ARTIFICIAL NEURAL NETWORK REGRESSION MODELS FOR THE PREDICTION OF BRAKE-RELATED EMISSIONS

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ABSTRACT: The growth of electric propulsion systems motivates the automotive industry to transfer the focus from exhaust to non-exhaust emissions, with special attention to brake-related emissions. The literature lacks well-established approaches that describe the particulate emissions through reliable analytical correlations. Moreover, the mechanisms of brake particulate formation entail highly stochastic phenomena, which cannot be captured by means of traditional deterministic modelling tools. Machine learning algorithms have been recently used as an alternative method to seek for a branched correlation between tribological properties (i.e. friction coefficient and wear rate), pad composition, environmental and operating conditions. In this regard, the presented work focuses on the study and identification of sophisticated stochastic meta-models for the prediction of the number of emitted brake particulate and associated uncertainty. Specifically, artificial neural networks are developed and validated against brake emission data collected in real driving conditions at Technische Universität Ilmenau. The developed algorithms are intended for multiple use: (i) in the course of real driving emissions (RDE) testing, to support the experimental data; (ii) while driving, to inform the driver about the brake-related emission levels; (iii) as an on-board optimisation tool that identifies the brake actuation rules to minimise the release of particulate emissions.

KEY WORDS: e.g. particulate number concentration, real driving conditions, correlation analysis, artificial neural networks

1. Introduction

The growth of electric propulsion systems motivates the automotive industry to transfer the focus from exhaust to non-exhaust emissions, with special attention to brake-related emissions. The brake operation is affected by parameters, such as friction and temperature, which have a strong impact on brake wear and particulate emissions [1]. The factors influencing the particulate formation are still not fully understood but from recent experimental analysis carried on advanced laboratory equipment, it is possible to state that: (i) parameters like the particulate number, particulate size distribution and the chemical compositions of the emitted brake dust particulate is mainly dependent on the system temperature and the frictional power [2]; (ii) brake pads that exhibit lower friction stability behaviour also produce higher amount of particulate [3]; (iii) the copper contributes to maintain a stable friction layer and therefore to achieve a lower emission of particulate [4]; (iv) the wear rate of the pad decrease with increasing contact pressure and sliding speed [5] until the disc temperature is about 200 °C; thereafter, they increase because high temperature values induce an unstable friction layer [1], [6].

The proper assessment of non-exhaust related emissions for the quantification of emission factors (EFs) represent a very awkward task and, at present, there is still no standardised testing protocol and/or measurement method. Common procedures of brake particulate characterisation generally draw upon: (i) receptor modelling that associates brake particulate to a range of sources that are easy to measure [7], [8]; (ii) direct measurements from the sources that involves the use of more sophisticated sensors [9].

These latter can be further divided into laboratory-based and on-road tests [10].

The literature lacks a well-established theory that describe the particulate emissions through reliable analytical correlations. Artificial neural networks have been recently used as an alternative method to seek for a branched correlation between tribological properties (i.e. friction coefficient and wear rate), pad composition, environmental and operating conditions [11]. In [12], ANN were used to estimate the brake linings’ coefficient of friction and wear rate as a function of brake pressure, sliding speed and surface temperature. In [13], the friction material composition was also included in the ANN to embed the variability induced by different lining compositions. In [14], an attempt was made at predicting the composition of ingredient of brake pad materials to optimise their frictional stability. Particularly, the network was used to find a correlation between the modifications in pad’s composition and change in the frictional characteristics. In [15], the friction material composition, its manufacturing conditions, and the brake’s operating conditions are used as input of ANNs to predict the friction coefficient.

