

# **Increasing Production Speeds via IR Control of Product Internal Temperature**

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## **Abstract**

Infrared temperature measurement methods of production processes are limited to material surfaces, and thus have a serious limitation in thermal process monitoring, particularly when considering speed increases. To overcome this limitation, a heat balance equation is derived in which the material surface temperature data is combined with other non-contact temperature data to calculate the internal temperature of the product, which in turn is used to optimize control to increase speeds. The derivation, via a mathematical model of unsteady heat transfer, is derived, using a novel application of the LaPlace Transform method, which is shown to be of an easily usable form for the design of measurement systems to estimate material internal temperature by non-contact means.

The model results are in agreement with experimental data in tire manufacturing, where hundreds of actual installations of such non-contact systems are successfully increasing tire production speeds by optimizing the vulcanizing time to the initial conditions of the green tire. The method is extended to any thermal process and examples for food processing, printing, and laminating are given.

## **Empirical Success with Tires**

A common general problem in thermal processing in production is lack of control of the initial condition of the materials to be processed. For example, the materials may be stored in a cold or hot warehouse, depending on the climate and season. Or the material may have been just delivered by truck, and there is no certainty as to bulk temperature. Or the materials may have come from another process in the same plant, but with a completely unknown time at room ambient.

From Figure 1, if the initial bulk temperature were known, the total time in an oven could be reduced by increasing conveying speed and adjusting oven temperature distribution. Without this temperature information, the oven time must be sufficiently long to accommodate the worst case initial condition. For non-metals, surface temperature alone, which can be obtained with infrared sensing devices, is not sufficient due the large gradient from the material surface to the interior. Accordingly, a more sophisticated technique is required which is based on easily measured variables, yet is robust enough to be used in the real world of factory conditions.

A specific application actually researched and installed was for tire vulcanizing for a major tire manufacturer. The process is similar to that illustrated in Figure 1, except that it is not continuous: tires are “cooked” under heat and pressure in special molds one at a time in individual presses. By measuring the “green” tire temperature immediately prior to vulcanizing, press time could be adjusted to maximize throughput, and thus increase plant capacity with almost no capital investment - a significant accomplishment compared to adding capacity to a \$100 million plant conventionally.

Initial experimental work immediately indicated employing surface temperature alone would result in unacceptable errors. Accordingly a method was proposed that solved a simple steady-state heat balance equation at the tire surface, and thus could provide the internal temperature with a simple non-contact device. This method is described in Figure 2 (patented by Exergen Corporation).

IR thermocouples with heat balance circuit were constructed, calibrated to the correct "K" value (Figure 2), and installed in the tire plant (Figure 3), with very good tracking of actual internal temperatures. Hundreds of tire presses were placed under the control of this method, and for 7 years they have

produced with excellent results, increasing throughput about 10%. This is especially effective in the summer when green tires are warmed by storage in hot warehouses prior to vulcanizing.

Despite the success, the underlying analysis was largely empirical, with no theoretical support for the simple steady-state model, and thus there was considerable uncertainty as to whether the method may be applied to other processes, or even to other tire plants. Accordingly, a more complete analytical model was required to provide the design parameters necessary for a successful application, and to minimize the time and cost of empirical investigations in actual plants.

## Mathematical Modeling

The model is derived by constructing the unsteady differential equations governing heat conduction as shown in Figure 4. The form of the solution includes all of the attributes needed to apply to the problem of determining internal temperature by non-contact measurement and a simple calculation:

1. The coefficient  $K_I$  necessary to program the IR device is clearly identified ( $=K$  of Figure 2).
2. The coefficient  $K_2$  which represents an uncontrolled initial condition error is clearly identified.
3. The coefficients emerge with conventional dimensionless heat transfer groups: the **Fourier No. (Fo)** characteristic heat conduction time, and **Biot No. (Bi)** ratio of surface transfer rate to conduction.

Figure 5 shows the variation of  $K_2$ ,  $K_I$  with  $Fo$  at the  $Bi$  calculated for the thermally processing tires, and for comparison, for food. Note that the experimental value  $K=0.31$  is directly predicted.

