AP Calculus AB Course Study Guide
Integration and Accumulation of Change

From Simple Studies, https://simplestudies.edublogs.org & @simplestudies4

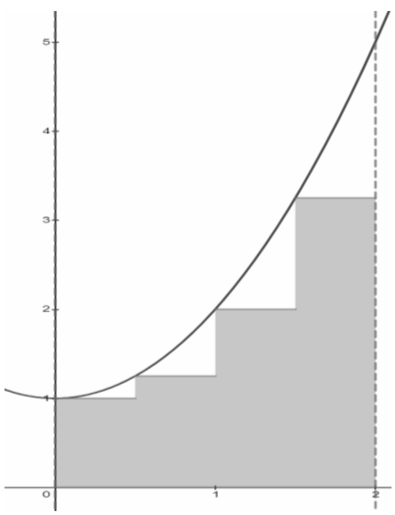
on Instagram

Riemann Sums

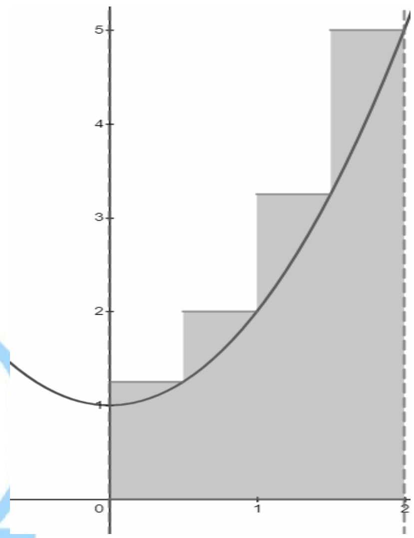
You use Riemann sums to find the actual area underneath the graph of f(x).

Example: f(x)=x^2+1 on [0,2] with 4 equal subintervals

Left Riemann Sum



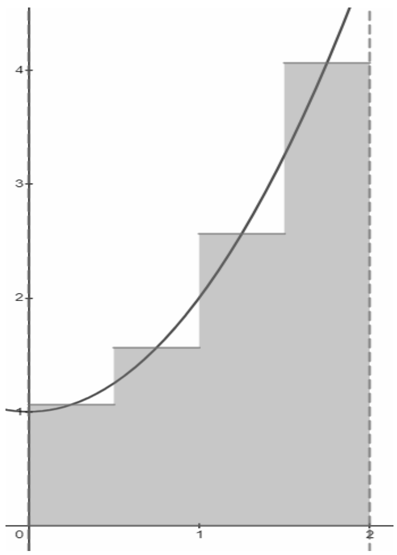
Right Riemann Sum



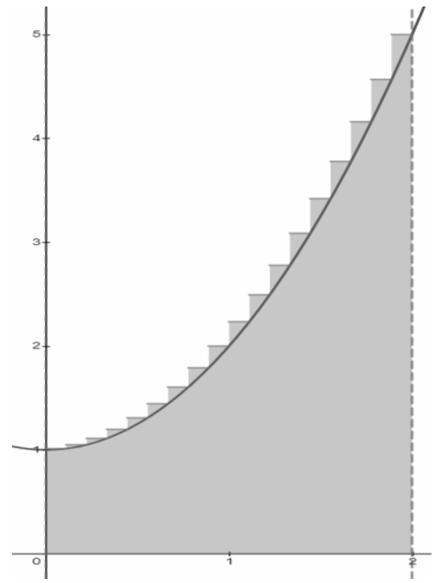
Left sum is an *underestimate*.

Right sum is an *overestimate*.

Midpoint Sum



Trapezoidal Sum



Fundamental Theorem of Calculus (Definite Integration)

$$\int_a^b f(x)dx = F(b) - F(a)$$

Picture Credits:

The area under the curve of derivatives of F from A to B is equal to the change in y-values of the function F from A to B, given f is:

- **Continuous in interval [a,b]**
- F is any function that satisfies F(x)=f(x)

Example:

$$\int 2x^2\,dx = x^3$$

What is an indefinite integration?