In this work, the authors address the estimation of brake-related particulate emissions in real driving conditions (RDE). This is done in the wake of the EU member States’ amendment adopted on May 3rd, 2018, to Directive 2007/46/EC, Commission Regulation (EC) No 692/2008 and Commission Regulation (EU) 2017/1151, which introduced the real driving emissions (RDE) tests as a mandatory part of the type-approval procedure for new passenger cars and light-commercial vehicles in EU. The RDE is based upon on-road emissions testing with the Portable Emissions Measurement
Systems (PEMS). Due to the high non-linearity and stochasticity of the problem at hands, the authors adopt a meta-modelling approach based on the machine learning techniques, namely artificial neural network (ANN), for the prediction of the brake-related particulate number (PN) under real driving conditions (RDE). The distinctive feature of ANNs is that they are trained to learn a weighted space of possible solutions rather than a single value. To the best of the authors knowledge, ANN have not been used previously to predict brake-related PN. In this work, two model configurations are tested:

- static ANN, considers as input the average conditions during each braking manoeuvre;
- dynamic ANN, accounts for the transient behaviour during each braking manoeuvre.

The functionality of the proposed ANNs is demonstrated against experimental data collected from a light commercial vehicle (LCV) equipped with cast-iron brake discs and copper-free ECE brake linings. Brake related particulate emission are acquired by means of a particulate emissions measurement system (PEMS) at Technische Universität Ilmenau. The sampling system is based on the mobile brake enclosure, whose effectiveness was demonstrated in a previous authors work [2], [16]. More than 800 braking manoeuvres, collected in real driving conditions, are used to train the ANNs. The reference RDE-conform driving cycle is conducted in the neighbourhood of TU Ilmenau and includes more than 100 km of mixed urban, rural and motorway sections. The variability among drivers and driving conditions is also accounted in this study. Preliminary results show that ANNs demonstrate good prediction performance. The models are trained and tested by means of the Ai4Uandi software-tool, based on Python programming language, created by Martin Schiele et al. at TU Ilmenau. Ai4Uandi is a Software, that enables researchers, medicals, economists and industrial companies to build Ai from scratch, by offering a very intuitive graphic user interface. The developed algorithms are intended for multiple use: (i) in the course of real driving emissions (RDE) testing, to support the experimental data; (ii) while driving, to inform the driver about the brake-related emission levels; (iii) as a tool to improve vehicle dynamics simulation studies. The paper is organized as follows. Section 2 deals with the addressed research methodology. A description of the type of RDE compliant test cycle, test vehicle and measurement equipment is provided. Section 3 gives a brief overview of the ANN theoretical background. Section 3 also addresses the data pre-processing and their analysis. Section 4 includes the simulation results and a benchmark with respect to the map based approach proposed in [5]. The authors also discuss the limitation of the proposed technique and open research questions. Finally, the conclusions are reported in Section 5.

2. Methodology

2.1. On-road tests

The measurement of the emitted brake dust particulate in real driving operation is complex due to external, continuously changing factors (e.g. flow conditions, driving dynamics, fine dust particulate from other sources). The studies conducted in the past mainly relate to laboratory test [17], [9]. In [18], the authors argue that the test reproducibility represents a critical factor which can be mainly ascribable to the employed test procedure, pad soak, environmental conditions, caliper residual torque, temperature control. In [17], it was instead demonstrated that the cycle-to-cycle repeatability is affected by the progressive wear of the brake pad-disc couple whereby, the variability tends to become lower for successive manoeuvres. The authors also concluded that the repeatability of the brake particulate measurements at disc temperature above 200°C is questionable owing to the formation of unstable friction layer. In [19], the authors conduct measurements of brake-related particulate emissions produced by a LCV on the chassis dynamometer. Although the experimental arrangement is very similar to a RDE-type test, the performed braking manoeuvres are not representative of real driving conditions and the particulate sampling system is of open type, which, as proved in a previous authors’ work, does not ensure good measurement repeatability [2].

In this work, on-road tests carried out on the LCV at TU Ilmenau allow investigating driver’s influence under different driving conditions. The drive cycles’ reproducibility under RDE conditions has been discussed in a previous authors’ work [2], where a good correlation between RDE and laboratory results has been quantitatively demonstrated. The RDE-compliant test cycle includes city, rural and highways sections, with an overall length of approximately 90 km in the immediate vicinity of TU Ilmenau. With respect to the driving cycle, the city section account for the 34%, the rural for the 33% and the motorway for the 33%. Under these conditions, the RDE test cycle complies EU Regulation 2016/427.

2.1. Measurement equipment

The test vehicle is a light commercial vehicle with a gross weight of 2286 kg and is equipped with cast-iron discs and copper-free ECE brake linings. A depiction of the test setup is reported in Figure 1. The particulate emitted by the front-left brake system is evacuated by means of the closed CVS-sampling system engineered and developed at TU Ilmenau [2], [16]. The vacuum created by a blower leads the emitted brake particulate through the measurement tunnel to the external environment. The PEMS-PN MAHA-AIP is used to measure particulate number concentrations (PNC) in the range 23 nm to 2.5 μm. The measurement principle is based on particulate condensation counting (CPC). The brakes are equipped with sliding thermocouples and pressure sensors. The vehicle is also equipped with the RaceLogic VBoX to log relevant kinematics quantities, such as vehicle speed and acceleration. The developed setup allows analysing the influence of driver, traffic, driving conditions, environment on the emissions level. Relevant vehicle and brake state variables, such as vehicle speed, vehicle acceleration, brake temperature and brake pressure are sampled at 10Hz, whilst PEMS provide the PNC at 1Hz sampling rate. The measurement system provides the particulate number concentration, expressed as number of particulate per cubic
centimetre. Knowing the volumetric flow rate of the sampling system, and supposing the PNC uniform across the tunnel section in the sampling point, the number of emitted particulate (PN) during a braking manoeuvre can be easily computed. For a matter of convenience, a logarithmic transformation is adopted to conduct a sensitivity analysis across the data. This leads to an adimensional quantity, herein referred to as emission factor (EF).

\[ EF (\text{J}) = \log_{10} \int_{t_{\text{brake}}} P\text{N} \, dt \]  

(1)

Table 1. Dataset used for the model identification (green) and validation (blue).

<table>
<thead>
<tr>
<th>Trip</th>
<th>Mean ( \sigma_x ) [m/s^2]</th>
<th>Std. ( \sigma_x ) [m/s^2]</th>
<th>N. braking [l]</th>
<th>Energy ( E_{\text{brake}} ) [MJ]</th>
<th>PN [#/km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.18</td>
<td>0.35</td>
<td>78</td>
<td>2.13</td>
<td>2.67 ( 10^3 )</td>
</tr>
<tr>
<td>2</td>
<td>0.91</td>
<td>0.42</td>
<td>112</td>
<td>5.63</td>
<td>2.04 ( 10^4 )</td>
</tr>
<tr>
<td>3</td>
<td>0.92</td>
<td>0.41</td>
<td>110</td>
<td>5.59</td>
<td>1.57 ( 10^3 )</td>
</tr>
<tr>
<td>4</td>
<td>0.97</td>
<td>0.43</td>
<td>82</td>
<td>2.80</td>
<td>0.45 ( 10^4 )</td>
</tr>
<tr>
<td>5</td>
<td>1.07</td>
<td>0.49</td>
<td>121</td>
<td>4.75</td>
<td>3.17 ( 10^4 )</td>
</tr>
<tr>
<td>6</td>
<td>0.97</td>
<td>0.44</td>
<td>84</td>
<td>2.75</td>
<td>3.32 ( 10^4 )</td>
</tr>
<tr>
<td>7</td>
<td>0.99</td>
<td>0.39</td>
<td>86</td>
<td>3.49</td>
<td>3.00 ( 10^4 )</td>
</tr>
<tr>
<td>8</td>
<td>1.21</td>
<td>0.44</td>
<td>102</td>
<td>4.41</td>
<td>2.77 ( 10^4 )</td>
</tr>
</tbody>
</table>

3. Artificial Neural Network

3.1. General remarks

ANNs differ from traditional modelling approaches since they are trained to learn the right solutions rather than being designed to model specific phenomena. Modelling of nonlinear relationships using the ANN is generally simpler in comparison with a nonlinear regression approach. Therefore, ANNs should be preferred to conventional modelling approaches when modelling highly nonlinear phenomena. ANNs require that the training examples span the whole domain of interest. As a rule of thumb, in ANNs, the operation of interpolation is always preferred to extrapolation. This issue can be overcome not only by adding knowledge to the training dataset but also by designing a neural network less confined to the available experimental data in order to avoid overfitting of training data. The nonlinearity of ANNs also implies the existence of many suboptimal solutions, which corresponds to local minima of the error function.

