IYGB GCE

Mathematics MMS

Advanced Level

Practice Paper M

Difficulty Rating: 3.5933/0.7479

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 15 questions in this question paper.
The total mark for this paper is 150.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Note that the actual exam papers are out of 100 marks (50 marks in each section) and typically contain 11 to 13 questions.

SECTION 1 - STATISTICS

Question 1

The annual car sales of a small car manufacturer, c, and the annual advertising expenditure, £a, has product moment correlation coefficient r_{ac}.

The data is coded as

$$x = c - 7000 \quad \text{and} \quad y = \frac{a}{1000},$$

and the summary is shown in the table below.

Year	2010	2011	2012	2013	2014	2015	2016	2017
x	52	340	511	621	444	700	805	921
y	120	126	134	138	132	146	153	160

a) Find, by a statistical calculator, the value of the product moment correlation coefficient between x and y, denoted by r_{xy}. **(1)**

b) State with full justification the value of r_{ac}. **(1)**

c) Interpret the value of r_{ac}. **(1)**

Question 2

The events A and B are such so that

$$\mathrm{P}(A) = \tfrac{3}{5}, \quad \mathrm{P}(B|A) = \tfrac{1}{6} \quad \text{and} \quad \mathrm{P}(A|B) = \tfrac{3}{13}.$$

Determine the value of $\mathrm{P}(A' \cap B')$. **(8)**

Question 3

The table below summarizes census information about the number of children in the households of an English town.

Number of children	0	1	2	3	4 or more
Percentage of households	23	32	35	7	3

A random sample of 20 households is selected from this town.

a) Determine the probability that the sample will contain ...

i. ... 3 households with no children. **(2)**

ii. ... more than half the households, with at least 2 children. **(2)**

iii. ... more than 15 but at most 19 households, with at most 2 children. **(2)**

A new random sample of n households is selected from this town.

The probability that this new sample contains a household with 4 or more children is more than 10%.

b) Determine the smallest value of n. **(5)**

Question 4

The table below shows the length of time, rounded to the nearest minute, spent by a group of patients for their dentist's check up visit.

One of the frequencies is given as a positive constant k.

Time (nearest minute)	2 - 6	7 - 11	12 - 16	17 - 31	32 - 36
Number of Patients	6	15	k	24	12

Determine the standard deviation of these times, given that the mean of these times is 18.6 minutes. **(8)**

Question 5

The volume of still water in Ebian bottles is Normally distributed with mean 502 ml and standard deviation 2.5 ml.

A bottle of Ebian still water is selected at random.

a) Determine the probability that the bottle will contain ...

i. ... less than 506 ml. **(2)**

ii. ... less than 495 ml. **(2)**

iii. ... between 495 ml and 506 ml. **(2)**

iv. ... exactly 500 ml. **(1)**

v. ... 502 ml, correct to nearest ml. **(2)**

The volume of sparkling water in Ebian bottles is Normally distributed with mean μ ml and standard deviation σ ml.

It is known that 2.5% of these bottles contain more than 503.5 ml and 2.5% of these bottles contain less than 498.5 ml.

b) Determine the value of σ. **(4)**

Question 6

William has been established over a long period of time that when he shoots an arrow at a target the probability of hitting it, is 0.35.

William buys a professional bow as he believes it will increase the probability of hitting the target.

Let X define the number of successful hits of the target, and assume that successful or non successful hits are independent of one another.

a) Determine a critical region based on 20 shots with the new bow.

The significance level must be as close as possible to 1%. **(6)**

William decides to carry out a significance test by shooting 150 arrows at the target using the new bow. He finds that 59 arrows hit the target.

b) Use a distributional approximation, to calculate an approximate p-value and hence state the conclusion in context. **(6)**

Question 7

The scheduled flight DM104, of a certain airline, can be delayed due to the following three reasons.

- aircraft technical problems, denoted by the event T
- weather conditions, denoted by the event W
- air traffic congestion, denoted by the event C.

These events are assumed to be independent of one another and the flight will be delayed if one or more of these events occur.

