DYNAMIC ERRATA

First update in October 2021
Second update in April 2022, marked in RED
Third update in April 2023, marked in GREEN
Fourth update in March 2024, marked in BLUE

FOR

Visual Differential Geometry and Forms
A Mathematical Drama in Five Acts

The current printing of VDGF implements corrections to all the errors that are listed in the STATIC INITIAL ERRATA—a separate PDF file on this VDGF.space website.

This DYNAMIC ERRATA, which will be updated periodically, lists errors that have continued to be discovered and reported by readers *after* PUP froze the current version of VDGF.

NOTE: Some errors were reported by multiple readers, and I am equally grateful to them all, but I have only cited the *first* reader to have reported each error.

Although general questions and comments are welcome at my USF email address (needham@usfca.edu), I would be very grateful if *errors* could be reported exclusively to the special address I created for this purpose, as announced in the Prologue of VDGF, namely, VDGF.correction@gmail.com.

PAGE	LOCATION	CORRECTION	FROM
xxvi	Final paragraph	Contrary to my wishes and my expectations, PUP did *not* include Professor Morgan's "remarkably generous assessment of VDGF on the back cover of this book." Instead, it may be found here, on the home page of VDGF.space.	T.N.
14	Line 3 of 1.6	Delete comma after "Euclidean Geometry"	T.N.
15	5th bullet	"…*similar triangles of different size do not exist!*"	Jeff Scargle
20	6 lines below (2.5)	"…(as it is everywhere in the region…"	Michele Polli
21	Line 6	"…according to the definition (2.1)…"	Juhani Pylkkänen
21	Line 4 of second paragraph	"…Ex. 22 on page 89)…"	Peter Bienstman
22	Line 1 of 2.3	(1.8) should be (1.3)	Joseph P. Skudlarek

33	First line after second equation	"Finally, by Pythagoras's Theorem…"	T.N.
43	First line of text	Replace "Equation (4.18) says …" with this: Provided that $f'(z) \neq 0$, equation (4.18) says	Petra Axolotl
43	Line 2 of last paragraph	"…arc of the circle through…"	Thomas Starbird
44	Second line after (4.20)	"…so too does their…"	Nicholas O'Dea
46	Footnote	The first full biography of Harriot was published after I completed VDGF, and in the future I would like to add it to the bibliography and to this footnote: *Thomas Harriot: A Life in Science* by Robyn Arianrhod Oxford University Press, 2019	T.N.
60	(5.13)	Last line: "…then it is a vertical half-line…"	Matthew Phillips
65	First paragraph	"…with other areas of mathematics…"	S. Blake Allan
65	Third paragraph	"That is about to change."	S. Blake Allan
69	Line 4	"…(recall (4.21) which…" –-instead of "[4.21]"	Alan Mehlenbacher
72	Line 2 of penultimate paragraph	"…seems clear that this is indeed the most general…"	Charles Weiner
73	Second sentence of second paragraph	Replace "As Euler was the first to prove (in 1775) the rigid…" with "Euler was the first to prove (in 1775) that the rigid…"	Nicholas O'Dea
86	First line of Ex. 12	Repeated word: "the the"	Haruyuki Kawabe
90	Ex. 25	"We know from [6.4]…"	Sang Park
97	Line 6	"… at the close of Act II …"	T.N.
98	Sentence before (8.1)	"… which we shall meet in a moment, …"	Eric LaMotte
106	Caption of [9.1]	Repeated word: "that that"	T.N.
111	(10.4)	"…in which these planes intersect…"	Charles Weiner
113	Line -12	Repeated word: "at time time t"	Prof. Wei'an Liu (Wuhan University)

115	Second line of fourth paragraph	"…standing in the middle of…"	Joseph P. Skudlarek
116	Three lines above 3rd displayed result	"…that is no longer orthogonal to the image ellipse…"	Tim Bending
116	Penultimate paragraph	"… let T_p be the tangent plane to …"	Eric LaMotte
117	11.2: Last line of first paragraph	in the **n** direction (if we choose **n** in the direction of $+\kappa_n$).	T.N.
117	11.2: Penultimate line of 2nd paragraph	"…in turn looks (locally) like an arc of a circle…"	Tim Bending
122	Third line after (11.7)	"…it makes it makes…" is repeated	Charles Weiner
131	First line of body	Repeated word: "the the"	Haruyuki Kawabe
134	Line 3	[12.2] should be [12.1]	Clayton Shonkwiler
136	Line -3	"…must be ultimately equal to…"	Ron R. Rickards
141	Line -5 of body	"…spontaneously spring back to its original…"	Ron R. Rickards
146	First new paragraph	"Imagine the polyhedron to be made up of…"	Ron R. Rickards
153	[15.3] in 15.3!	The font size for the vector components is much too large. [Strangely, my original PDF was correct.]	T.N.
154	Two lines above first bullet	"…fact that it makes no difference…"	Ron R. Rickards
156	Third paragraph	Repeated word: "that that"	T.N.
156	First line of (15.12)	Repeated word: "the the"	Haruyuki Kawabe
161	(15.22)	$\kappa(-\frac{\pi}{4} - \theta)$	Petra Axolotl
162	First line of last paragraph	"As discussed in the last chapter,…" should be "As discussed in Section 10.2,…"	Tim Bending
163	Line before (15.25)	"…this means that although…"	Clayton Shonkwiler
165	Line 4 of third paragraph	"If we instead stretch P…"	Alan Mehlenbacher

171	Line 6 of 16.4	"…plain bagels, and …"	T.N.
172	Line 2	Repeated word: "we we"	Prof. Liu Wei'an (Wuhan University)
172	Line 3 of second paragraph	"…undergoes this compression, we see that as the…"	Ron R. Rickards
173	Third paragraph	Both instances of $\mathcal{N}(s)$ should be $\mathcal{N}(\tilde{p})$	S. Blake Allan
173	Third paragraph	$\mathcal{P}(s) = +1,$ should be $\mathcal{P}(\tilde{p}) = +1,$	Tim Bending
175	Fourth paragraph	"…clear that its velocity…"	S. Blake Allan
175	Line -4	"rocking" should be "rocks"	Tim Bending
176	Line -5	Repeated word: "the the"	Haruyuki Kawabe
176	Last paragraph	Repeated word: "that that"	Clayton Shonkwiler
177	Two lines below (17.2)	Repeated word: "the the"	Haruyuki Kawabe
180	Last line of fourth paragraph	"…that region is therefore mapped to a point …"	Ron R. Rickards
194	Line before first equation	P_j	Clayton Shonkwiler
211	Line -4	"…must return to its…"	Thomas Starbird
217	First line of third paragraph	"However, (19.14) enables us to prove…"	Tim Bending
218	Penultimate line of first paragraph	"…purely local *geometric* measurements…"	Ron R. Rickards
222	Ex. 15 (i)	Change F to -F, thereby changing $\mathbf{n}$ to $-\mathbf{n}$	Petra Axolotl
223	Ex. 16	In part (vi), the equation should not be broken between lines. In part (vii), it should reference "(vi)" (not "(v)").	T.N.
224	Ex. 18: (20.4)	Delete the mysterious "g" from the denominator. [My LaTeX code did not contain this "g"!]	Petra Axolotl

224	Ex. 19: Last sentence of introduction	Replace "… the converse is false." with "… surfaces of equal but *variable* curvature need not be isometric. Thus the converse of the *Theorema Egregium* is false."	Qiao Lu
226	Ex. 25	Swap V and E in parts (i) and (ii).	Petra Axolotl
231	Footnote 6	"The 2017 Nobel Prize in Physics…"	T.N.
234	Line -3 of paragraph 4	"…we have chosen it to have the same unit length…"	Tim Bending
243	Framed equation after (23.3)	$\mathrm{D}_{\mathbf{v}}\mathbf{w} = \nabla_{\mathbf{v}}\mathbf{w} - [\mathbf{w}\cdot\mathrm{S}(\mathbf{v})]\,\mathbf{n}.$	John Stroughair
244	Second paragraph	"…that is parallel transported…"	Clayton Shonkwiler
251	First line of Section 25.4	"Consider [25.1]…"	Alan Mehlenbacher
254	Last bullet point	The bullet point should end right before, "Thus, combining …", which should itself begin a new paragraph.	T.N.
255	Line 2 of [25.1]	Missing a left-hand bracket. Should be: $\textit{equals the area}\ \widetilde{\mathcal{A}}\,(\widetilde{\Omega})$	Xinwen Wang
262	Line 1 of first text paragraph	"The *metric* then tells…"	Thomas Starbird
263	Footnote 1	Delete "it"	Thomas Starbird
264	Last paragraph	The apparent rotation along $\hat{\mathrm{f}}\,\hat{\mathrm{g}}$	Valter Sorana
265	Line -6 to -5 of text	"We now recognize the fact that the holonomy…"	Ron R. Rickards
266	End of first paragraph	Missing final period.	Ron R. Rickards
266	27.4: Line 1 of paragraph 6	"…projected onto the tangent plane…"	Nicholas Dreyer
269	Last line of Section 28.1	"This is the essence of Jacobi's discovery."	Alan Mehlenbacher
270	Three lines above second frame	$\lvert(\dot{\mathrm{r}}\,\delta\theta)\rvert$	Wei Liu [@NUDT]
271	Third line of [28.3]	"…of the sphere of radius…"	Ron R. Rickards

