# AP Calculus AB Course Study Guide Differential Equations 

From Simple Studies, https://simplestudies.edublogs.org \& @simplestudies4 on Instagram

## Differential Equations (Separate the integral)

Example: $\mathrm{dy} / \mathrm{dx}=\mathrm{e}^{3 \mathrm{x}} / 3 \mathrm{y}^{2}$

| You want to separate x and y | $3 \mathrm{y}^{2} \mathrm{dy}=\mathrm{e}^{3 \mathrm{x}} \mathrm{dx}$ |
| :--- | :--- |
| Integrate | $\int 3 \mathrm{y}^{2} \mathrm{dy}=\int \mathrm{e}^{3 \mathrm{x}} \mathrm{dx}(\mathrm{u}=3 \mathrm{x}, \mathrm{du}=3 \mathrm{dx}$, <br> $1 / 3 \mathrm{du}=\mathrm{dx})$ <br>  <br>  <br>  <br>  <br>  <br> $3 \mathrm{y}^{2} \mathrm{dy}=1 / 3 \int \mathrm{e}^{\mathrm{u}} \mathrm{dx}$ <br> $\mathrm{y}^{3}=1 / 3 \mathrm{e}^{3 \mathrm{x}}+\mathrm{c}$ |
| Make y by itself | $3 \sqrt{y^{3}=3} \sqrt{1 / 3} e^{3 x}+c$ |
|  | $y=3 \sqrt{1 / 3} e^{3 x}+c$ |

Example: $\mathrm{dy} / \mathrm{dx}=\mathrm{xy}$

| Separate $x$ and $y$. | $1 / y d y=x d x$ |
| :--- | :--- |
| Integrate. | $\int 1 / y d y=\int x d x$ <br> $\ln \|y\|=x^{2} / 2+c$ |
| Multiply each side by e. | $\|y\|=e^{x^{\wedge} 2 / 2} \cdot e^{c}$ |
| C will always be positive and will remain as <br> "c" even if <br> multiplied/divided/added/divided by | $\|y\|=e^{x^{\wedge} 2 / 2} \cdot \mathrm{e}^{c}$ |

## something.

## Differential Equation with Initial Condition

## Example:

The slope of a curve at each point ( $\mathrm{x}, \mathrm{y}$ ) is given by $2 \cos \mathrm{x}-\mathrm{x}$. Which of the following is an equation of the curve if its graph passes through the point $(0,1)$ ?

| Set up the equation. We know that the <br> "slope of a curve" equals to $d y / d x$ | $d y / d x=2 \cos x-x$ |
| :--- | :--- |
| Integrate. | $\int d y=\int 2 \cos x-x d x$ |
| $y=2 \sin x-x^{2} / 2+c$ |  |$|$|  | $\mathrm{c}=2 \sin (0)-(0)+\mathrm{c}$ <br> Using the points they gave us $(0,1)$, plug it <br> into the equation and solve for $c$. <br> Now that you have c, go back and plug it <br> into the original equation where you found <br> $y$. $\mathrm{y=2} \mathrm{\sin x-x}^{2} / 2+1$ |
| :--- | :--- |

## Slope Field

## A visual depiction of a differential equation of $d y / d x$.

- Example of what a slope field looks like:


Picture Credits:


