ECARDA

Mathematics at Allerton

Thoughts on facilitating some teacher development

Fifty years after starting my own career in mathematics education, I have been invited to work with a primary school on developing their teaching and learning of mathematics. This pamphlet records my preparation and includes some of the mathematical activities used.

Peter Lacey

May 2016

Musing over the preparation of some development work with teachers

I have been invited to work with teachers on their mathematics and its teaching fifty years after I started my own learning journey as a teaching majoring on mathematics.

The state educational landscape has altered beyond recognition since then but the disconnect between mathematics and the majority of the English population remains. Other than the language used, there is little change to the stream of reports bemoaning the state of English mathematics education and the consequent low levels of mathematical competence of school leavers. Is it really that bad? Have things really got worse? A lot of money has been invested in improving the teaching and learning of mathematics, not least on versions of a national curriculum, versions of end-of-key-stage testing and examinations, national strategies and ever more "focussed" school inspections. Has all this been wasted? I still smile at the recollection of a professor of education who told me that everything except the evidence points to a decline in standards of mathematics!

I bring to mind the 1984 – 1987 "Raising Achievement Project" (RAMP) which, amongst other things, paid to release teachers of mathematics from their classrooms for one day a week over the three years to reflect on, research and re-engineer their approaches to teaching. Did this make a difference? You bet it did. It transformed the teaching and learning of mathematics and had measurably positive benefits on the quality of educational experiences and learning outcomes. But it was judged to be too high cost and too long run. Conflations onto the twenty-day maths course spread over one or two terms had some benefits, but its codification and absorption into the national numeracy strategies killed off the critical ingredients of teacher reflection and research. It takes less time and money to tell teachers what to do and how to do it than to involve them in the creative and professionalising approach to deepening their own subject knowledge and developing appropriate pedagogies.

"Learning and understanding take time" is as true for young people in classrooms as it is for those older people in classrooms.

My invitation to work with this school in half-day blocks over more than one term opens possibilities of creating space for teachers to experiment and reflect.

For decades I have been struck by the simplicity and profundity of the statement made by Jacotot at the turn of the eighteenth century: "To teach is to cause to learn".

The first half-day session with the school focuses on stimulating the learning of mathematics and drawing teachers' attention towards the behaviours and strategies they deploy in order to generate that stimulation. We are prioritising getting the learning happening before we home in on exactly what is to be learnt. I am drawing on two of my publications: "Putting the Pieces Together" and "A Curriculum for Learning", both of which are in the publications section on the ECARDA website.

The second half-day session is an immersion in number with an emphasis on images available that enable learners to imagine and understand and manipulate numbers. I am drawing extensively on my publication: "Building Numeracy" from the ECARDA website. I am also using video extracts from the BBC Horizon programme, "Twice 5 and the Wings of a Bird". Critical to this session is drawing out those big ideas that underpin an understanding of number, both as teachers and learners, namely:

- that the number system is perfectly regular;
- that mathematics is shot through with infinity;
- that a lot can be gained for a little;
- equivalence;
- inverse;

[Acknowledgment to Geoff Faux for this perceptive short list]

The third or fourth sessions look at the inter-connectedness of mathematics and how this can be used to plan, navigate and chart learning journeys for groups of learners. Beyond the ingredients of mathematics published in programmes of study we look at recipes for combining, blending and cooking these ingredients to create healthy, nutritional and useful mathematical learning opportunities.

In the context of "how to teach" particular mathematics topics, we are considering the tasks or activities that teachers may present, use and mediate in the classroom.

I have chosen a small set of such activities to exemplify and illustrate the place of rich tasks in effecting learning.

I chose **NIM** as a limbering up exercise. It's easy to set up and can generate engagement, smiles and a quick focus on to winning strategies. Changing the number of rows and the number of tallies in each row exposes a common winning strategy.

None of these activity sheets is "mapped" onto the national curriculum programme of study nor onto the expected end-of-year age-related attainment targets or standards. Such mappings close down the possibility of pupils consolidating and extending their own learning beyond theses boundaries.

Moreover, these activity sheets appear more finished than the hand written originals taken from my filing cabinet. The hand written versions invited re-sculpturing and tweaks to be made before being used in the classroom. Feel free to overwrite and alter them so that they are designed for your learners. Indeed, you must! Avoid the "download and deliver" approach, so common in lessons with older students. Stick with the "design and teach" approach that more characterises the professional educator. Consider these activities as templates and not finished worksheets.

For example: In the **Four-in-a-row** activity – alter the numbers on the grid and in the pool. Alter the operation.   Likewise, in the **Railway** activity, alter the numbers on the stations and operation. In the **digital roots** activity break it into smaller steps. With **Palindromes**, you may invite pupils to search the internet to see conversations amongst mathematicians who have already worked on this problem.

In **Fill the gaps** you may want to have an activity just on the number square, or just on the tables square. The rotated/reflected extracts may be a step too far! Come to think of it – has a number square or table square realty got "a right way up"? I'm reminded of the description given to me of an upside down triangle!

With the **2D to 3D** activity you might want to start with a "backward" version: Building a 3-D model and then recording the top, front and side views. **Frogs** begs extending into unequal numbers on each side and the search for an algebraic rule. A practical start with pupils sitting on chairs representing the frogs and the spaces is recommended. With half the class on one side and half on the other a lively lesson is guaranteed! **Eggs** opens doors to combinations and patterns of combinations. More fundamentally, it requires agreements to be reached on what we mean by same and different ways. You may want to work this similarly to the remarks above about frogs and invite pupils to sit in different combinations on seats in the classroom.

Tees and Zees (or T's and Z's) is a fruitful starting point for exploring tessellation and replication. You might want to alter the generator shapes. Number chains such as **Happy numbers** or add the tens digit to double the unit digit take pupils into the territory of recording results in ways in which pattern becomes evident and inferences drawn. Change the rules of number chain production – or invite pupils to make their own**. Make 8** and **Game, Set and Match** illustrate possibilities using dominoes, whilst **Super Die** suggests challenges using dice. Why not design some starting points or problems using these resources which suitably engage and stretch your class?

Another game, '**Hex**' is included because, like Nim, it works and it's fun. In my teaching days, over 30 years ago, we had Hex leagues and knock-out competitions. Proficiency comes quickly with practice which is an attraction to players. Smaller or larger boards can be used. A 'doubles' game with some team players taking turns stimulates discussion on strategy and builds important inter-personal skills.

So, these activity sheets offer ideas to teachers rather than worksheets for learners. If you e-mail me, you can have the word versions so you can make revisions appropriate to your learners and the mathematical context. The invitation is to make such adaptions that they add value to the learning experiences of pupils in your class.

INGREDIENTS OF A RICH MATHEMATICAL ACTIVITY

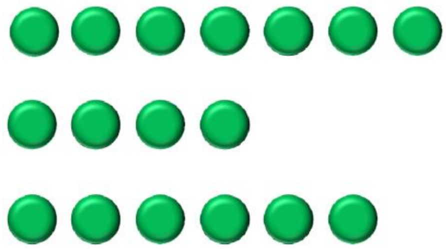
- It must be accessible to everyone at the start.
- It needs to allow further challenges and be extendible.
- It should invite learners to make decisions.
- It should involve learners in speculating, hypothesising and testing, proving or explaining, reflecting and interpreting.
- It should not restrict learners from searching in other directions.
- It should promote discussion and communication.
- It should encourage originality and/or invention.
- It should encourage ‘what if’ and ‘what if not’ questions.
- It should have an element of surprise.
- It should be enjoyable.

NIM

This is a game for two players.

Make some piles or rows of objects. You can make tally marks on paper.

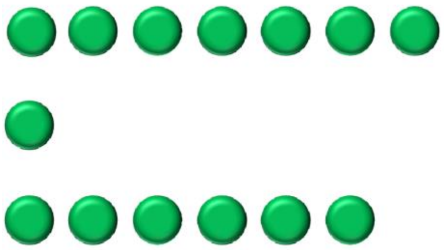
For example:



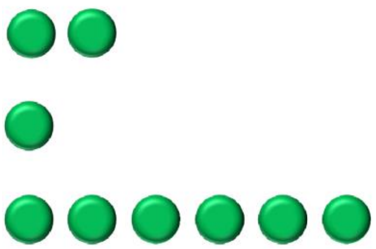
Decide who goes first and then take it in turns.

In your turn you may take as many objects away from any **ONE** row.

For example, player A has taken three counters from the middle row.



Next, player B may take 5 counters from the top row



They keep taking turns.

The player who takes the last counter wins.

Use different numbers of rows and different numbers of counters in each row.

Find a winning strategy.

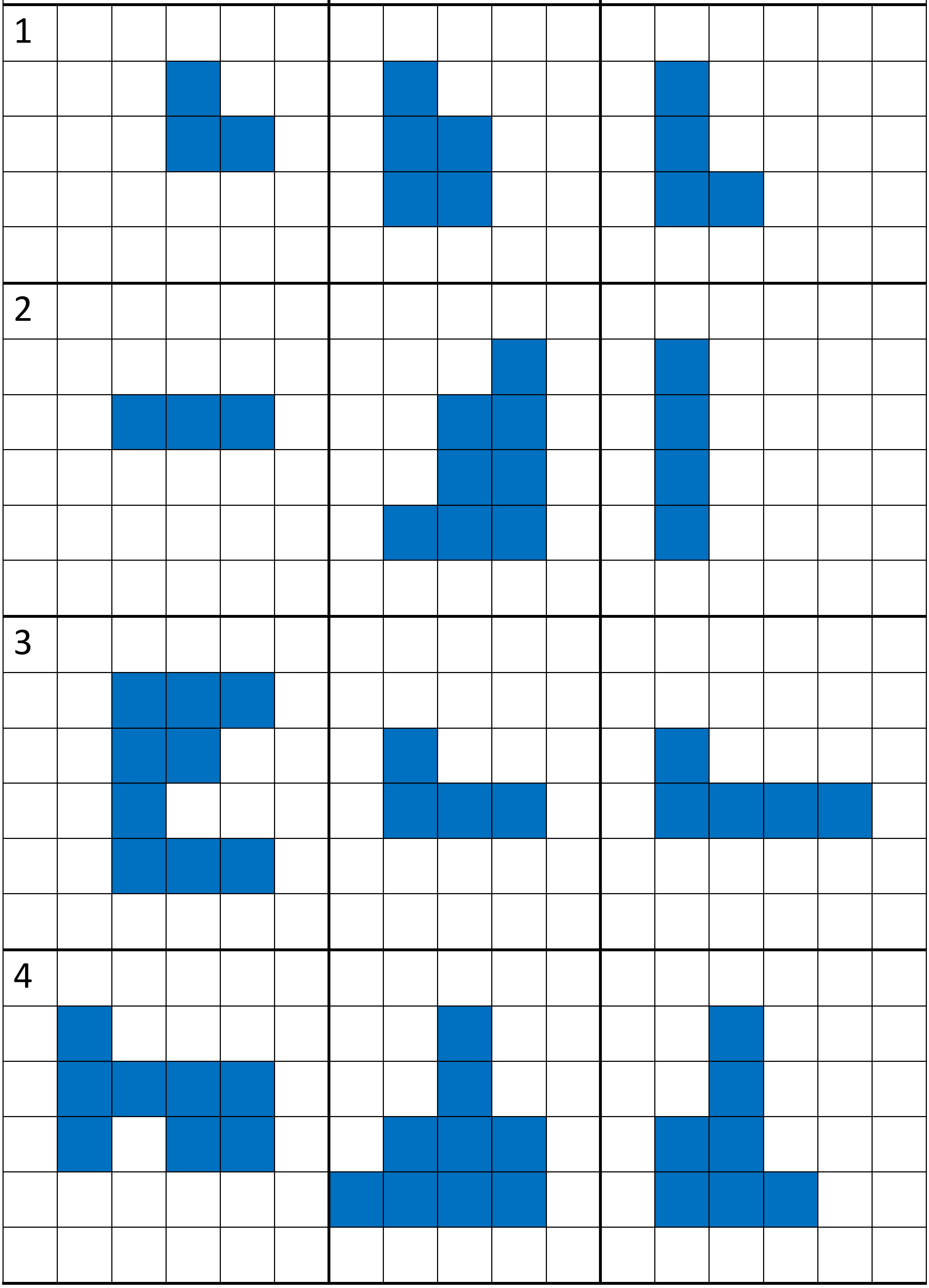
Does it matter who goes first?

Try games where the player who takes the last counter loses.

2-D to 3-D

Use cubes to build the shapes in 3 dimensions

VIEW FROM THE TOP	VIEW FROM THE FRONT	VIEW FROM THE SIDE



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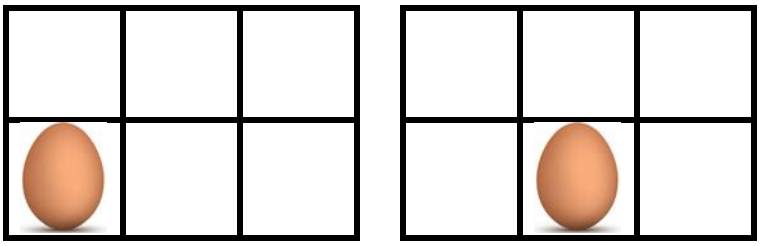
EGGS

An egg box like the one above can hold six eggs. It has 3 eggs in it.

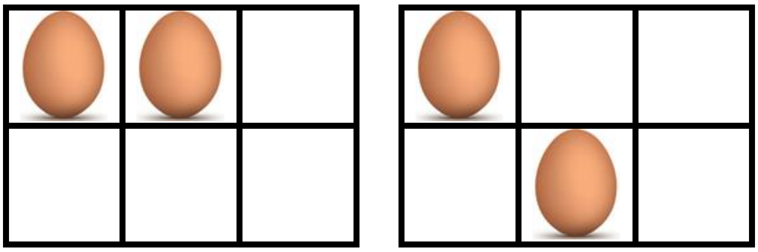
In how many different ways could you put one egg in an egg box?

For example:

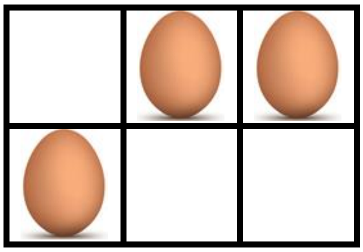


Explore and record the different ways with 2, 3, 4, 5 and 6 eggs.

For example, for 2:



And, for 3:



Look for patterns.

Describe your results.

Find out about Braille.

What about egg boxes that hold 9, 10 or 12 eggs?

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FILL THE GAPS

Blue are **number** square bits, orange are **table** square

1																
			7				23									
				18												
								44				67	68			

2	These may be rotated or reflected															
									23			89				
			12	22												
								35								
												59				

3																
				15												
							30					24				
									49					42		
		18														

4	These may be rotated or reflected															
	21							70					18			
			25													
						63		49				6		18		

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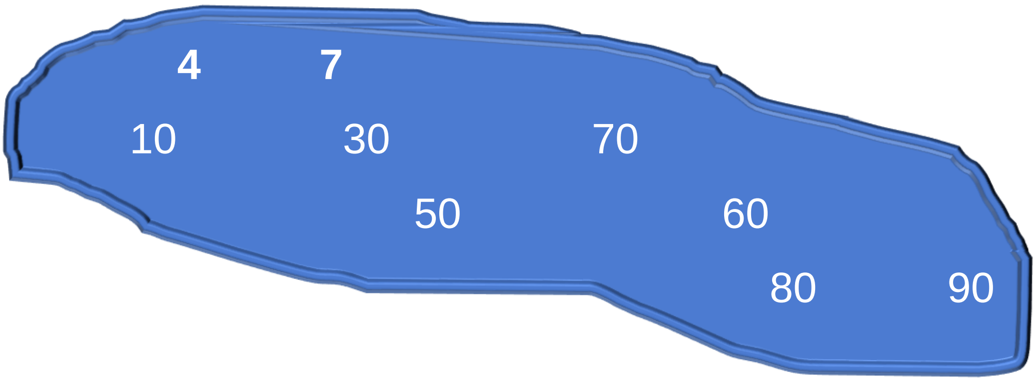
FOUR IN A ROW