Extending the model to the case of employing the IR sensors for speed increase, we employ the result:

$$T_c = K_I(T_s - T_\infty) + T_s + K_2(T_s - T_o)$$

Since we wish to maintain the surface temperature and internal temperature the same (or with a fixed relation), set  $T_c = T_s$ , which results in

$$\frac{(T_\infty - T_s)}{(T_s - T_o)} = \frac{K_2}{K_I} = (\overline{\Delta T})$$

Since  $K_2/K_I$  is a function only of material properties and characteristic time  $\tau$ , where  $\tau$  at constant material properties depends only on process speed, a new ratio can be formed as

$$\frac{V_{new}}{V_{old}} = \frac{(\overline{\Delta T})_{new}}{(\overline{\Delta T})_{old}}, \text{ where } \overline{\Delta T} = \frac{T_\infty - T_s}{T_s - T_o}$$

This expression, which we call the **Speed Boost Equation (SBE)**, can then be used as a control algorithm to maintain correctly balanced thermal input to produce consistent product temperature profiles from the surface to the center, at various speeds  $V$ . For large speed changes, for example >10%, material and heat transfer characteristics may become non-linear, and thus require renormalization of the value of  $K_2/K_I$  at more than one point until the final desired speed is reached.

The physical interpretation of the SBE is that the ratio of energy supplied by the heat source at  $T_\infty$  to the product with surface temperature  $T_s$ , divided by the energy level difference between the initial state  $T_o$  and final state  $T_s$  must be held constant at a constant speed. Accordingly, if the product temperature  $T_s$  is to be held constant, and the speed is held constant, the initial and source temperatures must be controlled to maintain the balance in the SBE.

For speed increase consideration as in the case of tires, with vulcanizing estimated at 200°C, if the initial temperature is elevated by 20°C we can immediately compute the speed increase as 10%, which agrees with the experimental result.

$$\frac{V_{new}}{V_{old}} = \left( \frac{T_{\infty} - T_s}{T_s - T_o} \right)_{new} \left( \frac{T_s - T_o}{T_{\infty} - T_s} \right)_{old} = \frac{(T_s - T_o)_{old}}{(T_s - T_o)_{new}} \approx \frac{(200 - 0)}{(200 - 20)} \approx 1.1$$

Note the importance of the denominator, or *preheat* term for speed increases. As in the case of tires, the source temperature for many thermal processes is limited by equipment or materials, while the product temperature is to be precisely held, the only variable available for speed increase is preheat, i.e. reducing the quantity  $\{T_s - T_o\}$ .

Applying the SBE to laminating processes, as illustrated in Figure 6, a 25% speed increase may be realized by increasing the heating roll temperature from 105°C to 120°C, holding all else constant. The same increase could be achieved by providing preheat to 48°C without changing the source temperature.

Figure 7 shows an example of a high speed color copier, which has as the heat source the fuser roll temperature, the product temperature is the copy itself, and the initial temperature is at the feed paper. Inks for color copies are particularly temperature sensitive, due to the strong viscosity dependence on temperature, and accurate control is very important to maintain quality at maximum possible speed (a highly competitive selling point for manufacturers). Applying the SBE with the appropriate IR sensors allows maximal speeds under all conditions, especially if a preheat stage is fitted to the design.

Figure 8 is another example from the graphics industry, showing how the temporizing roll in waterless printing may be controlled by using the SBE with the appropriate sensors. Manipulating the SBE and expressing the result as a control algorithm with conventional PID parameters gives the new equation

$$(T_w - T_s) = \frac{V}{K_2/K_1} (T_s - T_o) + PID(T_s - T_{setpoint})$$

for controlling the cooling water temperature  $T_w$ . This form is expressed as controlling differences, since an attribute of the preferred IR sensor types, infrared thermocouples (IRt/c™), is that they can be wired differentially to produce an extremely accurate delta T, as illustrated in Figure 9. For many applications, the required accuracy would limit the use of conventionally amplified IR devices, especially in the very sensitive denominator terms in the SBE, where small differences have a large effect.

## Conclusion

Employing IR sensing to accurately control thermal processing, especially to increase process speeds, must include provisions for the difference between the surface temperature, which can be directly measured, and the bulk material temperature, which must be indirectly measured. By employing a simple result of a complex mathematical model, the bulk temperature can be estimated from surface temperatures, ambient temperatures, material properties, and speed. By extending the model, a result we call the *Speed Boost Equation* can be directly employed with appropriate IR sensors to increase production speeds while maintaining material temperature characteristics.

