FRACTIONS

Whenever I ask pupils what they like least in mathematics, fractions nearly always top the list.

Working with pairs of numbers, where their order has meaning, and their combinations fit certain rules is not straightforward.

Fed up with pizzas and shaded flags I want to find a different learning pathway into fractions.

I want to tell them about the conventions of how we write fractions, but I want them to come to their own understanding of a possible meaning of an ordered pair of numbers and the concept of equivalence.

I want them to have secure footholds into adding and subtracting and even multiplying and dividing fractions.

This is a lesson design – it is not (yet) a record of a lesson taught.

It is intended to assist, not replace the teacher's own lesson planning.

WHAT CAN YOU SEE?

In this lesson design, the black type describes the teacher talking or thinking and the green bullets describe pupil talking or thinking.

Each pupil will need centimetre squared paper, a sharp pencil and a ruler.

You might want to use my pictures and diagrams to project onto the screen.

Show picture. “What do you see?”

- One to five
- One in ten
- Going up
- Going down
- Steepness
- Gradient
- It’s in the Yorkshire moors
- It’s in Wales
- The last on the right is odd

Can we separate the “goings up” from the “goings down”?

- They did

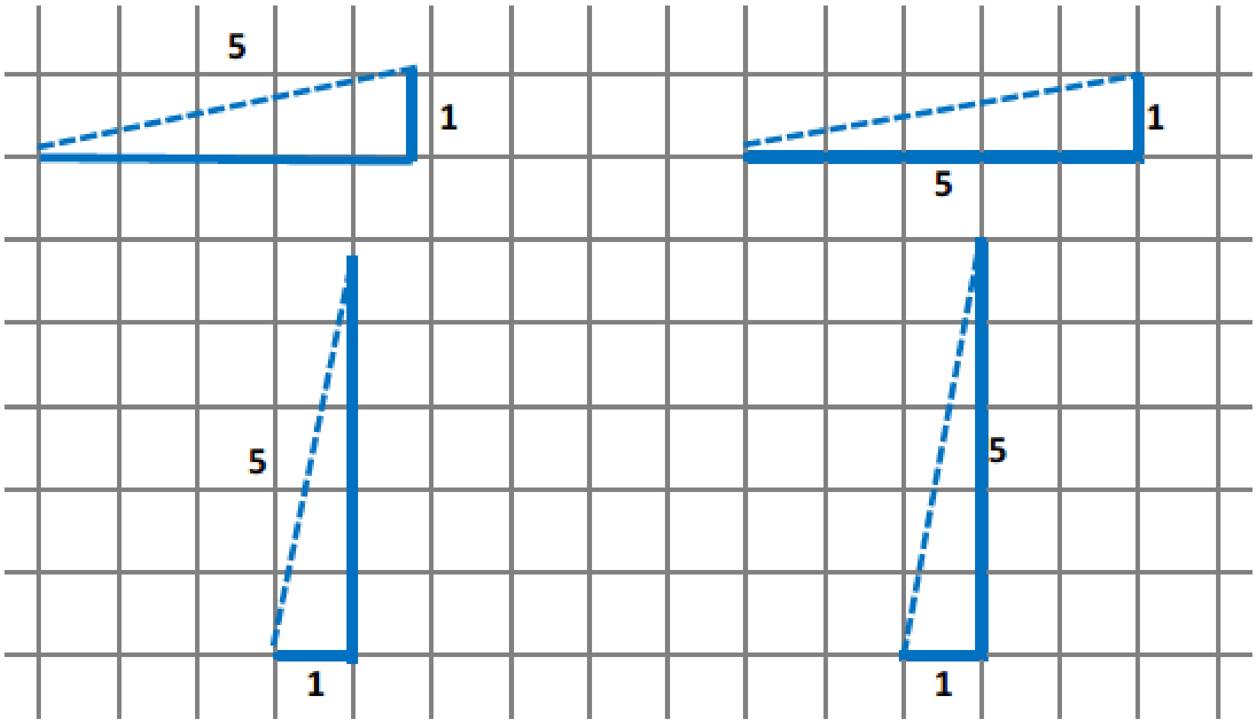
Which is the steepest?