It is given that $\mathrm{P}(T) = 0.1$, $\mathrm{P}(W) = 0.05$ and $\mathrm{P}(C) = 0.2$.

a) Find the probability that the next DM104 flight …

  i. … will be delayed due to exactly one reason. **(3)**

  ii. … will be delayed. **(3)**

b) Given that the next DM104 flight was delayed, find the probability that it was delayed due to **one** reason only. **(3)**

c) Given that the next DM104 flight had no technical problems, find the probability that it was delayed. **(3)**

Question 8

The heights of male students in a college are thought to be Normally distributed with mean 170 cm and standard deviation 6.

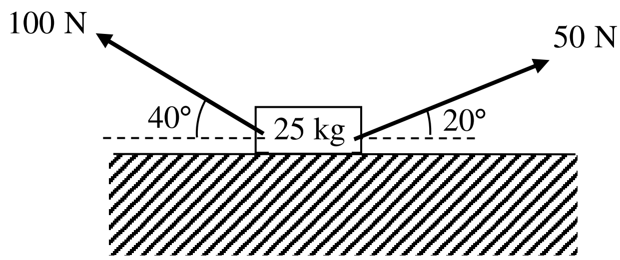
The heights of 4 male students from this college are measured and the sample mean was 180 cm.

Determine, at the 5% level of significance, whether there is evidence that the mean height of the male students of this college is greater than **175** cm.

(8)

SECTION 2 – MECHANICS

Question 9



The figure above shows a small box of mass 25 kg, pulled by two light inextensible strings along rough horizontal ground.

The tension in the rope inclined at $20°$ to the horizontal is 50 N.

The tension in the rope inclined at $40°$ to the horizontal is 100 N.

The box is modelled as a particle experiencing a constant frictional force, where the coefficient of fiction between the box and ground is 0.2.

Determine, by detailed calculations, whether the box remains in equilibrium. **(7)**

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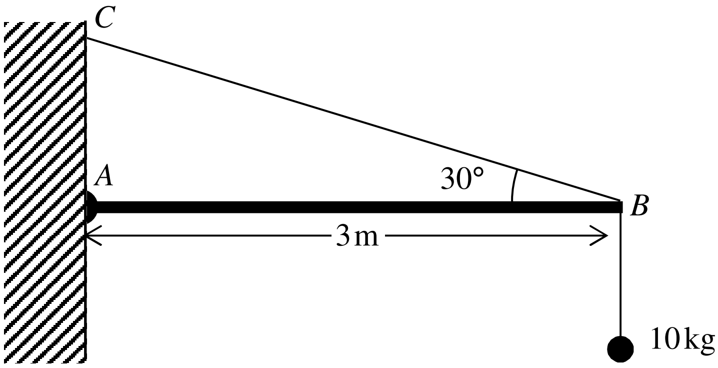
Question 10



The figure above shows a uniform rod AB, of length 3 metres and of mass 20 kg, smoothly hinged at the point A, which lies on a vertical wall.

A particle, of mass 10 kg, is suspended from the end B of the rod. The rod is kept in a horizontal position by a light inextensible string BC, where C lies on the same wall vertically above A.

The plane ABC is perpendicular to the wall and the angle ABC is $30°$.

a) Determine the tension in the string. **(4)**

b) Show that the reaction at the hinge has magnitude $98\sqrt{13}$ N. **(7)**

Question 11

The standard unit vectors $\mathbf{i}$ and $\mathbf{j}$ are oriented in the positive x direction and positive y direction, respectively.

Three forces $\mathbf{F}_1 = 4\mathbf{i} + b\mathbf{j}$, $\mathbf{F}_2 = 3a\mathbf{i} + 2b\mathbf{j}$ and $\mathbf{F}_3 = 10b\mathbf{i} + 3\mathbf{j}$, where a and b are scalar constants, are acting at the points $A(1,2)$, $B(4,-2)$ and $C(-3,-5)$, respectively.

a) Given that the resultant of the three forces is zero, determine the magnitude and direction of the total moment of these three forces about O. **(7)**

b) Find, by direct calculation, the magnitude and direction of the total moment of these three forces about C. **(3)**

Question 12

Relative to a fixed origin O, the horizontal unit vectors $\mathbf{i}$ and $\mathbf{j}$ are pointing due east and due north, respectively.

Two ships, P and Q, are sailing in a channel.