Author's note: The full mathematical development of the model, along with more detailed examples is available from the author by email to [fpompei@exergen.com](mailto:fpompei@exergen.com). For more data on the sensors employed in the applications, see [www.exergen.com](http://www.exergen.com).

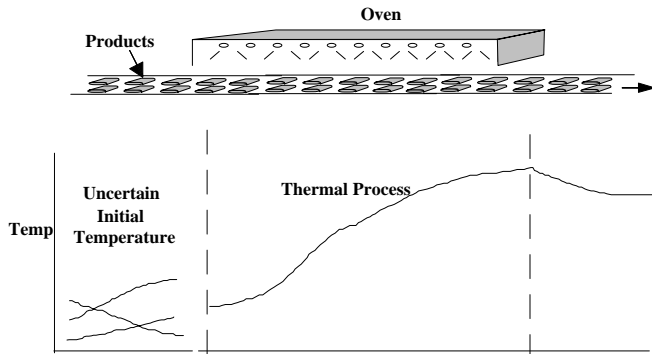
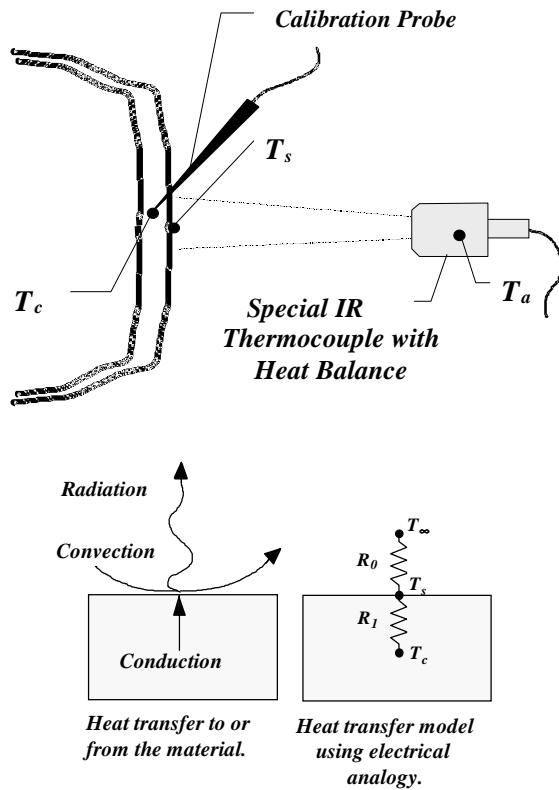


Figure 1. Illustration of time-temperature history for thermal processing products moving through an oven.



$$\text{IR thermocouple output signal} = T_c = K(T_s - T_\infty) + T_s$$

$$\text{where } K = \frac{R_1}{R_0}, \text{ determined empirically}$$

Figure 2. IR thermocouple probe developed with built-in capability to solve the steady-state heat conduction equation for the tire. Calibration probe is used in initial set-up to determine the value of the coefficient K.

