AP Calculus AB Course Study Guide
Differentiation: Composite, Implicit, and Inverse Functions

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The Chain Rule

The chain rule helps us **find the derivative of a composite function.** For the formula, g'(x) would be the chain.

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

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Example:

We're given this function:	$f(x)=(3x^2+8)^2$
Derive it by using the chain rule. • $g(x) = 3x^2+8$ • When you chain $3x^2+8$, $3x^2$ turns to 6x and 8 becomes 0.	$f'(x)=2(3x^2+8)\cdot(6x)$
This is your final answer. You bring 6x to the front to make the equation look neater and multiply it by 2.	$f'(x)=12x(3x^2+8)$

General Rule Power

$$\frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1}\cdot f'(x)$$

Picture Credits: calcworkshop

We use the general rule power when finding the **derivative of a function that is raised to the nth power.** In the formula given, f'(x) is the chain.

Implicit Differentiation

Step 1: Differentiate both sides of the equation with respect to x. Whenever you derive y, write dy/dx.	$y^3+y^2-5y+x^2=-4$ $3y^2(dy/dx)+2y(dy/dx)-5(dy/dx)-2x=0$
Step 2: Move all dy/dx terms on one side of the equation; move all other non dy/dx to the other side.	$3y^2(dy/dx)+2y(dy/dx)-5(dy/dx)=2x$
Step 3: Factor out dy/dx	$dy/dx(3y^2+2y-5)=2x$
Step 4: Solve for dy/dx by dividing $(3y^2+2y-5)$ on both sides.	$dy/dx=2x/(3y^2+2y-5)$

Inverse Trig Functions: Differentiation

$$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\cot^{-1}x\right)=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\sec^{-1}x\right)=\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2} \qquad \frac{d}{dx}\left(\csc^{-1}x\right)=\frac{-1}{|x|\sqrt{x^2-1}}$$

Picture Credits:

Example:

Function	h(x)=arcsinx-2arctanx

First, derive the inverse trig function arcsinx which is the same thing as $\sin^{-1}x$	h(x)=arcsinx h'(x)= $\frac{1}{\sqrt{1-x^2}}$
Next, derive the inverse trig function 2arctanx.	h(x)=2arctanx h'(x)= $\frac{2}{x^2+1}$
Last, combine them and you have your answer.	$\frac{1}{\sqrt{1-x^2}}-\frac{2}{x^2+1}$

Derivatives of Inverse Functions

Find:

(a) The derivative of the inverse function at x=a

(b) The equation of the tangent line to the inverse function at the point x=a

Using the function $f(x)=5-2x^3$, a=7

(a)	Original function	Inverse function
	f(x)	g(x)
	(-1,7)	(7,-1)
	$7=5-2x^3$	
	$2=-2x^3$	
	$-1=x^3$	
	X = -1	
Plug x=-1 into original function		
	f'(-1)	g'(7)
	$f'(x)=-6x^2$	
	f''(-1)=-6	g'(7)=-1/6

(b) $y-y_1=m(x-x_1)$

$y-(-1)=-1/6(x-7)$

$y+1=-1/6(x-7)$

Higher-Order Derivatives

Function	$f(x)=10x^3-3x^2+9x-7$
Finding the first derivative	$f'(x)=30x^2-6x+9$
Finding the second derivative	$f''(x)=60x-6$
Finding the third derivative	$f'''(x)=60$