A neural network is composed of nodes (neurons), responsible for the processing of the information through the layer (e.g., the kind of analytical dependence among the variables) and the branches (synapses) that instead are responsible for the transmission of the signal between the layers. The neurons between the input and the output layers constitute hidden layers that add non-linearity to the system and ramify the interactions among the variables of the previous layer. The transmission of the signals is performed in analogy with the way biological neural systems operate. The signals are generated in the neurons when the information coming from the previous layer exceeds a certain threshold (bias). Once the signal is generated, it is transmitted through the synapses to the next layer; the synapses modulate the relative importance of the signals flowing between two layers.

Figure 2 provides an example of ANN, where the input layer features \( N \) inputs, the two hidden layers have \( M^{(1)} \) and \( M^{(2)} \) neurons, respectively. The output of each \( j-th \) neuron to the first hidden layer, namely \( a^{(1)}_j \), stems from the weighted summation in eq. (2) that passes through the activation function eq. (3).
Particularly, $w_{ij}^{(1)}$ is the weight between input $i$ and neuron $j$. The choice in the activation function is particularly important to capture non-linearities in the modelled mechanisms. The sum in eq. (2) is biased by the factor $b_j^{(1)}$. The same algorithm holds for the output of the second hidden layer, where, in turn, the previous layer output $a_j^{(1)}$ is weighted by the factors $w_{jk}^{(2)}$. The sum in eq. (4) is biased by the factor $b_k^{(2)}$. Finally, the output layer performs an algebraic summation, where the outputs of the $M^{(2)}$ neurons are weighted by the factors $w_{kj}^{(3)}$ and biased by $b_j^{(3)}$, as per eq. (6). The ANN prediction is represented by the variable $y$.

$$
\phi_j^{(1)} = \sum_{i=1}^{N} w_{ij}^{(1)} x_i + b_j^{(1)}, \quad (2)
$$

$$
da_j^{(1)} = \varphi \left( \phi_j^{(1)} \right) = \frac{1}{1 + e^{-\theta_j^{(1)}}}, \quad (3)
$$

$$
\phi_k^{(2)} = \sum_{j=1}^{M^{(1)}} w_{kj}^{(2)} a_j^{(1)} + b_k^{(2)}, \quad (4)
$$

$$
da_k^{(2)} = \varphi \left( \phi_k^{(2)} \right) = \frac{1}{1 + e^{-\theta_k^{(2)}}}, \quad (5)
$$

$$
y = \sum_{k=1}^{M^{(2)}} w_{kj}^{(3)} a_k^{(2)} + b_j^{(3)}, \quad (6)
$$

Before its direct application, it is required to teach the network analytical relations between input and output values in order to ensure results with the possible lowest error. It is necessary to take into consideration that the number of training data pairs has a significant influence on the network’s generalization capability.

![Diagram](image)

Figure 2. Depiction of a fully-connected ANN featuring two hidden layers of cardinality $M^{(1)}$ neurons in the first and $M^{(2)}$ neurons in the second layer, respectively.
3.2. Data pre-processing

As demonstrated in previous authors’ works [2], [16], the developed RDE measurement equipment features a good repeatability cycle to cycle and satisfactory reproducibility when compared to laboratory test data. However, given the installation position of the evacuation box, PEMS is more sensitive to background particulate concentration (BGC). Therefore, before proceeding with the driving cycle features extraction, the BGC is estimated from the RDE measurements by considering the lowest emissions level when cruising. The BGC is thus mapped against the vehicle speed, leading to the results of Figure 3.

