AP Calculus AB Course Study Guide
Contextual Applications of Differentiation

From Simple Studies, https://simplestudies.edublogs.org & @simplestudies4 on Instagram

Particle Motion

- **s(t)** represents the **position function, aka f(x)**
  - t stands for time, s(t) is the position at a specific time.
- **v(t)** represents the **velocity function, aka f'(x)**
  - t stands for time, v(t) is the speed and direction at a specific time.
  - ***Velocity is the derivative of position.***
    - A particle is moving to the *right or up* when velocity is *positive*.
    - A particle is moving to the *left or down* when velocity is *negative*.
    - A particle's position is *increasing* when velocity is *positive*.
    - A particle's position is *decreasing* when its velocity is *negative*.
    - A particle is at rest or *stopped* when its velocity is *zero*
  - **Speed is the absolute value of the velocity**
- **a(t)** represents the **acceleration function aka f''(x)**
  - t stands for time, a(t) is the rate at which the velocity is changed at specific times.
- *Example:* $s(t)=6t^3-4t^2 \rightarrow v(t)=18t^2-8t \rightarrow a(t)=36t-8$

Particle Moving Away/Toward the Origin(x-axis)

Position s(t) and Velocity v(t)

Toward | Away

+	-	s(t)	-	+
-	+	v(t)	-	+

- A particle is moving towards the origin when its position and velocity have opposite signs.

- A particle is moving away from the origin when its position and velocity have the same signs,

Particle Speeding Up/Slowing Down

Velocity v(t) and Acceleration a(t)

Slowing Down Speeding Up

+	-	v(t)	-	+
-	+	a(t)	-	+

- A particle is speeding up (speed is increasing) if the velocity and acceleration have the same signs at the point.
- A particle is slowing down (speed is decreasing) if the velocity and acceleration have opposite signs at the point.

Related Rates

What is the purpose of related rates?

- The purpose is to **find the rate where a quantity changes**
- The rate of change is **usually with respect to time**

How to solve it?

1. ***Identify all given quantities to be determined.***
2. ***Make a sketch of the situation*** and label everything in terms of variables, even if you are given actual values.
3. ***Find an equation*** that ties your variables together.
4. ***Using chain rule, implicitly differentiate*** both sides of the equation with respect to time.
5. ***Substitute or plug in the given values*** and ***solve for the value*** that is being asked for.
    a. **Don't forget to put the correct units!*

The Different Types of Related Rates Problems

- ***Algebraic***
- ***Circle***
- ***Triangles***
- ***Cube***
- ***Right Cylinder***
- ***Sphere***
- ***Circumference***

Sphere: $V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Triangles: $a^2+b^2=c^2$

$A = \frac{1}{2} bh$

Circles: $A = \pi r^2$

$C = 2\pi r$

Cylinder: $V = \pi r^2 h$

$LSA = 2\pi rh$

$SA = 2\pi rh + 2\pi r^2$

Circumference: πd

Cube: $V = s^3$

$SA = 6s^2$

Cone: $V = \frac{1}{3}\pi r^2 h$

Rectangular Prism: $V = lwh$

$SA = 2lw+2lh+2wh$

Related Rates: Algebraic

Example: A point moves along the curve $y = 2x^2 - 1$ in which y decreases at the rate of 2 units per second. What rate is x changing when x = -3/2?

Identify all given quantities	dy/dt= -2 dx/dt= ? x=-3/2
Set up the given equation and differentiate both sides with respect to time	$y = 2x^2 - 1$ dy/dt = 4x dx/dt
Substitute the values in your new equation	-2 = 4(-3/2) dx/dt
Solve	-2 = -6 (dx/dt)
Put in the units	dx/dt=1/3 units/sec

Related Rates: Circle

Example: The radius of a circle is increasing at a rate of 3cm/sec. How fast is the circumference of the circle changing?

Identify all given quantities	dr/dt = 3 dc/dt=?
Set up the equation and differentiate both sides with respect to time. Use the equation for circumference	c=2πr dc/dt=2π dr/dt
Substitute the given values in	dc/dt=2π(3)
Put in the units	dc/dt=6π cm/sec

Related Rates: Triangle

Example: A 13ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the ladder be moving away from the wall when the top is 5ft above the ground?

