Inspired Instruction

Algebra
Creating Equations
Population Growth/Decay
High School

Rationale

- Understanding how populations grow and shrink is a fascinating concept for many students. Mathematically, population growth or decay helps students understand nonlinear functions in a real-world context. By applying nonlinear functions to their own communities, students can visualize the importance of functions in predicting patterns that affect their lives. In this project-based lesson, students will determine the population growth or decay of their town or city.

Goal
- To apply knowledge of creating equations based on exponential and quadratic functions

Standards
- **A-CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Objectives
- Students will determine whether their community's population has been increasing or decreasing and will create equations to model future population growth.

Materials
- Pencils
- Calculator
- Internet access or census data

Procedures
- Introduce the lesson by asking students if anyone knows the population of their town or city. Students may know general estimates. If a student says he or she knows the precise population, ask how him or her to describe how they know.
- Ask students to consider whether their town or city is increasing or decreasing in population, and how they can determine this information. Prompt students to consider all of the variables involved in figuring out a population increase or decrease (i.e. time, initial population, etc…).
- Use the census data available at census.gov to determine the 2000 and 2010

populations of the students' town or city.

- Split the class into small groups of 3–4 students in each group. Ask students to use the populations of 2000 and 2010 to determine the average rate of growth or decay each year. Remind students that a rate can be a percent, and that the annual rate can be found by dividing the rate of change from 2000 to 2010 by 10. Make sure that students first recognize whether the population is increasing or decreasing.
- Ask the groups to figure out how they could estimate the population one year after the 2010 census. Then ask them to estimate the population five or 10 years after 2010.
- Students may naturally try to set up an equation to determine the population estimates. However, encourage students to brainstorm how they would create an equation to represent the population after *n* years.
- Closing: Have students use their equations to determine the population of their town or city in 50, 100, or even 1,000 years. Once they've solved for the population, ask them to use their equations to predict the number of years that their town or city will reach a specified population. In both cases, students will be solving their equation in one variable to find the answer.

Teacher Tips

- Some students may need assistance working with exponents. For students struggling to derive an appropriate equation, be sure to direct students to recognize why an exponent would be useful in this situation. Encourage students to consider which situations that an exponent should be used.
- Some students may need to review how to change a percent to a decimal. Because most annual population changes are small, students may need to correctly convert the number formats from a percent less than 1, such as 0.35% to 0.0035.
- Encourage students to use a variable that matches the first letter of the unknown quantity.
- After the activity has ended, show students the formula for exponential growth/decay. Let students brainstorm other real-world applications for the exponential growth and decay formula besides population.

Extension Activities

- Have students research the populations on their own towns or cities, using online resources. Students can also use the different populations of various cities, countries, or the world. Have them predict future growth or decay of those areas based on what they learn.
- Students can use the population data from other years, including the census data from 1970, 1980, and 1990, to determine the population trends over a greater period of time.
- Students can graph their exponential function to illustrate the effect on population over a great period of time. Students can compare their graphs to graphs of linear functions.
- If students used a function with exponential growth, ask them to create another function with decay using a real-world population. (You can use the populations of cities or countries experiencing a population decay.) If students used a function with exponential

decay, ask them to create another function with growth using a real-world population.

- Students can set up their population equations to determine when the population will reach very large or very small populations, such as 10,000,000 for an expanding city or 10 for a shrinking small city.
- Introduce a scenario that students can model with a quadratic equation instead of an exponential equation. For example, explain that the acceleration of an object on Earth due to gravity is 9.8 meters per second. Students can work to generate an equation using an initial velocity and initial height. For example, a model rocket is launched from a height of 100 m with a velocity of 200 m/s. The time it will take for it to come back to Earth at a height of 0 m can be modeled by the equation $0 = (-9.8)t^2 + 200t + 100$. Let students investigate on their own or in small groups to generate a similar or related quadratic function. Students can also graph their quadratic functions to compare them with the patterns in exponential and linear functions.
- Introduce a scenario that students can model with a simple rational equation instead of an exponential equation. Ask students to model the average cost of a common household appliance that would last for *n* years. Students could estimate or research the purchase cost of the item and the cost of the electricity to run it for the annual cost for the item. Next, have students develop an equation setting the annual cost of the appliance equal to the expression showing the number of years that they would own the appliance. The final equation should contain a fraction, such as $1{,}000 = \frac{(600 + 900n)}{n}$, where *n* represents the number of years.