![Figure 3. Measured background concentration and its linear mapping against the vehicle cruising speed.](image)

Afterwards, the raw data are reduced in dimensionality by extracting meaningful information during braking manoeuvres. Figure 4 reports an example of braking features extraction from one of the recorded RDE cycle. For a matter of clarity, the variables of interest are underlined in orange during braking occurrence. The numbering refers to n-th braking manoeuvre and the black cross symbol refers to the corresponding average quantity. To this effect, it is worth mentioning that the feature extraction is limited to the deceleration manoeuvres, while brake-related emissions might also occur during acceleration manoeuvres, due to the release of previously deposited particulate [19].

In the case of static ANN, the input variables are condensed into stationary quantities, characterizing the specific brake applications. Hence, the average pressure, the initial braking speed, the final braking speed, the average disc temperature and the brake duration are computed for each brake application. In the case of dynamic ANN, the braking manoeuvres are allocated to an array of fixed length. This step is referred to as sequencing. The length of this array has been chosen based on the longest braking manoeuvre registered along an RDE cycle. All other manoeuvres, being characterized by a different duration, are scaled accordingly. To account for different braking manoeuvres, the duration of a n-th braking is also provided as input to the network. The training is performed upon scaling input and output down to the same range. This step is necessary to ensure even importance among braking manoeuvres. Thereafter, the resampling takes place based on the assumption that all manoeuvre types are uniformly distributed.

This step is fundamental to mitigate model over-fitting with respect to the braking manoeuvres occurring more often. A scheme of the data pre-processing procedure is reported in Figure 5.

![Figure 4. Example of data feature extraction from one trip. Braking manoeuvres are marked in orange; the black cross symbols represent average quantities. a) reports the vehicle speed; b) brake pressure; c) disc temperature; d) PNC from PEMS.](image)

![Figure 5. Scheme of the data pre-processing to train the proposed ANNs.](image)
3.3. Data analysis

Assuming that PNC is normally distributed, a two-sigma benchmark can be performed leading to the results in Figure 6.

The results show that initial braking speed and brake pressure well correlate with the PNC only in the area of more severe braking manoeuvres, characterized by high initial speed at the beginning of braking. This suggests that a simple map-based approach, as proposed in [5], might not perform accurately in the case of more gentle braking manoeuvres, which account for more than 80% of the RDE-compliant driving cycle. This result also suggests that other input must be considered to reduce the prediction error and increase the variance accounted for. The ANNs represent a promising solution, as they are designed to handle multi-input single-output problems. To this effect, in addition to the vehicle speed and brake pressure, other inputs are included in the regression model, namely the disc temperature and the brake duration. A sensitivity analysis conducted across the experimental data shows that initial speed ($V_{in}$) and brake duration ($\Delta t$) strongly correlated to EF (Figure 7). Expectedly, the friction energy ($E_f$) also exhibits a remarked correlation being it proportional to the product of the applied pressure ($p_b$), the vehicle speed and braking duration. The Spearman coefficients are reported in Figure 7, where data are equally scaled for ease of representation. Although brake pressure and disc temperature show a weak Spearman correlation with EF, their combined effect might still play a role in determining the EF. Therefore, the authors decided to include also brake pressure and disc temperature in the regression model. Depending on the type of regression model, the input will be provided in a stationary or dynamic form, as later reported in

![Figure 6](image-url)  
*Figure 6. Two-sigma benchmark of the measured PNC for a specific braking condition. The interpolating surface identifies the expected emission level for a specific braking condition, whilst the black bars identify two-sigma values (95%).*

![Figure 7](image-url)  
*Figure 7. Spearman correlation matrix among the variable under analysis. On the diagonal, the histogram of the main variable under analysis.*
3.4. ANN training and validation