280	7200	320	800	4800	28
4200	300	2400	40	900	1800
560	70	420	3500	5600	600
500	5400	3000	490	120	630
2100	1500	200	2700	350	4000
210	4500	700	360	6300	240



Multiply together any two numbers from the pool to capture a ring

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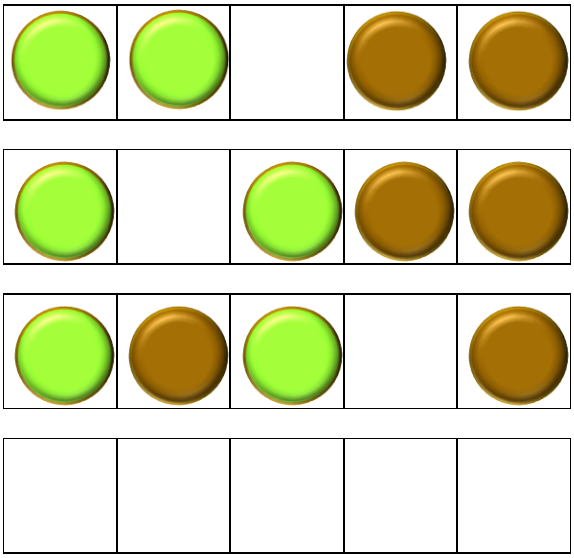
FROGS

A frog may jump onto an empty space. in front of it or may leap over one other frog directly in front of it as long as there is a space behind it.



The counters on the grid show how the green and brown frogs start and then may move.

Green starts by moving into the space in front.

Brown then leaps over green into the space behind.

Keep going until the two brown are on the left hand side and the two greens are on the right hand side.

Now try it with three frogs at each side and then more.

Find the fewest moves.

Find patterns and rules.

GAME, SET and MATCH

0	3	1	1	0	2	2
1	1	2	0	3	4	0
2	2	1	5	4	1	6
4	0	0	3	2	3	3
6	5	1	3	2	5	5
6	6	3	2	5	5	0
6	4	4	6	0	5	6
5	4	6	4	3	4	1

Use all 28 dominoes and set them out to match the grid above.

