Why did God send deadly snakes?

Obviously, it was because the people spoke against God (21:5-6). However, they had often done so in the past as well without such severe consequences. (See Exodus 15:22-25, 16:1-5, and 17:1-7.) The difference in Numbers 21:4-9 seems to be that they did not complain because they lacked food or water. Rather, they loathed the food, the manna, that God had provided (21:5). So their complaining was like a thankless child throwing food on the floor, rather than a small child crying because of hunger.

Why didn't God totally remove the snakes?

After many had died, the people repented and asked for the venomous snakes to be removed (21:7). Rather than doing so, however, the Lord provided a way for individuals who were bitten to be saved (21:9). In Exodus, the frogs that plagued Pharaoh died and became a stinking mess, but for God's people in the wilderness the provision was better. The snakes which represented sin continued to bite the people, but each one could turn and look with faith at the bronze serpent in order to be spared. This was in line with Jesus' explanation of salvation in John 3:14-16 and Paul's teaching on the same in Second Corinthians 5:21.

Why is the fiery serpents passage so short?

It is only five verses. Yet, it seems to be important since the Lord Jesus cited the incident in John chapter three and Paul cited it in First Corinthians chapter ten. Moreover, Moses included it in his summary of key wilderness events in Deuteronomy 8:14-17. Why then is the original passage so short? We do not know, but the important, water-from-the-rock passage, Exodus 17:1-7, is also short. Some may wish to know what the rock that was struck looked like, but describing it would have distracted from the miracle. Likewise, describing the snakes in detail would probably have been distractive as well.

What kind of venomous snakes were they?

It is impossible to tell with certainty, and groundless speculation is unhelpful. Nevertheless, there are some indications that the snakes may have been desert horned vipers which are known for their stealth and quick strikes. (These may be referred to as fiery flying serpents in Isaiah 14:29.) Alternatively, they may have been common sand vipers which also hide in the sand but become more openly aggressive as their annual hibernation period approaches. Their being sent by the Lord (Num. 21:6) may have involved their natural aggressiveness at that time of year.

What are the applications?

Obviously, we each should personally believe in the One foreshadowed by the bronze serpent (John 3:14-15). Looking in Numbers 21:9 is much like believing in John 3:15. In addition, of course, we as believers should not complain about God's provision like the Israelites did. (Compare Numbers 21:5 and First Corinthians 10:9.) Each snake bite had to be taken seriously. Likewise, a sinful, unthankful heart of unbelief toward the Lord needs to be repented of quickly.