First, set up your triangle with all the sides.	With the right triangle, the hypotenuse should be c=13, the opposite being b=5, and we don't know the adjacent side because it's not given.
Use the Pythagorean theorem for triangles to get the adjacent side.	$A^2+B^2=C^2$ $A^2+(5)^2=(13)^2$ $A^2+25=169$ A=12
List out all given quantities.	A=12, B=5, C-13 db/dt=-2 (It is negative because it is slipping down) dc/dt=0 (Not moving) da/dt=?
Use the Pythagorean theorem as your equation and differentiate it.	$A^2+B^2=C^2$ 2A(da/dt)+2B(db/dt)=2C(dc/dt)
Substitute the values and solve.	2(12)(da/dt)+10(-2)=0 24da/dt=20 da/dt=5/6
Add your units	da/dt=5/6 ft/s

Related Rates: Cube

Example: The volume of a cube is increasing at a rate of $10cm^3$/min. How fast is the surface area increasing when the length of an edge is 30cm?

Identify all given quantities.	dv/dt=10 s=30 dSA/dt=?
Set up the equation for a cube and differentiate it with respect to time.	$v=s^3$ $dv/dt=3s^2(ds/dt)$
Substitute the values in.	$10=3(30)^2(ds/dt)$ ds/dt=1/270
Since you're looking for surface area, use the surface area formula and differentiate.	$SA=6s^2$ dSA/dt=12s(ds/dt)
Substitute what you got earlier for ds/dt and s.	dsa/dt=12(30)(1/270)=4/3
Don't forget your units.	dsa/dt=4/3 cm^2/min

Related Rates: Right Cylinder

Example: The radius of a right circular cylinder increases at the rate of 0.1cm/min and the height decreases at the rate of 0.2 cm/mm. What is the rate of change of the volume of the cylinder, in cm^3/min, when the radius is 2cm and the height is 3cm?

Identify your quantities.	dr/dt=0.1 dh/dt=-0.2 dv/dt=? r=2 h=3
Set up the equation for a right cylinder and differentiate both sides with respect to time.	$v=\pi r^2 h$ $dv/dt=(\pi r^2)(dh/dt) + (2\pi r)(dr/dt)h$
Substitute in your values and solve.	$dv/dt=(4\pi)(-0.2)+(4\pi)(0.1)(3)=0.4\pi$

Related Rates: Sphere

Example: As a balloon in the shape of a sphere is being blown up, the volume is increasing at a rate of $4in^3$/s. At what rate is the radius increasing when the radius is 1 inch.

Identify the quantities given.	dv/dt=4 dr/dt=? r=1
Set up the equation for the sphere and differentiate both sides with respect to time.	$v=4/3\pi r^3$ $dv/dt=4\pi r^2(dr/dt)$
Substitute in your values and solve.	$4=4\pi(dr/dt)$ $dr/dt=1/\pi$ in/s

Related Rates: Circumference

Example: What is the value of the circumference of a circle at the instant when the radius is increasing at 1/6 the rate the area is increasing?

Identify the given quantities.	dr/dt=⅙ (da/dt) c=?
Set up the equation for finding the area of a circle and differentiate both sides.	$A=\pi r^2$ $da/dt=2\pi r\ (dr/dt)$
Substitute in your values.	$da/dt=2\pi r(⅙\ da/dt)$
You can cancel out da/dt and solve for r.	$r=3/\pi$
Use the circumference equation now and plugin and solve.	$c=2\pi r$ $c=2\pi(3/\pi)$ $c=6$

L'Hopital's Rule

Let f and g be continuous and differentiable functions on an open interval (a,b). If the limit of f(x) and g(x) as x approaches c **produces the indeterminate form 0/0 or ∞/∞ then,**

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

Example:

$$\lim \quad \underline{\ln x} = \underline{\ln\infty} =$$
$$\underline{\infty}$$

- The answer is **indeterminate so we have to use L'Hopital's rule.**

Using L'Hopital's Rule

$$\lim \quad \ln x \quad = \quad \lim \quad 1/x \quad =$$