HAPPY NUMBERS

Choose a number less than 50

For example, 35 **35**

Square each digit: (3 X 3 = 9) and (5 X 5 = 25) ↓

Sum the squares: (9 + 25 = 34) **34**

↓

Repeat the process

(3 X 3) + (4 X 4) = 9 + 16 = 25 **25**

↓

Keep going

(2 X 2) + (5 X 5) = 4 + 25 = 29 **29**

↓

Keep going

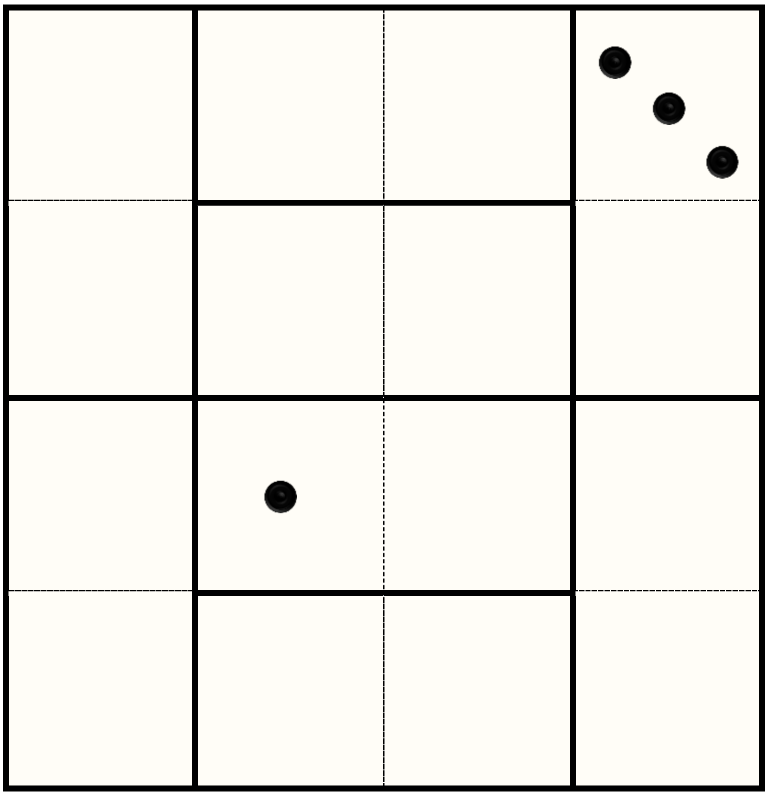
Continue the chain

If the chain ends in 1, then your **starting number** is **happy.**

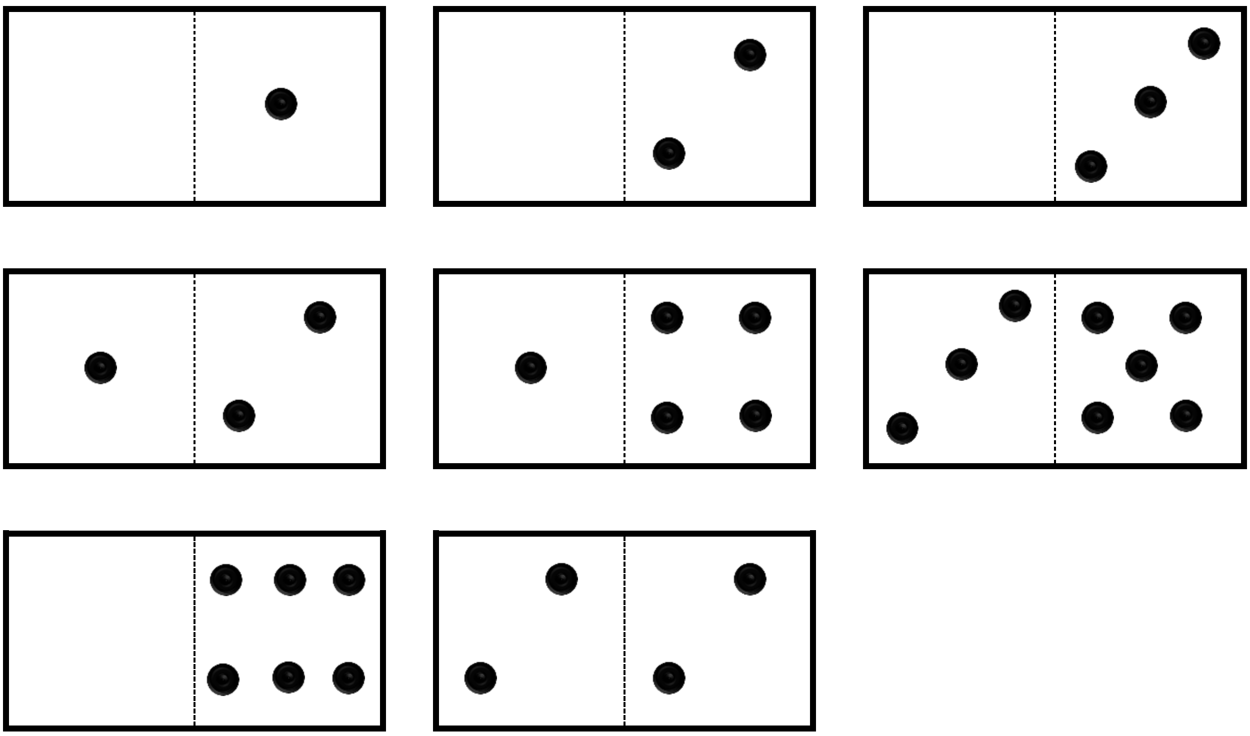
If it doesn't end in 1, then your **starting number** is **sad**.

Find as many happy numbers as you can.

MAKE 8



Arrange the dominoes from the set below to make the arrangement above so that each row and each column adds up to 8.



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PALINDROMES

A palindromic number reads the same from left to right as right to left.

So, 373, 8118, 22 and 5 are all palindromic numbers.

84 and 3259 are not palindromic.

Try this out:

Start with any two-digit number, say	47	
Now reverse the digits	74	
Add the two together	121	which **is** palindromic after **one** reverse.

Another:

Start with	49	
First reverse	94	
Add the two together	143	which is **not** palindromic.
Second reverse	341	
Add the two together	484	which **is** palindromic after **two** reverses.

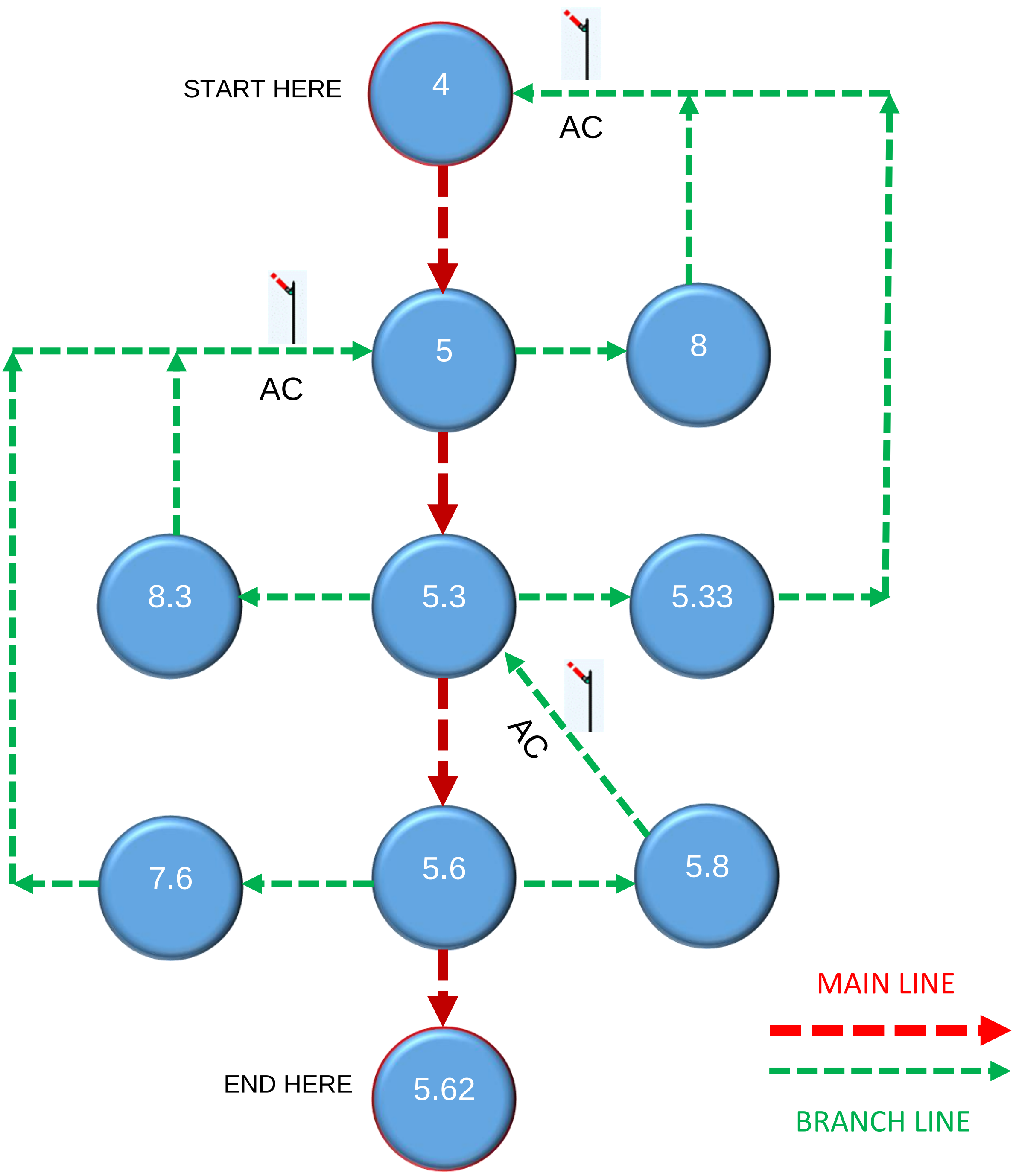
Do all numbers eventually end up palindromic?

