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## **Report on the PhD Thesis “Certification of entangled quantum states and quantum measurements in Hilbert spaces of arbitrary dimension” by Shubahayan Sarkar**

The PhD thesis by Shubahayan Sarkar focuses on the concept of quantum certification. This is a central concept in the field of quantum information science and technologies: given a quantum device, it is crucial to provide tools and method to certify its correct functioning or detect the presence of relevant quantum phenomena. The problem is especially challenging because many parameters are needed to determine a quantum state. Therefore, one should often be able to certify the property of interest without having complete information about the quantum system of interest. Another important point is related to the assumptions used to certify. This is particularly relevant in cryptographic scenarios, where assumptions that may not be met in practice open the door to security loopholes. The device-independent scenario and variants of it are designed to alleviate this problem by minimising the number of assumptions in the certification process. The main theme of the thesis is therefore extremely timely and the obtained results significantly contribute to improve the state of the art in the question.

The thesis starts with a few-page introduction in which the structure, motivation and main results of the thesis are presented. It is useful because it provides a clear overview of what follows.

Chapter 2 provides an introduction to the main concepts and tools used in the thesis. It starts with standard elements of quantum theory and quantum information theory, such as quantum states, quantum dynamics or measurements. Then, it provides a description of the two main ingredients used in the thesis for certification, namely quantum steering and Bell nonlocality, and how they are used for self-testing, one-sided device-independent entanglement certification and randomness certification. This chapter is rather detailed and serves as a solid basis for the next ones.

Chapter 3 contains one of the main results of the thesis: the proof of self-testing for the so-called Greenberger-Horne-Zeilinger state of an arbitrary number of systems and dimension. The proof is based on the maximal violation of Bell inequalities previously derived by the PhD supervisor and co-workers. These results are quite remarkable since, as also mentioned in the thesis document, most of the results on self-testing are derived for qubit systems and there exist few results valid for arbitrary dimension. The construction provided in this Chapter is one of the few exceptions. Moreover, the results are obtained for the minimal scenario consisting of only two measurements per system.

While Chapter 3 is obtained in the fully device-independent scenario, where devices are completely uncharacterized, in the two next chapters the student works in the one-sided device-independent scenario where one of the two devices is completely characterized (trusted) and the other is not. This asymmetric scenario is meaningful in situations in which there exist users with different power and resources. The quantum property used in this scenario is no longer Bell nonlocality but quantum steering, a weaker form of quantum correlations.

In Chapter 4, a scheme to certify quantum measurements based on steering is shown. The construction in fact applies to a subset of measurements that are called “genuinely incompatible” which contain the so-called mutually unbiased measurements, a type of measurements that find different applications in quantum information theory. The student also shows that the certification protocol is robust against noise, which makes it relevant for a possible experimental implementation. It is worth mentioning that the certification of mutually unbiased bases in the device-independent scenario remains an open problem. The results of this chapter show that this is possible when one of the two devices is trusted.

Finally, in Chapter 5, a scheme to certify any bipartite pure state of arbitrary dimension based on steering inequalities is provided. The construction is also expanded to certify any rank-one extremal measurement. Finally, these results are used to design schemes for maximal randomness certification in steering scenarios, in particular to obtain the maximum randomness attainable in systems of a given dimension. Interestingly, the results of the thesis show that for some values of the dimension, the amount of randomness is independent of the entanglement present in the state.

The last Chapter ends with the discussion of the results in the thesis and a list of open questions. The discussions are nicely organised into two categories, from a fundamental and practical perspective, where the student proves to have a good understanding of the implications of the thesis results. He also provides a list of relevant open questions which describe ideas that deserve further investigation.

All the results in the thesis are, to my knowledge, new. They have led to four manuscripts, two accepted for publication and two under review. The student appears as first leading author in all of them, which is remarkable. I have no doubts that the presented dissertation meets the formal requirements for a PhD thesis and recommend admission of the Candidate to the subsequent stages of the procedure, including the public defence.

Finally, I would like to conclude this report by saying that the obtained results significantly advance our current understanding of certification methods. Most of the existing results so far were obtained for qubit systems or in the case of two two-outcome measurements where one can invoke the so-called Jordan’s lemma, a rather specific technique that cannot be applied to the general case. Going beyond this firm territory is much more difficult because we lack techniques. The present thesis makes significant progress in this almost unexplored direction. I therefore expect that the derived results will find applications in subsequent works by the student and/or other researchers. Based on all these points, my assessment on the thesis work is openly positive and I recommend that it is considered for a distinction.

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