- One in ten
- Forty percent
- No, hold on …….

Time to explain a convention: when talking about gradients we read 1:5 as “one in five”. What do you think the two individual numbers are describing in the middle picture in the second last row – point.?

- You go along one and up five
- You go along five and up one
- Um

Which one is it? I need to tell you something about the meaning of “along”.



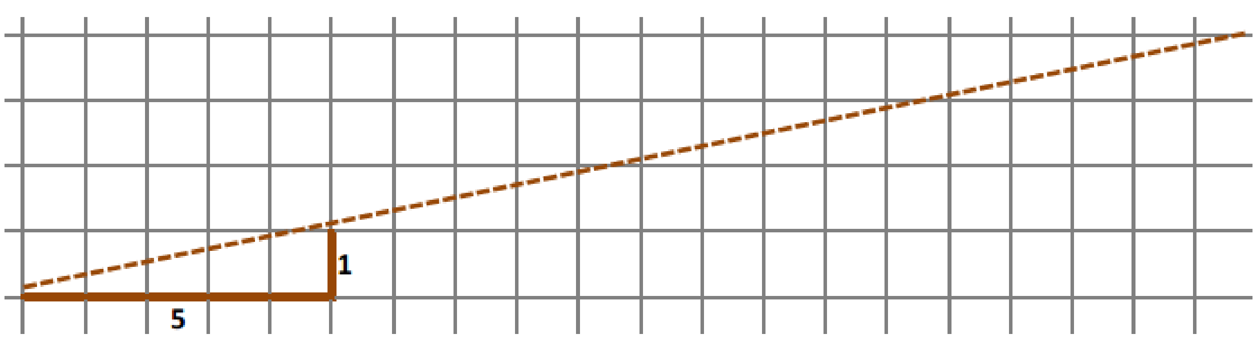
The two diagrams on the left-hand side show the length of five is “along the road”. (The diagonal)
The two diagrams on the right-hand side show the length of five is “along the horizontal or vertical”.

For road gradients the convention is that the two numbers refer to **horizontal** and **vertical**.

Now back to the question – looking at the two drawings on the right-hand side – is the road sign 1:5, one in five describing the top picture (along 5 and up 1) or the bottom picture (along 1 and up 5)?

- A car couldn’t go up the road in the bottom picture
- It’s like a cliff
- It must mean along 5 and up 1

You are right, it’s another convention - so now you know what a gradient of 1 in 5 looks like and how we write it and say it. If you want, think of it as I need to go along 5 to get an uplift of 1.



The two numbers are needed to describe the steepness or gradient. If I wanted the cliff face, I would need to say 5 in 1 because it is 1 along and 5 up. The order of the two numbers has meaning. Think about that and look at the road signs again and work out order of steepness. Draw them out on your squared paper.

- General discussion
- Questions about the percentages
- The bigger the percentage the steeper?

What do you think 25% means in the middle picture on the right-hand side?

- It's a quarter
- It's 1 in 4
- You go along 4 to go up 1
- It's steeper than 1 in 5

What about 40%?

- Well its 40 in 100
- That means you go along 100 to go up 40
- It doesn't fit on my paper
- It's the same s as 4 in 10

What do you mean by "it's the same as…"?

- Discussion on how the same steepness can be expressed in different ways.

I log in my thoughts how I can link this to a future examination of the variants and invariants under the transformation of enlargement - the affine transformation.