Given y' or f '(x), the anti-derivative can be thought of as the **original** function, f(x). Integration is **used to find the original function.**

- The operation of finding all solutions to this equation is called **antidifferentiation or indefinite integration.**
- **Detonated by an integral sign: ∫**

$$v = \int f'(x)\, dx = f(x) + c$$

- f '(x) = integrand
- dx = variable of integration
- f(x) = antiderivative
- c = constant of integration
- ∫ = integral

Reminder: **ALWAYS** add **+C** when you're solving for an INDEFINITE integral!

Reminder: Differentiation and integration are inverses!

Basic Integration Rules (w/ examples)

***K=constant/number**

Power Rule	$\int x^n dx = \frac{x^{(x+1)}}{(n+1)} + C$; $n \neq -1$	$\int x^2 dx = \frac{x^{(2+1)}}{(2+1)} = \frac{x^3}{3} + C$
Constant Rule	$\int kdx = kx + C$	$\int 7dx = 7x + C$
Multiple Of A Constant	$\int k\, f(x)dx = k \int f(x)dx$	$\int 7xdx = 7 \int x\, dx = 7\, \frac{x^2}{2} + C$
Sum & Differences	$\int [f(x) \pm g(x)]dx =$ $\int f(x)dx \pm \int g(x)dx$	$\int [3x+5]dx = \int 3xdx + \int 5dx =$ $3\frac{x^2}{2} + 5x + C$

Antiderivative Trig Function

$\int \cos x\,dx = \sin x + C$	$\int \sec x \tan x\,dx = \sec x + C$
$\int \sin x\,dx = -\cos x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$

Examples with trig functions

$\int 2\sin x\,dx = 2\int \sin x\,dx = 2(-\cos x + c) = -2\cos x + c$

C is still a constant when

- ***HINT:*** How I memorize antiderivatives by using derivatives of trigonometric functions.
  - EX: **d/dx sinx = cosx** and for the antiderivative, you just switch the two trigonometric functions and add +C since it's an indefinite integration.
  - EX: **d/dx cscx = -cscxcotx** and for the antiderivative, just switch the two trigonometric functions and add +c since it's an indefinite integration. Also, if the derivative was negative, then the anti-derivative is also negative!

Integration by U-substitution

Example 1: $\int x^2(x^3-7)^3dx$

First step: Find *u* ***HINT:*** Usually *u* is the one in the parentheses or the one where if you derive it, the derivative equals something else in the function.	$U = x^3 - 7$
Second step: Derive *u* ***HINT:*** Notice when you divide each side by 3, x^2 is the same as the function in the original function	$du = 3x^2dx$
Third step: Once you derive, divide each side by 3.	$\frac{1}{3}du = x^2dx$

Fourth Step: Substitute *u* and *du* back into the original function	$\int \frac{1}{3} u^3 \, du$
Fifth Step: Factor out the ⅓ and integrate	$\frac{1}{3} \int u^3 \, du \rightarrow \frac{1}{3} (u^4/4) = \frac{u^4}{12} + C$
Sixth Step: Go back and replace *u*	$\frac{(x^3-7)^4}{12} + C$
REMINDER: Don't forget to go back and replace *u* and add +C!	

