AP Calculus AB Course Study Guide
Analytical Applications of Differentiation

From Simple Studies, https://simplestudies.edublogs.org & @simplestudies4 on Instagram

Mean Value Theorem

If f(x) is a function that is **continuous on the closed intervals [a,b] and differentiable on the open interval (a,b),** then there must **exist a value *c* between (a,b)**

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Picture Credits: andymath

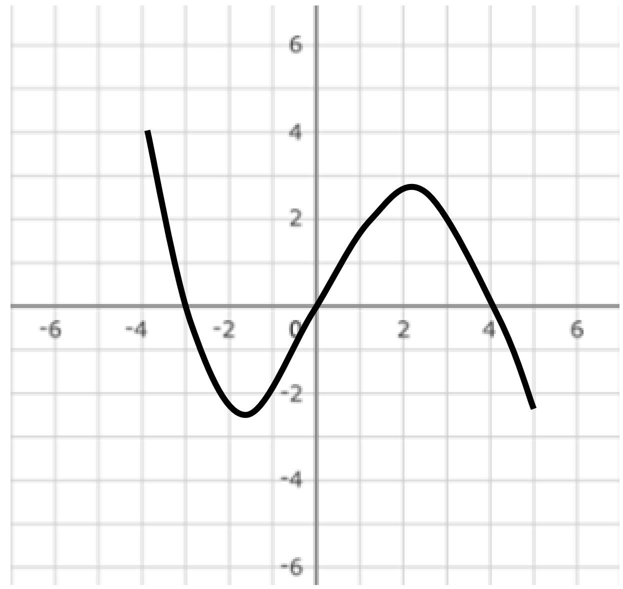
- *Example:* Confirm $f(x)=x^3$ on [0,3] and find a value that satisfies this theorem
    - $f'(x)=3x^2$
    - $3x^2 = \frac{f(3) - f(0)}{3-0} \rightarrow 3x^2 = \frac{(3)^3-(0)^3}{3} \rightarrow 3x^2 = 9 \rightarrow x \pm \sqrt{3}$

Function Increasing or Decreasing

- f(x) is *increasing* when **f'(x) is *positive***
- f(x) is *decreasing* when **f'(x) is *negative***

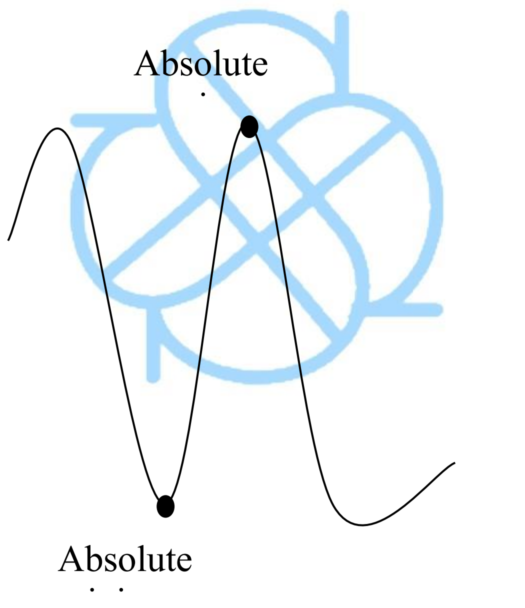
Example: Given f'(x), where is f(x) increasing and decreasing on the interval [-4,6]?

- f(x) is increasing $(-\infty, -3) \cup (0,4)$ because f'(x) is positive
- f(x) is decreasing $(-3,0) \cup (4,5)$ because f'(x) is negative



Extreme Value Theorem

If f(x) is **continuous on a closed interval [a,b],** then f(x) has **both a minimum and maximum on the interval.**



Example: Locate the absolute extrema on the function on the closed interval.

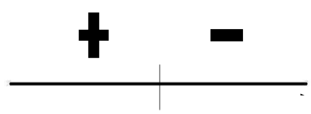
- f(x)=x^3-12x [0,4]
- f'(x)=$3x^2$-12
- 0=$3x^2$-12
- x = -2,2
- f(0) = 0, f(2) = -16, f(4) = 16
    - f(x) has an absolute minimum at x=2 and an absolute maximum at x=4

First Derivative Test

For first derivative tests, **derive the function once and set it to 0.** After that, **find the zeros** and plug them into a number line. Using your derived function, plug-in numbers before and after your constant (the zeros of the function) to see if it becomes negative or positive, as shown below.

- If it's *positive, constant, negative* then it's a **relative maximum**
- If it's *negative, constant, positive* then it's a **relative minimum**

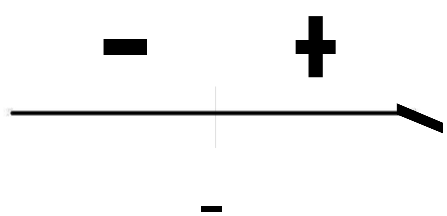
- f(x) has a relative maximum at x=c



- f(x) has a relative minimum at x=c

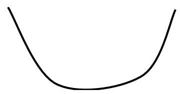
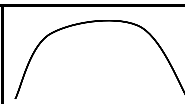
Example:

- $f(x)=x^2+6x+10$
- $f'(x)=2x+6$
- $0=2x+6$
- $x=-3$
- f(x) is decreasing $(-\infty,-3)$
- f(x) is increasing at $(-3, +\infty)$
- f(x) has a relative minimum at x=-3



Concavity

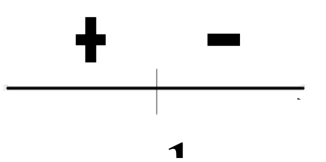
- The graph of f is concave *up* when f’(x) is *increasing*
- The graph of f is concave *down* when f’(x) is *decreasing*
- If **f’’(x) is positive** then the graph of f is ***concave up***
- If **f’’(x) is negative** then the graph of f is ***concave down***

f(x)		
f’(x)	increasing	decreasing
f’’(x)	positive	negative

- **Points of inflection**
    - Occurs when **f(x) changes concavity**
    - Determined by a **sign change for f’’(x)**

Second Derivative Test

Example:



- y=$-x^3+3x^2$-2
- y’=$-3x^2$+6x
- y’’=-6x+6
- 0=-6x+6
    - x=1
- Like the first derivative test, after you find your second derivative, **find the zeros of f(x).**
- In this case, 1 is the zero, so it goes on the number line.
- **To see if it’s concave up or concave down, use -1 and 2 to plug into the second derivative function.**
- When you plug in -1 you get a positive number.
- When you plug in 2 you get a negative number.

f(x) is concave up on (-∞,1) because f’’(x) is positive on that interval.

f(x) is concave down on (1, +∞) because f’’(x) is negative on that interval.

Critical Numbers

- Critical numbers are **points on the graph of a function where there’s a change in direction.**

- To **find critical numbers**, you use the **first derivative of the function and set it to zero.**

Example:

- $f(x)=2\sec x+\tan x$
- $f'(x)=2\sec x\tan x+\sec^2 x$
- $0=\sec x(2\tan x+\sec x) \rightarrow 0=\sec x \rightarrow 0=1/\cos x \rightarrow x=3\pi/2,\ \pi/2$
- $0=2\tan x+\sec x \rightarrow -\sec x=2\tan x \rightarrow -1/\cos x = 2\sin x/\cos x \rightarrow -1=2\sin x \rightarrow \sin x=-½ \rightarrow$ $x=7\pi/6,\ 11\pi/6$
- The critical numbers for this function are $x = 3\pi/2,\ \pi/2,\ 7\pi/6,\ 11\pi/6$