# IYGB GCE

## Mathematics FP2

## Advanced Level

### Practice Paper K

Difficulty Rating: 3.5933/1.6620

### Time: 1 hour 30 minutes

**Candidates may use any calculator allowed by the regulations of this examination.**

**Information for Candidates**

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 8 questions in this question paper.
The total mark for this paper is 75.

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

I.Y.G.B. www.madasmaths.com

**Question 1**

A Cardioid has polar equation

$$r = 1 + 2\cos\theta, \; 0 \le \theta \le \frac{\pi}{2}.$$

The point $P$ lies on the Cardioid so that the tangent to the Cardioid at $P$ is parallel to the initial line.

Determine the exact length of $OP$, where $O$ is the pole. **(7)**

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**Question 2**

$$f(x) = \sinh x \cos x + \sin x \cosh x, \; x \in \mathbb{R}.$$

**a)** Find a simplified expression for $f'(x)$. **(3)**

**b)** Use the answer to part **(a)** to find

$$\int \frac{2}{\tanh x + \tan x} \, dx.$$ **(4)**

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**Question 3**

$$I = \int_1^4 \frac{3}{(x+9)\sqrt{x}} \, dx.$$

**a)** By using a suitable substitution find an exact value for $I$. **(6)**

**b)** Show clearly that the answer to part **(a)** can be written as $2\arctan\frac{3}{11}$. **(3)**

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**Question 4**

Find a solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 20\sin 2x,$$

subject to the boundary conditions $y = 1$ and $\frac{dy}{dx} = -5$ at $x = 0$. **(10)**

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**Question 5**

$$f(r) = \frac{2}{r(r+1)(r+2)}, \quad r \in \mathbb{N}.$$

**a)** Express $f(r)$ into partial fractions. **(2)**

**b)** Hence show that

$$\sum_{r=1}^{n} f(r) = \frac{1}{2} - \frac{1}{(n+1)(n+2)}.$$ **(6)**

**c)** Find the value of the convergent infinite sum

$$\frac{1}{5\times6\times7} + \frac{1}{6\times7\times8} + \frac{1}{7\times8\times9} + \ldots$$ **(3)**

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**Question 6**

$$y = 2x\arcsin 2x + \sqrt{1-4x^2}, \quad -\tfrac{1}{2} \le x \le \tfrac{1}{2}.$$

Show clearly that

$$\frac{d^3y}{dx^3}\left(y - x\frac{dy}{dx}\right) = x\left(\frac{d^2y}{dx^2}\right)^2.$$ **(10)**

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**Question 7**

Evaluate the integral

$$\int_0^{\infty} \frac{2}{1+2x} - \frac{x}{1+x^2} \; dx \,,$$

showing clearly the limiting processes used.

Give the answer in the form $\ln N$, where $N$ is a positive integer. **(7)**

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**Question 8**

$$f(z) = z^6 + 8z^3 + 64 \,, \; z \in \mathbb{C} \,.$$

**a)** Given that $f(z) = 0$, show that

$$z^3 = -4 \pm 4\sqrt{3}\mathrm{i} \,.$$ **(3)**

**b)** Find the six solutions of the equation $f(z) = 0$, giving the answers in the form $z = r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. **(6)**

**c)** Show further that …

**i.** … the sum of the six roots is zero. **(1)**

**ii.** … $\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{6\pi}{9} + \cos\frac{8\pi}{9} = -\frac{1}{2}$. **(4)**

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