Upon performing the data pre-processing in accordance with the procedure of Section 3.2, the proposed ANNs can be trained. For the sake of clarity, a qualitative representation of the ANNs proposed in this study is reported in Figure 8. Neurons are represented by yellow nodes; input and output are reported in the violet boxes. Both ANNs are fully connected, that is each layer communicate with the previous and following. Although static and dynamic ANNs feature similar structures, they differ in the way input data are provided. The static ANN requires stationary input which are reported in Figure 8 with the hat symbol. The index $k$ corresponds to the $k$-th brake application. In this scenario, the set of stationary inputs is as follows:

$$\begin{align*}
\tilde{U}_k, \tilde{V}_k, \tilde{W}_k, \ldots & = [V_{in,k}, V_{1,k}, \tilde{P}_{b,k}, T_{b,k}, \Delta t_k] 
\end{align*}$$

(7)

In the case of dynamic ANNs, dynamic inputs must be provided in a discrete form. The time varying input are included as set of arrays of fixed length. Each $i$-th element of the variables array counts as a separate input for the generic $k$-th brake application. The input variable array length is fixed based on the longest braking manoeuvre. All other manoeuvres are scaled accordingly in order to fill the vector elements. To account for the dynamics of the generic braking manoeuvre, the brake duration is included as last input. Expectedly, given the higher amount of input variables, a dynamic ANN lead to a higher parameter cardinality and, thus, increased computational burden. The input vector for the dynamic ANN is defined as follows:

$$\begin{align*}
[u_k(t), v_k(t), \ldots] & = [V_k(t), p_{b,k}(t), T_{b,k}(t), \Delta t_k] 
\end{align*}$$

(8)

In both cases, the output represents the stationary brake-related particulate number emitted upon completing the $k$-th brake application.

$$y_k = EF_k = log_{10} \int_{t_0}^{t_{0+k+\Delta t_k}} PN(t) \, dt$$

(9)

The problem at hand is an example of supervised learning. The networks weights and biases are identified by means of the backpropagation algorithm. For a deeper insight, the interested reader is referred to a specialized book.

4. Results and discussion

4.1. ANN test results

The functionality of the proposed meta-models is herein assessed by comparing the model responses and the experimental data with respect to trip 8. Moreover, ANNs are compared with the map-based approach proposed in [5]. Herein, this latter is referred to as LUT (Look-up-Table) and has the form reported in Figure 6, where the emitted PN pro braking is related to the initial braking speed and average brake pressure. The results reported in Figure 9 show that all models exhibit very good correlation with the target values. Nevertheless, ANNs feature a much higher correlation coefficient when compared to the map-based approach. As discussed in Section 3.3, the inputs used in [5] are not sufficient to render the brake-related emissions under all operating conditions. The overall PN prediction error, with respect to Trip 8, is reported in Table 2.
The proposed models are benchmarked using the performance indexes of Table 3. The training time is defined as the time required to perform ANNs training on the used computer. Space in memory relates to the size of the model. The average EF prediction error is computed by means of the nRMSE. EF refers to the target and $\tilde{E}$ to the model’s response. The $k$ index ranges among the $N_b$ braking applications characterizing Trip 8. Relevant to this study is the cumulative PN prediction error which relates to the model capability of correctly predicting the emitted PN with respect to the whole driving cycle. $PN$ refers to the target and $\tilde{PN}$ to the model’s response. At last, the variance accounted for reflects the model capability of explaining the variance in the target signal. If the residual variance accounted for is close to one, the model exhibits a high correlation with experimental data. The performance indexes are then normalized with respect to the model exhibiting the best performance. This leads to the result in Figure 10, where a factor closer to the unitary value represents higher performance.

The time dependence of dynamic ANN explains the low cumulative error, as reported in Table 2. Moreover, because of dynamic ANN capability of following transient manoeuvres, it also exhibits a higher VAF index. However, compared to static ANN, dynamic ANN requires a much larger number of parameters to be stored in memory, which negatively affects its real-time capability. As anticipated in the previous sections, the map-based approach accounts only partially for the variance in the brake-related emissions (low VAF index). The map-based approach requires limited space in memory to the detriment of prediction error. The results suggest that a static ANN complies criteria of error minimization and variance accounted for.

