AP Calculus AB Course Study Guide
Differentiation: Definition and Basic Derivative Rules

From Simple Studies, https://simplestudies.edublogs.org & @simplestudies4 on Instagram

What is a derivative?

- **Derivative:** The slope of the tangent line at a particular point; also known as the **instantaneous rate of change.**
- The derivative of f(x) is **denoted as f'(x)** or

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(Picture credit to Bartleby)

Derivatives as Limits

f'(c) = lim f(x) - f(c)

x → c x - c

f'(x) = lim f(x+h) - f(x)

x →

Steps to find derivatives as limits:

1) Identify the form of the derivative first (look at the image above)… is it form a? b? c?
2) Identify f(x)
3) Derive f(x) using the corresponding equations next to each form
4) Plug in the "c" value if applicable.

Example:

lim x^3 -

$(2...)^3$

1) This is form A.
2) $f(x) = (x)^3$

3) Derive → $f'(x) = 3x^2$

4) Plug in the limit → $f'(2) = 3(2)^2 = 12$

- **Differentiable:** A function f(x) is differentiable at **x=a if f'(a) exists.**

Rules of Differentiation

Constant Rule	$\frac{d}{dx}[c] = 0$
Power Rule	$\frac{d}{dx}[x^n] = nx^{n-1}$
Constant Multiple Rule	$\frac{d}{dx}[c\, f(x)] = c\, f'(x)$
Sum and Difference Rule	$\frac{d}{dx}[f(x) + g(x)] = f'(x)+g'(x)$ $\frac{d}{dx}[f(x) - g(x)] = f'(x)-g'(x)$
Sine and Cosine	$\frac{d}{dx}[\sin x] = \cos x$ $\frac{d}{dx}[\cos x] = -\sin x$

Constant Rule

If the function is just a **number, then it would equal 0** because there's nothing to derive.

Example: $f(x) = 5$

$f'(x) = 0$

Constant Multiple Rule

Example: $f(x) = 5x^4$

$f'(x) = 5 \cdot 4x^3$

$f'(x) = 20x^3$

Sum and Difference Rule

Example: $f(x) = 3x + 5x^3$

$f'(x) = 3 + 15x^2$

Power Rule: $\frac{d}{dx}$ $[x^n] = nx^{n-1}$

Example: $f(x)=3x^2$

$f'(x) = 2 \cdot 3x^{2-1}$

$f'(x) = 6x$

Product Rule: $f(x) = f(x) \cdot g(x)$

$f'(x) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

Example: $f(x) = 2x^2 \cdot 5x$

$f'(x) = 4x \cdot 5x + 5 \cdot 2x^2 = 20x^2 + 10x^2 = 30x^2$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

(Picture Credits: andymath_

Example: $f(x) = 2/4x$

$f'(x) = (4x)(0)-(2)(4)/(4x)^2 = 0 - 8 / 16x^2 = -1/2x^2$

Derivatives of Trigonometric Functions:

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

Picture Credits: monkarlomonskie

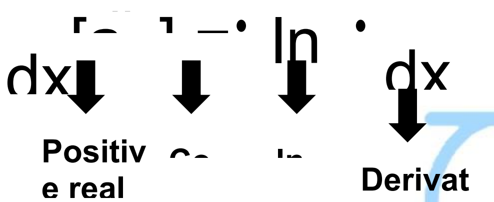
- **HINT:** *If the original function starts with C, then the derivative is negative!*
    - Example: cosx, cotx, & cscx

Derivative Rule for LN

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

- HINT**:** [Derive over copy]
    - *Example:* h(x) = ln(2x^2 + 1)
        - First derive 2x^2 + 1. That would be 4x! And then put that over the original function, which would be 2x^2 + 1.
        - Your answer would then be 4x/(2x^2 + 1)

Deriving Exponential Functions



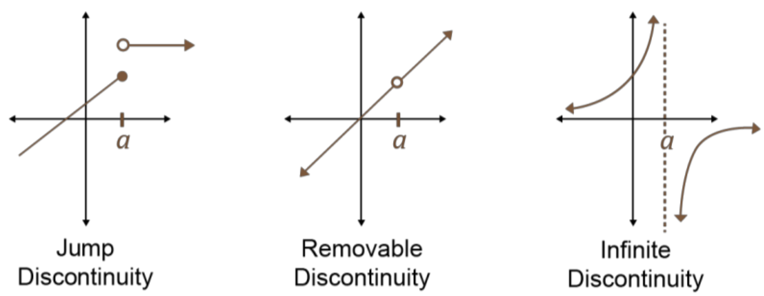
Continuity

A function f is continuous at “c” if:

- **The value exists-** The value of the function is defined at “c” and f(c) exists.
- **The limit exists -** The limit of the function must exist at “c”.
    - ***The left and right limits must equal.***
- **Function=limit.** The value of the function at “c” must equal the value of the limit at “c”

Discontinuity

- **Removable** → discontinuity at "c" is called removable if the function can be continuous by defining f(c)
- **Non-removable** → discontinuity at "c" is called non-removable if the function cannot be made continuous by redefining f(c)



Picture Credits: calcworkshop

Differentiability

In order for a function to be differentiable at x = c:

- The function ***must be continuous at x = c***
- Its ***left and right derivative*** must equal each other at x = c

Example:

$$f(x) = \begin{cases} 2x+1, & x > 2 \\ x^3-x-1, & x \le 2 \end{cases}$$

- **Continuity** → $2x+1 = x^3-x-1$ → $2(2)+1 = (2)^3-2-1$ → $5 = 5$
- **Differentiability** → $2 = 3x^2-1$ → $2 = 3(2)^2-1$ → $2 \neq 11$
  - This function is continuous but not differentiable!