274	Second line of second paragraph	"…surfaces of revolution…"	Tim Bending
276	Line 5 of last paragraph	"…are then repelled from each other…"	Ron R. Rickards
277	Line 6 of paragraph 3	"…emanate from different points and lie within…"	Ron R. Rickards
283	Line above double-frame	$\mathbf{w}_o$ should be $\mathbf{w}(p)$	S. Blake Allan
286	Second paragraph of 29.5.1	Swap the letters u and v in $\mathbf{u}(a)\,\delta u$ and $\mathbf{v}(p)\,\delta v$	S. Blake Allan
289	Three lines above first equation	Replace, "Then the holonomy…" with "Taking **U** to have unit length, the holonomy…"	Petra Axolotl
294	Line 1 after 2nd double frame	Forgot to divide by 2: "… from 81 to 27."	Qiao Lu
294	Line 1 after (29.13)	Forgot to divide by 2, again: "… from 27 to 9."	Qiao Lu
296	Six *text* lines from bottom	"…again assume it is a unit vector…"	Tim Bending
299	[29.10] caption	Delete duplicate source attribution.	T.N.
299	Line 10 of 29.6.1	"… now there are …"	Filip Stappers
304	(29.27)	The result is missing a concluding period.	S. Blake Allan
321	First line of penultimate bullet	"…then the solution only applies to the vacuum region…"	Ron R. Rickards
322	Line 2 of #1	"… long before he had …"	Filip Stappers
323	30.8: Third line of paragraph 3	"…to the direction of the wave…"	Ron R. Rickards
327	End of (30.16)	$= 4\pi G\,\mathbf{T}(\mathbf{v},\mathbf{v}).$	Petra Axolotl
329	Penultimate paragraph	"…these flashes can pass through the matter of the core,…"	Wei Liu [@NUDT]
330	[30.9] The Birth of a Black Hole	In the "corrected" printing (addressing the STATIC INITIAL ERRATA) this figure is *misprinted*, whereas it was printed *correctly* in the original release! [NOTE: The figure is reproduced correctly in the *Kindle* edition of the corrected "printing".] In the faulty version, the majority of the (crucial) null cones have simply	T.N.

		vanished, most confusingly along the trajectory of the "massive particle", and the few null cones that remain have been badly mutilated. UPDATE: The very *latest* reprint is reported to have restored this figure to its original, correct form.	
335	Ex. 5(i) Line 2	"…allows us to…"	Thomas Starbird
338	Ex. 11	In part (ii): "…if we also take these pair symmetries into account…" In part (iii): Defining $B_{ijkl} \equiv R_{ijkl} + R_{jkil} + R_{kijl}$.	Tim Bending
340	Ex. 15	I neglected my own "CONVENTION WARNING" on page 305! My definition of **R***icci* is the *negative* of both MTW's and Penrose's, and therefore, using my conventions, the correct formula for the Weyl curvature is $C_{ij}{}^{kl} \equiv R_{ij}{}^{kl} + 2R_{[i}{}^{[k}\,g_{j]}{}^{l]} - \frac{1}{3}R\,g_{[i}{}^{k}\,g_{j]}{}^{l}.$ i.e., $C_{ijkl} = R_{ijkl} + \frac{1}{2}(R_{ik}\,g_{jl} + R_{jl}\,g_{ik} - R_{il}\,g_{jk} - R_{jk}\,g_{il}) + \frac{1}{6}R\,(g_{jk}\,g_{il} - g_{ik}\,g_{jl})$	Tim Bending
341	Ex. 15	In part (iii), delete a comma: "…implies that in vacuum, the Weyl tensor…"	T.N.
349	Line 5	"…to complete the interpretation …"	T.N.
355	Three lines up from second framed equation	"…, as illustrated."	Charles Weiner
356	Bottom frame	Missing space before "is the Cartesian …"	T.N.
357	Penultimate displayed equation	$+ \left[\partial_y f\right] \mathbf{dy}\}$	S. Blake Allan
364	Last paragraph	"…is indeed represented by a familiar…"	Clayton Shonkwiler
367	Second line of penultimate paragraph	"In order to follow the same path as before…"	Ron R. Rickards
386	Line 4 of 35.2	"Let us begin with the simplest …"	Filip Stappers