Are there any patterns or rules?

Which numbers below 50 need the most reverses?

What about 3-digit numbers?

RAILWAY



Try to get from 4 to 5.62 by passing through all the main line stations.

Put your counter on 4 and **enter** 4 on your calculator.

Press **+** on your calculator; **enter** a number; press **equals**. Move to that station.

If you end up on a branch line go through the station, follow the green line and press **clear** as you pass the signal to get back onto a station on the main line. **Enter** that number on your calculator.

If you go completely off the rails, press **clear**, then go back to start.

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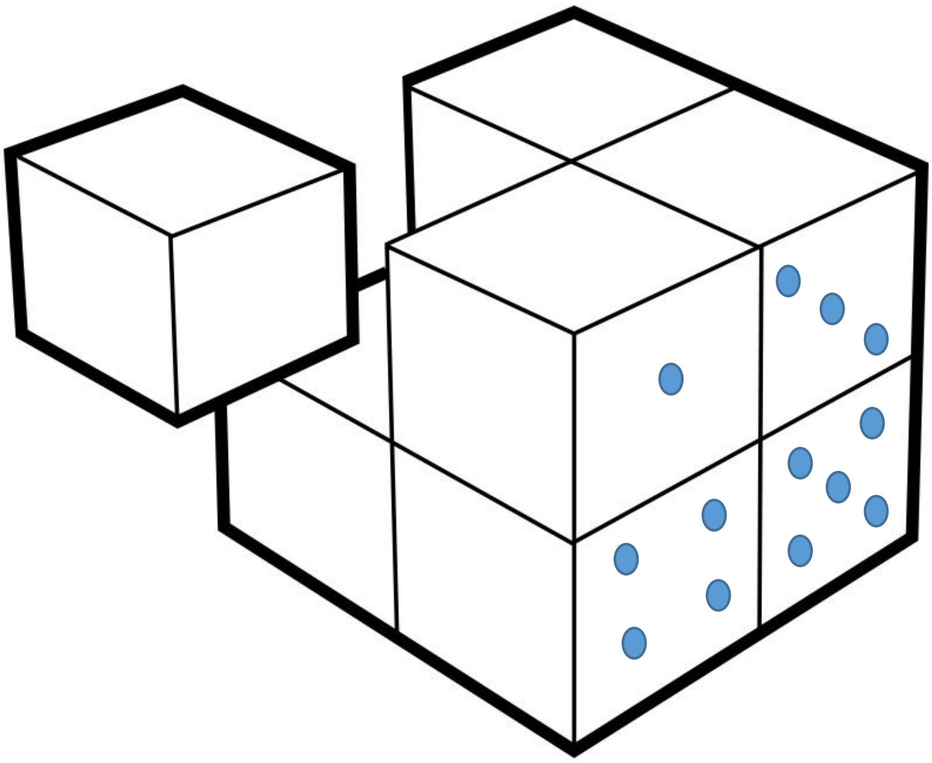
SUPER DIE

As a group, use eight dice and put them together to make a cube (super die) so that any face shows four different numbers.

The picture below gives one example, with one face completed.

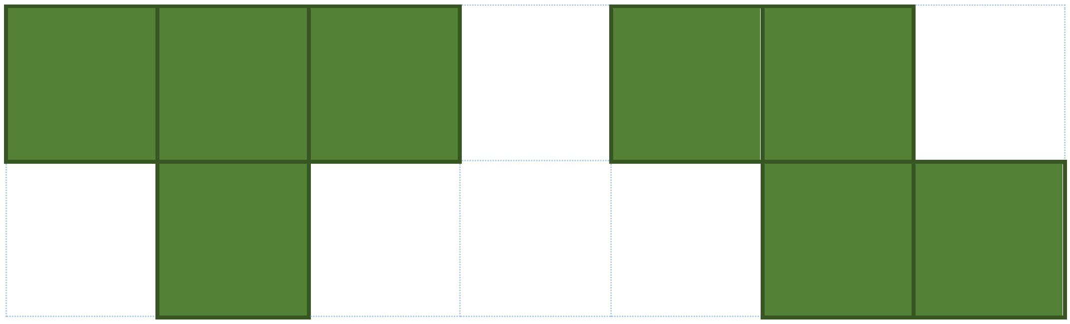


Add the total number of dots showing on all six faces of the super die.

