domain doesn't exist, then it can be continuous at these points.

Restoring Continuity to functions

If a function has a removable discontinuity, we can restore continuity to it via algebra.

Example: Find the value of K such that the function is continuous



$$f(1) = \lim_{x \rightarrow 1} f(x)$$

$$f(1) = \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$f(1) = \lim_{x \rightarrow 1^-} f(x)$$

$$3(1)-2 = \lim_{x \rightarrow 1^-} 2k-x$$

$$1 = 2k-1$$

$$2 = 2k$$

$$1 = k$$

How do we even get a derivative

Conditions:

- 1) Functions must be continuous at the specific point you are trying to find the derivative ΔT
- 2) The limit of the slope on the left of the point must be equal to the slope right of the point

Why do we care about the derivative?

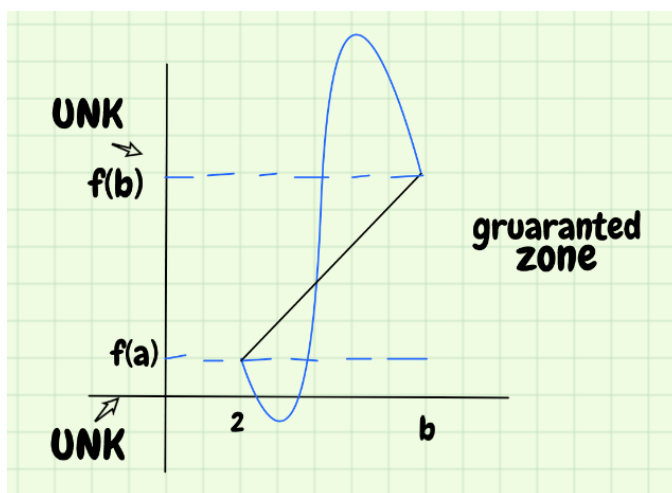
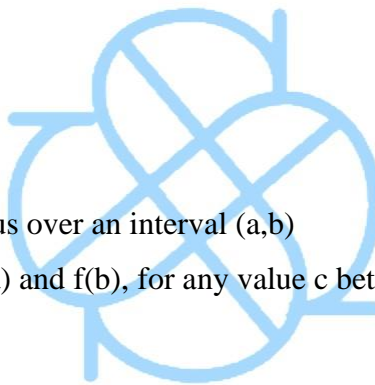
- 1) Measure of change (velocity, acceleration, slope, steepness of the graph)
- 2) Specific to value of x
- 3) Provide a good analogue or approximation of what's going on at a point

The intermediate value theorem

Focuses on the function $f(x)$

Condition: $f(x)$ must be continuous over an interval (a,b)

Result: Given a and b produce $f(a)$ and $f(b)$, for any value c between a and b , $f(c)$ is guaranteed to be between $f(a)$ and $f(b)$



Let's take those two points and try to get something useful about the rate of change . Instead of asking what happens in between, we generally just find the slope between the two points directly-secant slope(average rate of change)(average difference quotient)

A secant slope or line gives a general idea of what happened. Details are lost.

When we want the details, you need specific information about specific parts. The tangent or line gives specific details about each point, not a collection of points

The IVT Revisited

$f(x)$ represented the amount of money in Ahiti's wallet at x days. If $f(x) = 2x^2 - 7x + 3$, what does the IVT tell us in the context of the problem for $x=0$ and $x=1$.

$$f(0)=3; f(1)= -2$$

The IVT tells us that between day 0 and day 1, Ahiti had the guaranteed values between -2 and 3 dollars of money in his wallet.

If we are talking about slopes, we love two options.

- 1) Average slope uses 2 points
- 2) Tangent slope.... Uses 1 point(this is also your derivative at the point)

1.5 Infinite Limits

Definition of V.A (Vertical Asymptote):

If $f(x)$ approaches to infinity(positive or negative) as $x \rightarrow c$ from left or from right, then the line $x=c$ is a vertical asymptote.

EX: $f(x) = \frac{1}{x-3}$ $x=3$ is a V.A(vertical asymptote)

$$\lim_{x \rightarrow 3^-} f(x) =$$

$$\lim_{x \rightarrow 3^-} \frac{1}{x-3} = \frac{+}{2.9-3} = \frac{+}{-} = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} \frac{1}{x-3} = \frac{+}{+} = +\infty$$