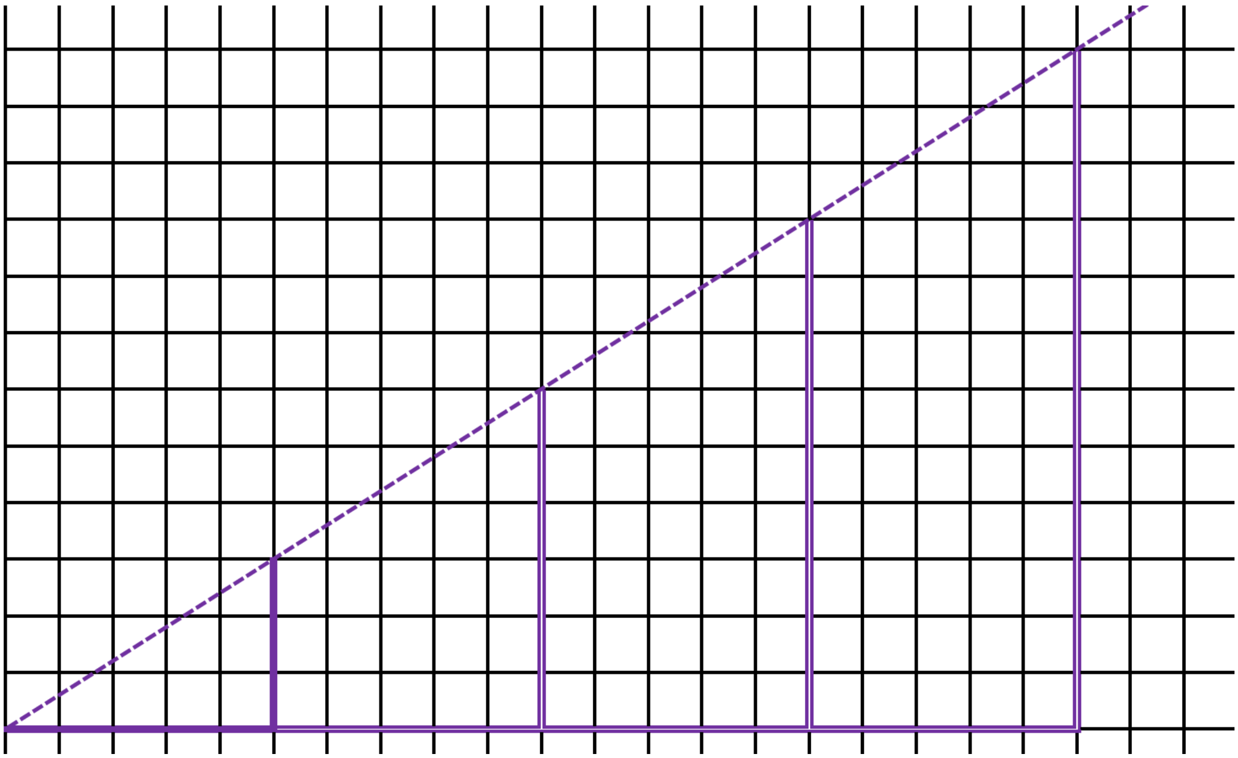
I nudge the lesson on.

So, the road signs typically express gradients as a ratio form "1 in something" or as a percentage form "something in 100".

Another way we write the ratio 1 in 5 is as a fraction 1/5 where 5 is the distance along and 1 is the height raised. If we move into this fraction world we could think of any fraction as a gradient. For example, here is a picture of 3/5. It shows five along and 3 up.

But I could also describe the gradient as 6/10 – point

Tell me what you see.



- 15 along and 9 up
- 20 along and 12 up

So, the same gradient can be expressed as 3/5, 6/10, 9/15, 12/20

In mathematics, we say that these fractions are equivalent. What do you notice?

- We are doubling or trebling both horizontal and vertical and the gradient is the same

That's true, and more – we can scale up both horizontal and vertical by any factor and the gradient remains the same.