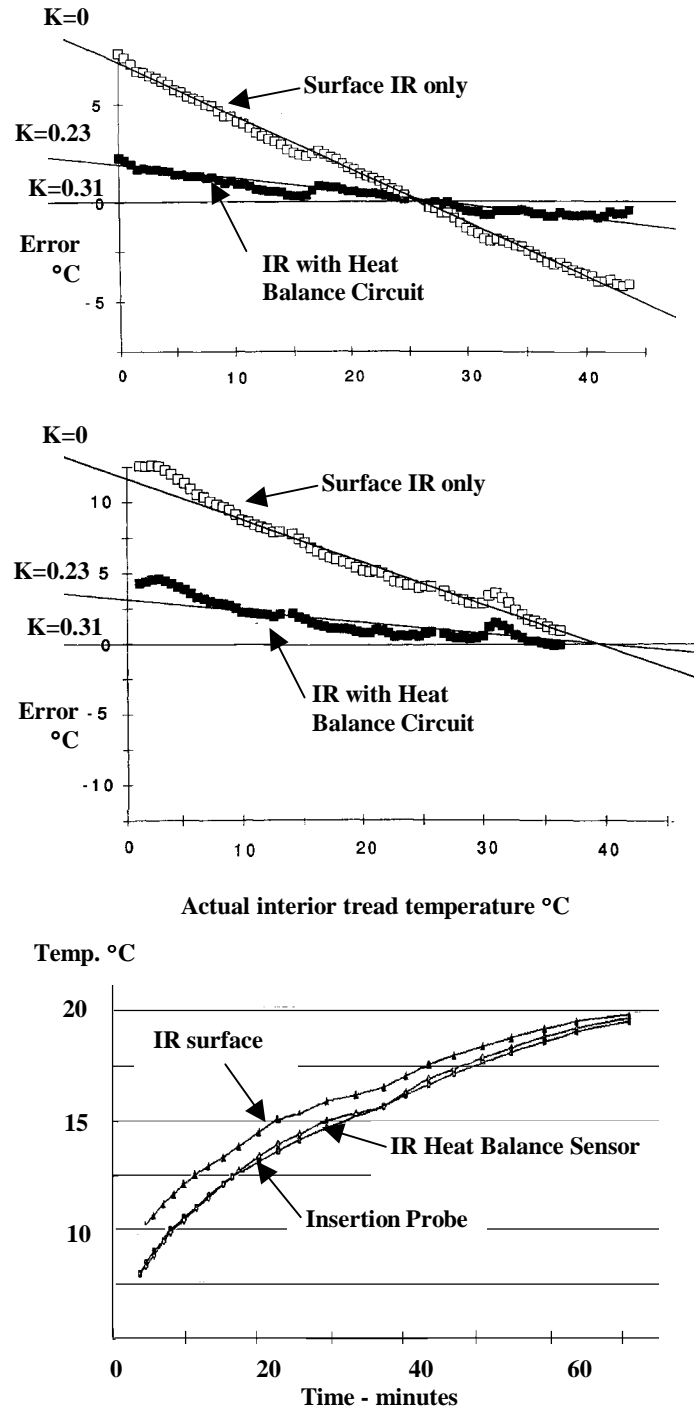


Figure 3. Measured error between Heat Balance IR Thermocouple and actual internal tire temperatures for initial estimate of K=0.23. Final value of K=0.31 reduces error to near zero. Top: data for tires removed from freezer and placed in room ambient. Center: data for tires removed from freezer and placed in oven. Bottom: Temperature vs. time for tires removed from freezer into room temperature.

Find  $T_c = T_c(T_s, T_\infty, t, \text{material properties})$

$$\nabla^2 T = \frac{\rho c_p}{k} \frac{\partial T}{\partial t} \Rightarrow \frac{\partial^2 T}{\partial x^2} = \frac{1}{\kappa} \frac{\partial T}{\partial t}$$

Apply Laplace Transform to convert

partial differential equation to ordinary differential equation

p.d.e. becomes an o.d.e.

$$s\bar{T}(x, s) - T(x, t=0) = \kappa \frac{d^2 \bar{T}(x, s)}{dx^2} \Rightarrow \frac{d^2 \bar{T}}{dx^2} - \frac{s}{\kappa} \bar{T} = -\frac{T_o}{\kappa}$$

Solution to new o.d.e.:

$$\bar{T}(x, s) = A(s)e^{-x\sqrt{s/\kappa}} + B(s)e^{x\sqrt{s/\kappa}} + \frac{T_o}{s}$$

After applying boundary conditions and manipulating, the solution becomes:

$$\bar{T}(x, s) = \frac{1}{s}(T_s - T_o) \cosh\left(a\sqrt{\frac{s}{\kappa}}\right) + \frac{1}{s\sqrt{s/\kappa}} \frac{h}{k}(T_s - T_\infty) \sinh\left(a\sqrt{\frac{s}{\kappa}}\right) + \frac{T_o}{s}$$

No inverse transform due to positive exponential (cosh and sinh), i.e. does not converge for  $t \rightarrow \infty$

Deriving a method of inverting the Laplace transform for steady state solution:

$$\bar{T}(-a, s) = \int_0^\infty e^{-st} T(-a, t) dt \Rightarrow \text{set } s = \tau^{-1} \Rightarrow \bar{T}(-a, \tau^{-1}) = \int_0^\infty e^{-t/\tau} T(-a, t) dt$$