Can you make a different total?

What is the highest total you can make?

Ts and Zs



Cut out 6 **T** shapes and 6 **Z** shapes.

Using **T** shapes only:

Make a 4 by 4 square

Using **T** shapes and **Z** shapes:

Make a 4 by 5 rectangle.

Make a 4 by 6 rectangle

Make a 4 by 7 rectangle

Look for patterns…..

Can you make predictions for a 4 by 10 rectangle?

Which rectangles can you make using only **T** shapes?

Make up some problems of your own and look for patterns.

FROM DIGITAL ROOTS TO DIGITAL ROUTES

The digital root of any number is the sum of the digits, continued until a single digit is reached

For example:

The digital root of 25 is 2+5 = 7

The digital root of 49 is 4 + 9 = 13, continued to 1+3 = 4

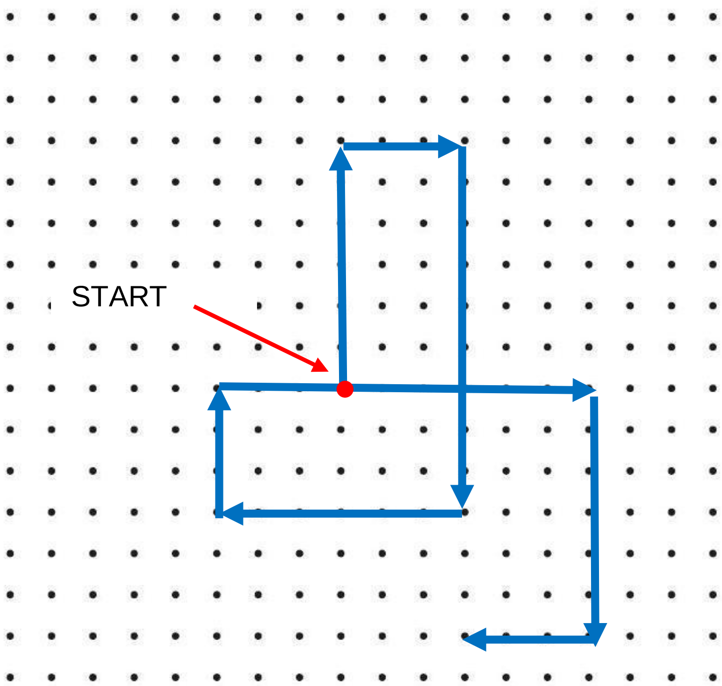
The digital root of 6 is 6.

We can take a multiplication table and find its digital roots.

For example:

1 X 6 = 6		→	6
2 X 6 = 12		→	3
3 X 6 = 18		→	9
4 X 6 = 24		→	6
5 X 6 = 30		→	3
6 X 6 = 36		→	9
6 X 7 = 42		→	6
6 X 8 = 48	→12	→	3

Keep going



The picture on the right shows how we can represent these digital roots.

Imagine walking the journey: Starting at the red spot travel up 6 units and then **turn right**. Draw the line as you go.

Now walk 3 units and **turn right**. Now walk 9 units and **turn right.**

Keep going even if you overlap or crossover until you get back to start.

Exlore other times tables. Make a display. Talk about what you notice.

PAPER FOR DIGITAL ROOTS TO DIGITAL ROUTES

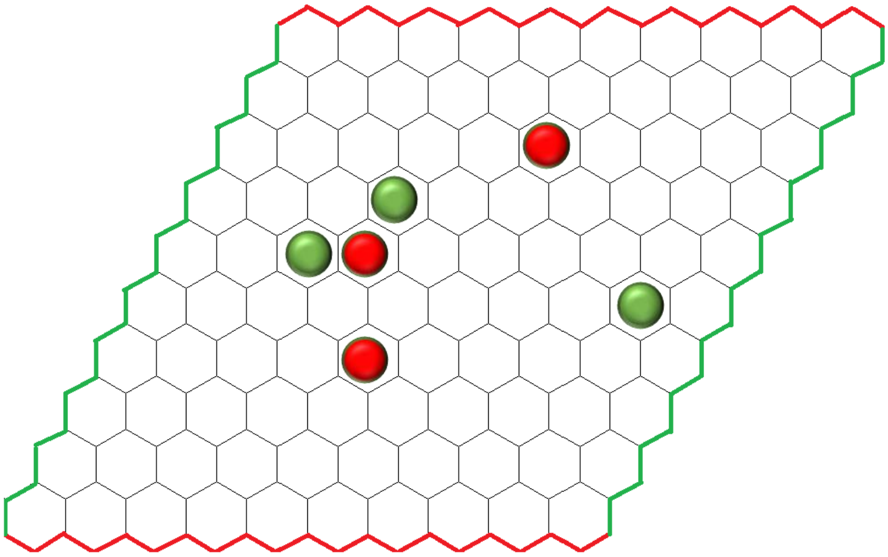
It helps to start somewhere around the middle of the page

THE GAME OF HEX

Two players: one has green counters, the other red.