### Table 2. Overall PN prediction error. Trip 8.

<table>
<thead>
<tr>
<th>Target Static ANN</th>
<th>Dynamic ANN</th>
<th>LUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.77E+09 [#/km]</td>
<td>2.52E+09 [#/km]</td>
<td>2.66E+09 [#/km]</td>
</tr>
<tr>
<td>(-9.07%)</td>
<td>(-3.95%)</td>
<td>(-32.68%)</td>
</tr>
</tbody>
</table>

### Table 3. Key performance indexes used to evaluate models’ performance.

<table>
<thead>
<tr>
<th>KPI</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training time</td>
<td>Time required to complete the ANNs training procedure</td>
</tr>
<tr>
<td>Space in memory</td>
<td>Size expressed in MB of the stored ANN parameter file</td>
</tr>
<tr>
<td>Average EF prediction error</td>
<td>$\text{nRMSE} = \sqrt{\frac{1}{N_b} \sum_{k=1}^{N_b} (EF_k - \tilde{EF}<em>k)^2} = \frac{1}{N_b} \sum</em>{k=1}^{N_b} EF_k$</td>
</tr>
<tr>
<td>Cumulate PN prediction error</td>
<td>$\Delta PN = 1 - \sum_{k=1}^{N_b} PN_k / \sum_{k=1}^{N_b} PN_k$</td>
</tr>
<tr>
<td>Variance accounted for</td>
<td>$VAF = 1 - \frac{\text{var}(PN - \tilde{PN})}{\text{var}(PN)}$</td>
</tr>
</tbody>
</table>

In consideration of its satisfactory PN prediction performance, the authors indicate static ANN as the optimal solution for the prediction of brake-related emissions.

### Figure 9. ANNs test results comparing the measured and predicted PN pro braking from Trip 8.

The method based on ANN correlates well with the experimental data; however, it might not well reproduce the emissions behaviour when the actual brake operating conditions are far from the investigated ones. Moreover, ANNs are purely black box, therefore they are not able to describe the actual phenomenology of the tribological contact. Hence, although the propose approach can reproduce the effect of brake-related particulate emissions, it is not capable of describing its causes. The functionality of the proposed ANNs has been proved with respect to cast-iron discs with copper-free ECE brake linings. Future research shall demonstrate the applicability of the proposed method to coated discs and different brake linings type. ANNs represent a flexible modelling framework, which enables the continuous training of the model parameters.

### Figure 10. ANNs benchmark analysis with respect to the defined KPIs.

#### 4.2. Limitations of the study

The method based on ANN correlates well with the experimental data; however, it might not well reproduce the emissions behaviour when the actual brake operating conditions are far from the investigated ones. Moreover, ANNs are purely black box, therefore they are not able to describe the actual phenomenology of the tribological contact. Hence, although the propose approach can reproduce the effect of brake-related particulate emissions, it is not capable of describing its causes. The functionality of the proposed ANNs has been proved with respect to cast-iron discs with copper-free ECE brake linings. Future research shall demonstrate the applicability of the proposed method to coated discs and different brake linings type. ANNs represent a flexible modelling framework, which enables the continuous training of the model parameters.
when new data are available. In accordance with previous literature examples, ANNs can predict with satisfactory accuracy the effect of braking conditions on tribological performance, provided that a demanding experimental campaign is a justified mean. It is worth remarking that although brake-related emissions might also occur during acceleration manoeuvres [19], they were not accounted in the present study.

5. Conclusion

In this work artificial neural networks are deployed to predict the brake-related particulate emissions in RDE-conform driving conditions. To this effect, the model identification is performed on the RDE data collected from a light commercial vehicle at Technische Universität Ilmenau. The distinctive feature of the neural network regression models is that they are trained to learn a weighted space of possible solutions rather than a single value. Two model configurations are tested and compared against the map-based approach. The most promising solution is found under criteria of error minimisation and computational burden. The results are graphically presented and supported by performance indexes.

References


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