Integrate by parts

$$\bar{T}(-a, \tau^{-1}) = T(-a, t)\tau(-e^{-t/\tau}) + T'(-a, t)\tau^2(-e^{-t/\tau}) + \dots + T^{(n)}(-a, t)\tau^{n+1}(-e^{-t/\tau}) \Big|_0^\infty + \dots$$

evaluate the exponential at  $t = \infty, 0$

$$= \tau T(-a, t) \Big|_0^\infty + \tau^2 T'(-a, t) \Big|_0^\infty + \tau^3 T''(-a, t) \Big|_0^\infty + \tau^4 T'''(-a, t) \Big|_0^\infty + \dots$$

Select leading order term only, i.e. all time derivatives are zero, then the solution has no time dependence

$$\bar{T}(-a, s = \tau^{-1}) = \tau T(-a) \Rightarrow T_c = T(-a) = \frac{1}{\tau} \bar{T}(-a, s = \tau^{-1})$$

which is valid, as long as the Laplace Transform,  $\bar{T}(-a, s) = \int_0^\infty e^{-st} T(-a, t) dt$  converges as  $t \rightarrow \infty$

which is always true if the  $T$  solution of interest has no time dependence

Substitutes  $s = \tau^{-1}$  into the solution of the p.d.e. in the Laplace Transform:

$$\begin{aligned} \bar{T}(-a, \tau^{-1}) &= \tau(T_s - T_o) \cosh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) + \tau\sqrt{\tau\kappa} \frac{h}{k}(T_s - T_\infty) \sinh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) + \tau T_o \\ T_c &= \frac{1}{\tau} \bar{T}(-a, \tau^{-1}) = (T_s - T_o) \cosh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) + \sqrt{\tau\kappa} \frac{h}{k}(T_s - T_\infty) \sinh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) + T_o \\ &= \sqrt{\tau\kappa} \frac{h}{k} \sinh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) (T_s - T_\infty) + T_s + \left(\cosh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) - 1\right)(T_s - T_o) \\ &= K_1(T_s - T_\infty) + T_s + K_2(T_s - T_o) \quad \text{where } K_1 = \frac{1}{a} \sqrt{\tau\kappa} \frac{ha}{k} \sinh\left(a\sqrt{\frac{1}{\tau\kappa}}\right), K_2 = \left(\cosh\left(a\sqrt{\frac{1}{\tau\kappa}}\right) - 1\right) \\ &\quad \text{and } \frac{\tau\kappa}{a^2} \text{ is the Fourier No., } \frac{ha}{k} \text{ is the Biot No.} \end{aligned}$$

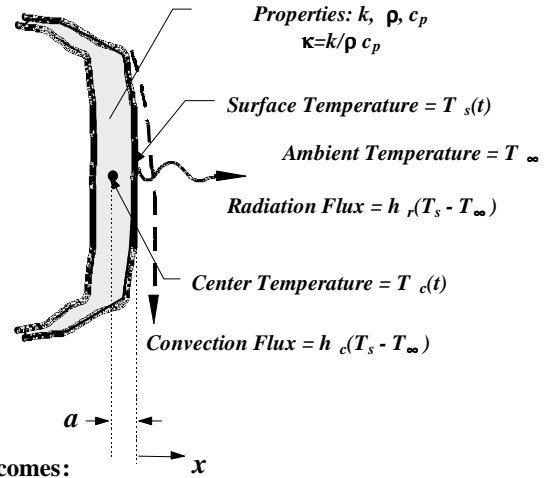


Figure 4. Summary of a general mathematical model to calculate the internal temperature  $T_c$  by solving the unsteady state heat conduction equation via Laplace transform with new inversion technique.

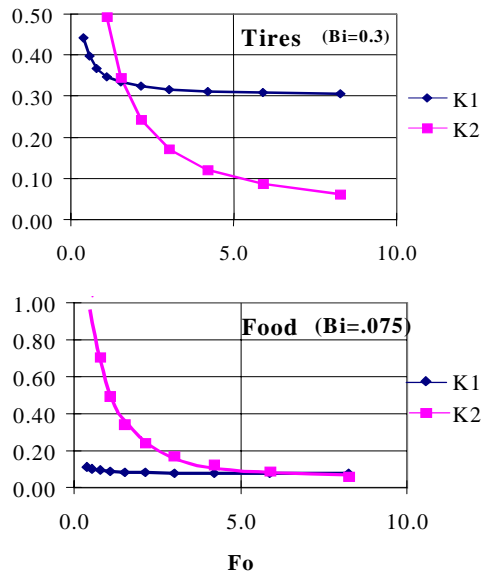


Figure 5. For tires  $K_1 (=K)$  asymptotes to the experimental value 0.31 at  $Fo \sim 2$ . For food  $K_1$  is nearly independent of  $Fo$ .