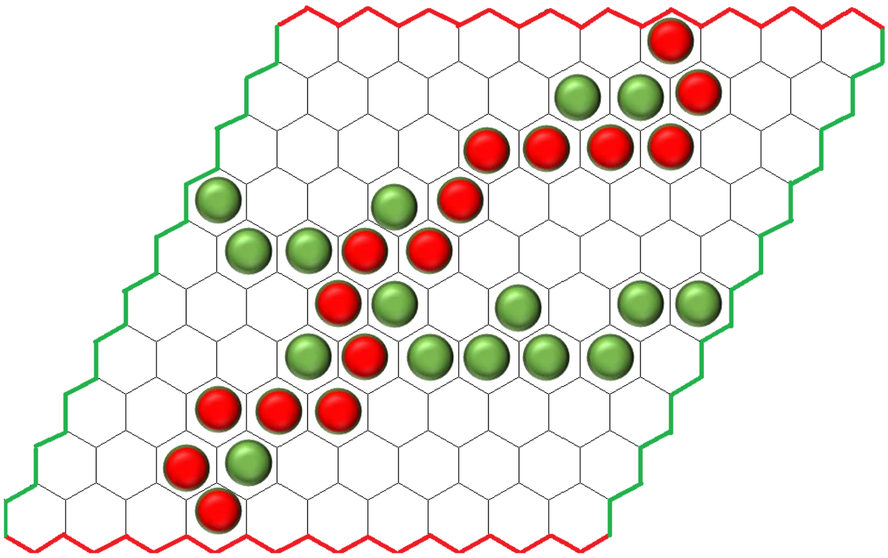
You will need a full-sheet size hex board.

Take it in turns to place one counter each turn anywhere on the hex board.



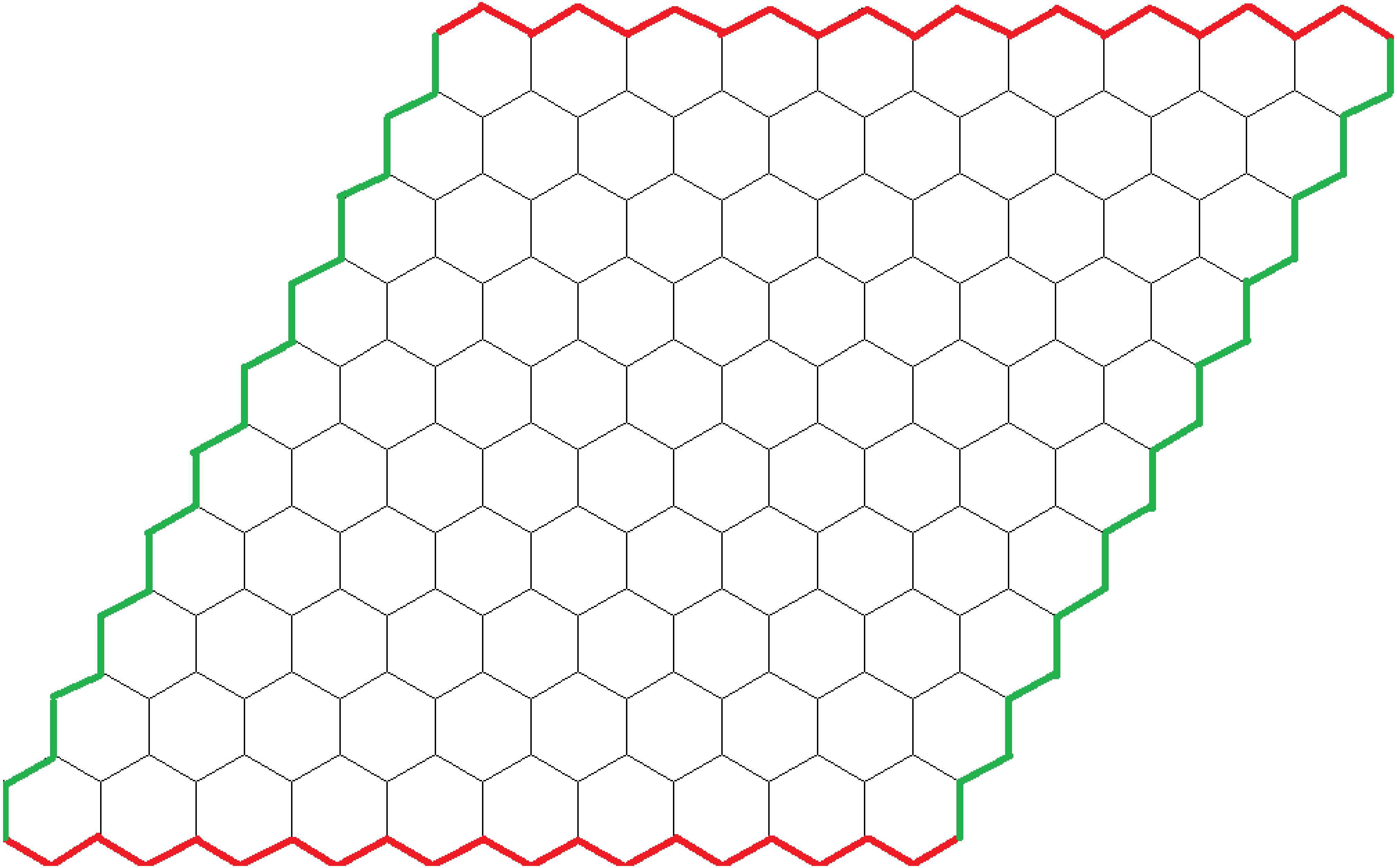
The first person who forms an unbroken chain linking their own coloured sides is the winner.

The picture below is an example of a win for red.



Play some games and find some winning strategies.

HEX BOARD



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