Initially, P is at the point with position vector $(6\mathbf{i}-2\mathbf{j})$ km and Q is at the point with position vector $-5\mathbf{i}$ km. The velocity P is $12\mathbf{j}$ km h^{-1} and the velocity of Q is $(8\mathbf{i}-6\mathbf{j})$ km h^{-1}.

a) Determine the speed of Q. **(2)**

The respective position vectors of P and Q, at time t hours, are $\mathbf{p}$ and $\mathbf{q}$.

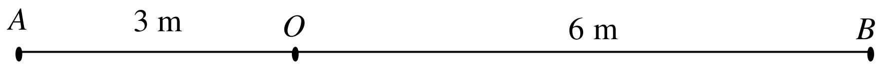
b) Find expressions for $\mathbf{p}$ and $\mathbf{q}$. **(3)**

c) Calculate the distance between P and Q when $t=2$. **(4)**

d) Determine the value of t when P is north of Q. **(3)**

Question 13

A particle is travelling along a straight line with constant acceleration a ms^{-2}.



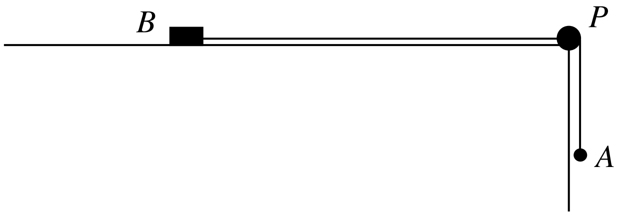
The points A, O and B lie in that order on this straight line, as shown in the figure above. The distance AO is 3 m and the distance OB is 6 m.

The particle is initially observed passing through O with speed u ms^{-1} and 4 s later is observed to be passing through B with speed 7 ms^{-1}, in the direction OB.

a) Find in any order the value of a and the value of u. **(5)**

b) Prove that the particle never passes through A. **(5)**

Question 14



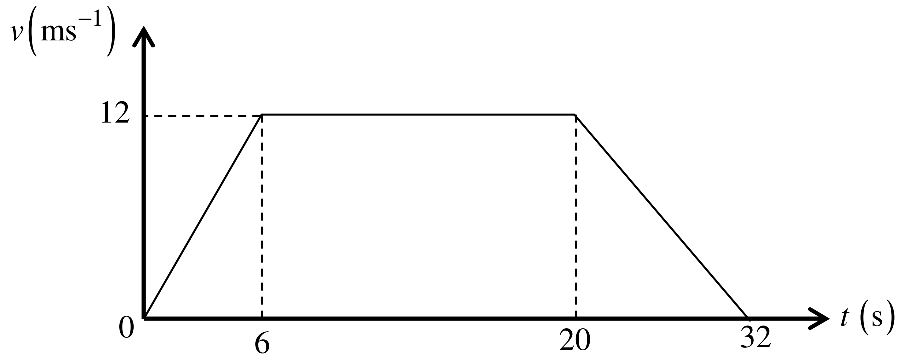
A particle A of mass 2 kg is connected to small box B of mass 3 kg by a light inextensible string. The string passes over a light smooth pulley P, which is located at the end of a horizontal house roof. The box is held on the roof with the particle hanging vertically at the end of the roof, as shown in the figure above.

The system is released from rest with the string taut, so that the distance BP is 4 m. On release, the motion of B takes place over a smooth section of the roof.

After B has moved for 2.5 m the roof becomes rough and the coefficient of friction between B and the roof is 0.75.

Calculate the speed with which B hits P. **(14)**

Question 15



The figure above shows the speed time graph (t, v) of a car travelling along a straight horizontal road between two sets of traffic lights.

The car starts from rest at the first set of lights and accelerates uniformly for 6 s, reaching a speed of 12 ms^{-1}. This speed is maintained for 14 s, before the car decelerates uniformly for 12 s, coming to rest as it reaches the second set of lights.

The distance of the car, $s(t)$, measured from the first set of traffic lights is given by

$$s(t) = \begin{cases} f_1(t) & 0 \le t < 6 \\ f_2(t) & 6 \le t < 20 \\ f_3(t) & 20 \le t < 32 \end{cases}$$

where $f_1(t)$, $f_2(t)$ and $f_3(t)$ are functions of t.

Determine simplified expressions for $f_1(t)$, $f_2(t)$ and $f_3(t)$. **(11)**