392	Line below (36.1)	"This expression is now antisymmetric…"	Ron R. Rickards
393	Line 1 of second paragraph	"…variation in the vector fields…"	Thomas Starbird
393	Line 1 of second paragraph after (36.2)	"The key to simplifying (36.2)…"	T.N.
395	Line 5 of 6-line equation	$= \quad (\mathbf{d}\phi) \wedge \Psi - [\phi_i \, \mathbf{d}x^i] \wedge [\mathbf{d}\Psi_{jk} \wedge \mathbf{d}x^j \wedge \mathbf{d}x^k]$	Wei Liu [@NUDT]
396	36.4.2	Both lines immediately beneath the first two framed equations should not be indented.	Ron R. Rickards
399	$\mathbf{d}\Psi \quad =$	In the 2nd and 3rd lines, $\partial_2\Psi^3$ should be $\partial_3\Psi^3$,	Petra Axolotl
400	First frame	$\mathbf{d}\varphi = \mathbf{d}^2 f = 0 \qquad \Longleftrightarrow$	Petra Axolotl
400	Second frame	$\mathbf{d}\Psi = \mathbf{d}^2\varphi = 0 \qquad \Longleftrightarrow$	Petra Axolotl
405	First line	"…definition of the integral of the 1-form…"	Clayton Shonkwiler
406	Line 4 of 37.1.3	"… is simply the net change…"	Filip Stappers
409	Line 4 of 37.2.2	Let us define $\mathbf{\Omega}(\epsilon\mathbf{u}, \epsilon\mathbf{v}, \epsilon\mathbf{w})$ to be the integral of Ψ over Π:	Filip Stappers
410	End of penultimate paragraph	Last line missing closing period.	Ron R. Rickards
412	Last line	The last line should not be indented.	Ron R. Rickards
414	Second line of first paragraph	"…must be taken to be an oriented…"	Ron R. Rickards
415	Second line of penultimate paragraph	"We shall collectively refer to the parallelograms…"	Ron R. Rickards
416	Penultimate line of	Remove the comma after "two"	Ron R. Rickards

	second paragraph		
427	First line	(37.21) should be (37.19)	S. Blake Allan
428	3 lines above 2nd equation	Should just be \lambda, NOT \lambda_1	Xinwen Wang
434	Penultimate paragraph of 38.2.2	**IDEA 6** is printed in too large a font size.	S. Blake Allan
437	Third line of body	Repeated word: "the the"	Haruyuki Kawabe
439	(38.9)	Sum over j (not k)	T.N.
440	First line after (38.12)	"…is that it characterizes…"	S. Blake Allan
442	Caption of [38.3]	**[38.3]** *Geometric proof that* $\boldsymbol{\theta}^1 = \mathbf{d}r$, $\boldsymbol{\theta}^2 = r\,\mathbf{d}\phi$, *and* $\boldsymbol{\theta}^3 = r\sin\phi\,\mathbf{d}\vartheta$.	T.N.
446	End of second line below (38.20)	Should be "avoid", not "avoid-ing"	Ron R. Rickards
455	38.12.1: Second line of first paragraph	"…the following property of an arbitrary…"	Ron R. Rickards
459	(38.55)	Insert a *minus sign* on the left-hand side, in front of the Riemann tensor: $$-R^i{}_{jkl}\,\boldsymbol{\theta}^k \wedge \boldsymbol{\theta}^l = \boldsymbol{\Omega}^i{}_j = \mathbf{d}\boldsymbol{\omega}^i{}_j + \boldsymbol{\omega}^i{}_m \wedge \boldsymbol{\omega}^m{}_j.$$ Sadly, this was a genuine convention error on my part (not a typo), so *all* the Riemann tensor components in Section 38.13 should have their signs reversed, too. [This is equivalent to swapping the last two indices--see new corrections below.] Fortunately--for my book, and for our *Universe*!--this does not affect the *conclusion* of my climactic section: the black hole geometry that Schwarzschild discovered in 1915 is *indeed* a solution of Einstein's Vacuum Field Equation, because -0 = 0!	John Stroughair & (independently) Petra Axolotl
460	First (unframed) equation	Missing a minus sign: $0 = -\boldsymbol{\Omega}^i{}_j \wedge \boldsymbol{\theta}^j =$	T.N.
460	Last line	(38.58) should be (30.12)	S. Blake Allan