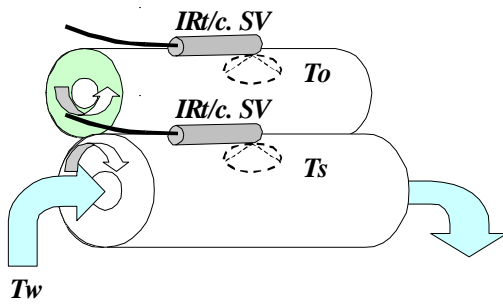


Figure 8. High performance cooling system with both  $T_s$  and  $T_o$  monitored with IRT/c sensors.

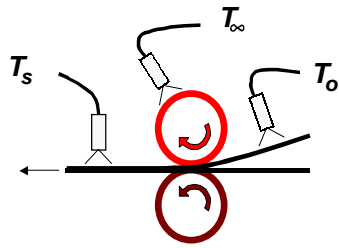
## Laminating

Existing Set-up:

$$T_{\infty} = 105 \text{ C}$$

$$T_s = 85 \text{ C}$$

$$T_o = 25 \text{ C}$$



New Set-up:

$$T_{\infty} = 120 \text{ C}$$

$$T_s = 85 \text{ C}$$

$$T_o = 25 \text{ C}$$

$$\overline{\Delta T} = \frac{T_{\infty} - T_s}{T_s - T_o}$$

Potential Speed Increase: 25%

Figure 6. Speed Boost for laminating process

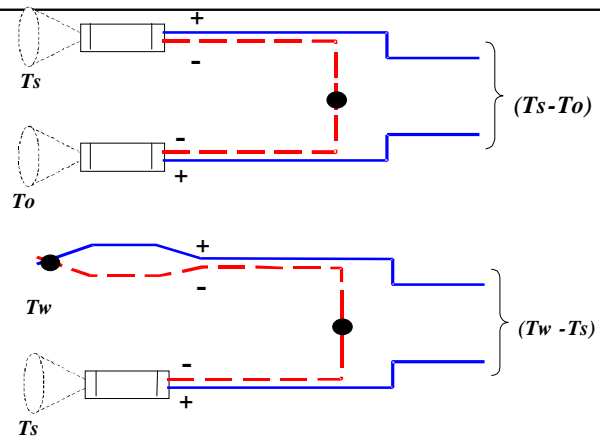


Figure 9. Infrared thermocouples wired differentially (top) and with contact t/c to provide highly accurate delta T.

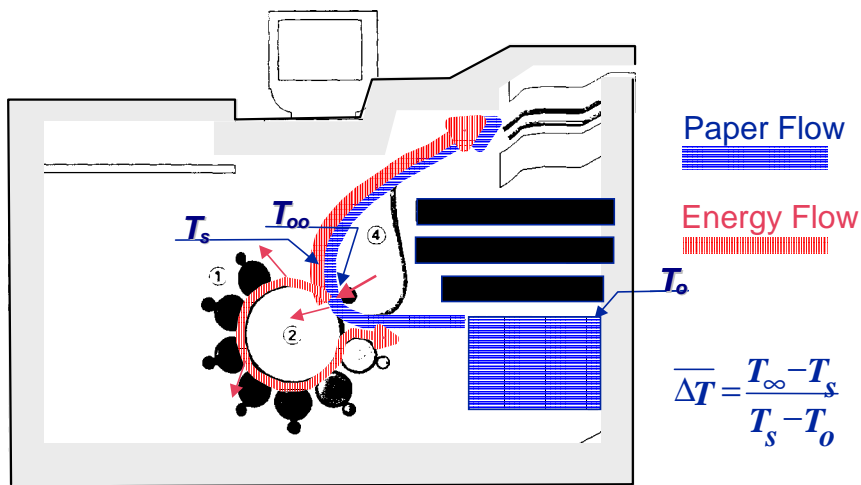


Figure 7. Speed Boost applied to high speed copier by placing IR sensors at feed paper, fuser roll, and printed copy as it discharges from fuser roll.