Example 2: $\int \cos(8x)dx$

First Step: Find *u*	U = 8x
Second Step: Derive *u*	du = 8dx
Third Step: Divide both side by 8 so dx is by itself	$\frac{1}{8}$ du = dx
Fourth Step: Substitute *u* and *du* back into the original function	$\frac{1}{8} \int \cos(u)du$
Fifth Step: Integrate by using indefinite integrals trig functions	$\frac{1}{8}$ sin(u) + c
Sixth Step: Replace *u*	$\frac{1}{8}$ sin(8x) + c

Natural Log Function for Integration (Log rule for integration)

Use this rule when 'x' becomes DNE

$$\int 1/x\ dx = \ln|x|+c$$

$$\int 1/u\ dx = \ln|u|+c$$

Ex. $\int 2/x\ dx$

$2\int 1/x\ dx$

2 ln|x|+c

Ex. $\int \sec^2 x / \tan x\ dx$

U = tanx
du = $\sec^2 x dx$

$\int 1/u\ du$

ln|u|+c

ln |tanx|+c

Integrals of the 6 Basic Trig Functions

$$\int \sin u\, du = -\cos u + C \qquad \int \cos u\, du = \sin u + C$$

$$\int \tan u\, du = -\ln|\cos u| + C \qquad \int \cot u\, du = \ln|\sin u| + C$$

$$\int \sec u\, du = \ln|\sec u + \tan u| + C \qquad \int \csc u\, du = -\ln|\csc u + \cot u| + C$$

Picture Credits: kerrierich

- ***HINT:*** For ∫ tan u du, I memorized it like this: ∫ tan u du = ∫ sin u/cos u du because of the trigonometric identities. After that, I just did u-substitution with cos u being u.
- If you work it out, it looks like this:

Step One: Use trig identities	∫ tanu du = ∫ sinu/cos u du
Step Two: Use U-Substitution	U = cos u du = -sinu du -du = sinu du
Step Three: Substitute *u* and *du* back into the original function	−∫ 1/u du
Step Four: Use the log rule for integration	-ln|u|+c
Step Five: Don't forget to go back and replace u	-ln|cosu|+c

Example 1: $\int \tan(5x)dx$

Step One: Find *u*	U = 5x
Step Two: Derive	du = 5dx
Step Three: Divide both sides by 5	$\frac{1}{5}$ du = dx
Step Four: Substitute *u* and *du* back into the original function	$\frac{1}{5} \int \tan u \, du$
Step Five: Use the integrals of the 6 basic trig functions	$-\frac{1}{5} \ln\lvert\cos u\rvert + c$
Step Six: Replace u	$-\frac{1}{5} \ln\lvert\cos(5x)\rvert + c$

Integration rule for "e"

$\int e^x dx = e^x + c$	$\int e^u du = e^u + c$

- With e, it's just the same thing as regular u-substitution but with the additional 'e'.

Example 1: $\int e^{3x+1}dx$

Step 1: Find *u*	U = 3x+1
Step 2: Derive	du = 3dx
Step 3: Divide both sides by 3 because you want dx by itself	$\frac{1}{3}$ du = dx
Step 4: Substitute *u* and *du* back into the original function	$\frac{1}{3} \int e^u du$

Step 5: Integrate using the integration rule for "e"	$\frac{1}{3} e^{u}+c$
Step 6: Replace *u*	$\frac{1}{3} e^{3x+1}+c$

Example 2: $\int e^{2x}/1+e^{2x}$

Step 1: Find *u*	$U = 1+e^{2x}$
Step 2: Derive	$du = 2e^{2x}dx$
Step 3: Divide both sides by 2	$\frac{1}{2} du = e^{2x}dx$
Step 4: Substitute *u* and *du* back into the original function	$\frac{1}{2} \int 1/u \, du$
Step 5: Use the "log rule" for integration	$\frac{1}{2} \ln\|u\|+c$
Step 6: Replace *u*	$\frac{1}{2} \ln\|1+e^{2x}\|+c$

Integration Rule for Exponential Functions

$$\int a^{x}dx= (1/\ln a)a^{x}+c$$

Example: $\int 7^{-x}dx \rightarrow -\int 7^{u}du \rightarrow -1/\ln(7) \cdot 7^{u} + C \rightarrow -7^{-x}/\ln 7 + C$

- $u=-x \rightarrow du=-dx \rightarrow -du=dx$