So, we can write: 3/5 = 6/10 = 9/15 = 12/20 = 15/25 = 18/30 = 21/35 = 24/40 (the class callas out the next in the sequence)

- What about 8 along and 5 up? – that looks to be on the line
- It is slightly above if you look carefully

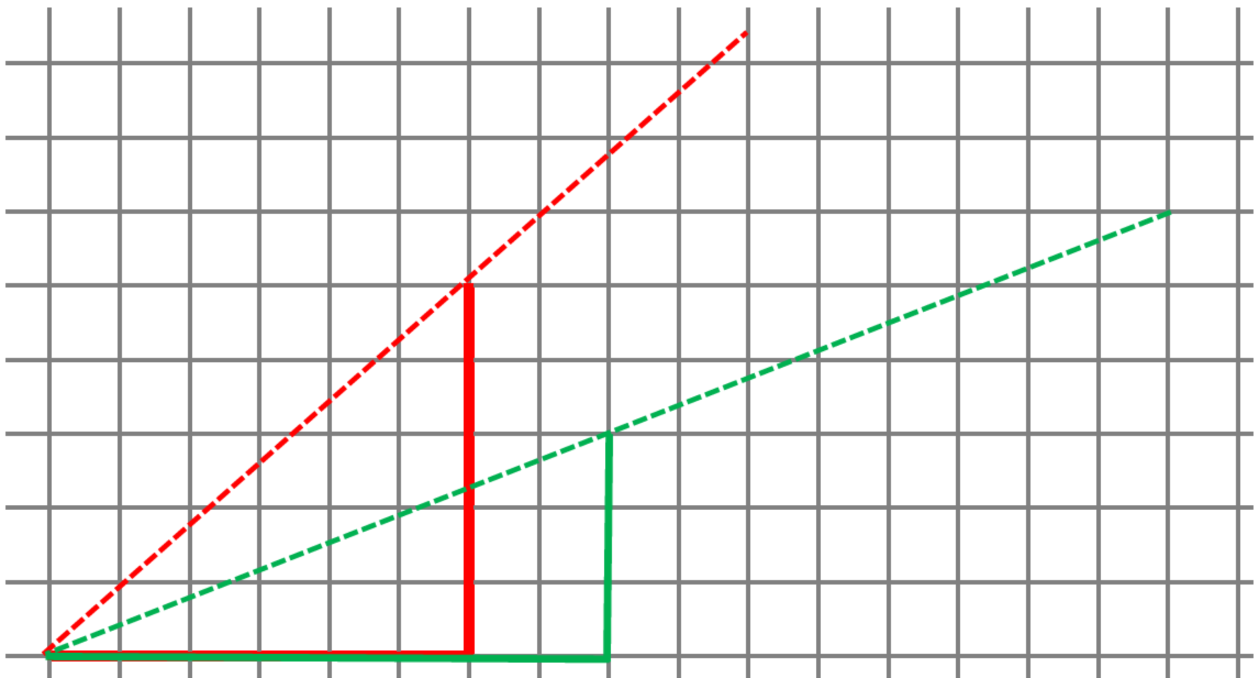
Let's check it out by looking at the equivalences of 5/8

5/8 = 10/16 = 15/24 = 20/32 = 25/40 (the class calls out the next in the sequence)

And? What do you see?

- You have to keep going until you have the same horizontal distance and then see which is higher
- 3/5 is smaller than 5/8 because the line is below that point
- 3/5 is smaller that 5/8 because 24/40 is smaller than 25/40

What about the picture below?



The red dotted line is steeper than the green

- The red line shows 5/6
- The green line shows 3/8
- So, 5/6 is bigger than 3/8

If we made the green slope “twice as steep”, what would it look like?

- Interesting discussion!
- Eventually, same horizontal by twice the vertical

And what about half the horizontal and same vertical? You had better draw these out.

- They do

So, twice 3/8 is 6/8 or 3/4

And what about “halving the steepness”. Think about that.

I’ve reached another take-off point but I need them to consolidate so I set them a written practice exercise on ordering fractions and finding equivalents.

They should be ready to move into adding and subtracting fractions. Doubling and halving could lead to trebling and thirding so there is a pathway into multiplying fractions.

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