463	Two lines above double-framed equations	Swap the last two indices of the Riemann tensor: $$\boldsymbol{\Omega}^{i}{}_{j} = R^{i}{}_{jlk}\,\boldsymbol{\theta}^{k} \wedge \boldsymbol{\theta}^{l}$$	T.N.
463	$R_{rr} \quad =$	Change "t" to "r" in $$R^{\phi}{}_{r\phi t}$$ AND swap the last two indices–see next two corrections.	Petra Axolotl and T.N.
463	Double-framed equations	Swap the last two indices of the Riemann tensor: $$R^{t}{}_{rrt} = -\frac{f''}{2} = +\frac{2GM}{r^3}$$ $$R^{t}{}_{\phi\phi t} = R^{t}{}_{\vartheta\vartheta t} = R^{\phi}{}_{rr\phi} = R^{\vartheta}{}_{rr\vartheta} = -\frac{f'}{2r} = -\frac{GM}{r^3}$$ $$R^{\vartheta}{}_{\phi\phi\vartheta} = -\left[\frac{1-f}{r^2}\right] = -\frac{2GM}{r^3}.$$	T.N.
463	Last four equations	Swap the last two indices of the Riemann tensor, thereby restoring my definition of **R***icci* on page 305 (which is the *negative* of MTW's and of Penrose's): $$R_{tt} = R^{r}{}_{ttr} + R^{\phi}{}_{tt\phi} + R^{\vartheta}{}_{tt\vartheta} =$$ $$R_{rr} = R^{t}{}_{rrt} + R^{\phi}{}_{rr\phi} + R^{\vartheta}{}_{rr\vartheta} =$$ $$R_{\phi\phi} = R^{t}{}_{\phi\phi t} + R^{r}{}_{\phi\phi r} + R^{\vartheta}{}_{\phi\phi\vartheta}$$ $$R_{\vartheta\vartheta} = R^{t}{}_{\vartheta\vartheta t} + R^{r}{}_{\vartheta\vartheta r} + R^{\phi}{}_{\vartheta\vartheta\phi} =$$	T.N.
464	Line 3 of second paragraph	"For this discovery, Penrose was awarded half of the 2020 Nobel Prize in Physics; …" AND Change corresponding Penrose index entry on page 498	T.N.
466	Exercise 3 (ii)	$$[\tilde{\varphi}_1, \tilde{\varphi}_2] = [\varphi_1, \varphi_2]\,[R_\theta].$$	Petra Axolotl
467	Ex. 12	(36.9) should be (36.8)	S. Blake Allan
467	Last line	Delete space after "closed"	Ron R. Rickards
470	Exercise 17(i)	$$^{+}\mathbf{F} = \frac{1}{2}\left[\mathbf{F} - i\star\mathbf{F}\right] \quad \text{and} \quad ^{-}\mathbf{F} = \frac{1}{2}\left[\mathbf{F} + i\star\mathbf{F}\right].$$	Petra Axolotl

480 AND 485	*Further Reading* AND Bibliography	There is a new 2022 3rd edition of Banchoff and Lovett's, *Differential Geometry of Curves and Surfaces*	Alan Mehlenbacher
483 AND 489	*Further Reading* AND Bibliography	Weintraub's *Differential Forms* has a 2014 2nd edition with a new subtitle: *Theory and Practice*	Alan Mehlenbacher
485	Bibliography	There are missing entries for the following books, which are referenced in the *Further Reading* section: Chinn and Steenrod's *First concepts of topology* Wheeler's *A Journey into Gravity and Spacetime* Hartle's *Gravity* Zee's *Einstein Gravity in a Nutshell*	Alan Mehlenbacher
485	Bibliography	There are missing entries for the following essay and books, which are referenced in the *Further Reading* section: Berry's essay, *The Quantum Phase, Five Years After* (contained in the anthology edited by Shapere and Wilczek, which *is* in the Bibliography) O'Neill's *Semi-Riemannian Geometry* Tapp's *Differential Geometry of Curves and Surfaces*	Geoffrey Seton
487	Bibliography	Henderson's *Differential Geometry* has a self-published 2013 3rd edition, available from Project Euclid.	Alan Mehlenbacher
487	Bibliography	Hilbert's *Geometry and the Imagination* should include its second author: Cohn-Vossen, S. [His name is correctly included in the original *Further Reading* reference.]	Alan Mehlenbacher
488	Bibliography	Misner, Thorne, and Wheeler: "A marvellous new hardback edition…"	David Drysdale
489	Bibliography	I had advance knowledge of Schutz's 3rd edition of *A First Course in General Relativity*, but ultimately it was not published until *2022*, (after VDGF), *not* in 2021.	Geoffrey Seton
489	Bibliography	Weeks's *The Shape of Space* should be 3rd ed.	Geoffrey Seton
492	First entry	Page 456 should be a second entry under, "Cartan's Structural Equations: Second Equation"	T.N.
492	contraction	Missing index entries for "contraction": "of 1-form and vector, 347" "of tensors, 365—366"	Friedrich Hartmann