

## Importance of Side Resistance in a 3D Stability Analysis

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### Abstract

Present 3D limit equilibrium (LE) methods do not incorporate shear resistance from near vertical sides parallel to the direction of slide movement. Consequently, the computed 3D factor of safety (FS) is underestimated and the shear strength parameters from an inverse analysis are overestimated. The present study uses continuum mechanics to calculate the magnitude of side shear resistance along near vertical sides of a translational slide mass. Results of the parametric study show use of an earth pressure coefficient ( $K_\tau$ ) that is in-between at-rest ( $K_\theta$ ) and active ( $K_A$ ) earth pressure and Mohr-Colombo strength criteria provides a reasonable estimate of the side shear resistance and 3D/2D FS ratios that are in agreement with finite element (FE) and finite difference (FD) a continuum analyses. Based on these findings, charts showing the influence of shear resistance on 3D/2D FS ratios for various slope inclinations and geometries are presented herein.

### INTRODUCTION

Two-Dimensional (2D) limit equilibrium (LE) analyses are based on a plane strain condition that assumes the slide mass or cross-sections, is infinite in the direction perpendicular to slide movement and therefore 3D effects (end effects) are negligible compared to the shear resistance mobilized along the failure surface. This assumption is acceptable if the width of the slide mass is large compared to its height, i.e., ratio of width (W) to height (H) of the slide mass is greater than six (Arellano and Stark 2000). However, most, if not all, landslides are not infinitely long and vary perpendicular to slide movement. Therefore, application of a 2D analysis to a 3D problem is not accurate but believed to be conservative/sufficient for engineering purposes because the end effects are neglected. Past research, e.g., Hutchinson and Sarma (1985); Cavounidis (1987); Hungr (1987); Duncan (1996), shows that 3D analyses yield greater FS values than those calculated using 2D analyses for the critical failure surface, all other things being equal. 2D analyses are conservative because the resistances along the out-of-plane faces of the slide mass are neglected in the analysis. This conservatism may be acceptable for slope designs but in the case of inverse analyses of landslides, 2D analyses may result in unconservative values of back-calculated shear strength by as much as 30% (Stark and Eid 1998). 3D analyses also allow modeling of changes in slope geometry and material properties across the slide mass.

For a translational landslide, Stark and Eid (1998) show three-dimensional (3D) LE methods do not incorporate the effects of shear resistance along vertical or near vertical sides of the slide mass parallel to the direction of slide movement. Consequently, the computed 3D factor of safety (FS) is underestimated which results in an overestimate of inverse analysis of shear strength parameters. To overcome this limitation, Stark and Eid (1998), Arellano and Stark (2000), and Eid et al. (2006) suggest different techniques to incorporate the side shear resistance in 3D LE computations. These three techniques estimate the magnitude of side shear force using at-rest earth pressure ( $K_0$ ) and Mohr-Coulomb strength criteria.

The present study uses finite element (FE) and finite difference (FD) continuum analyses to calculate the magnitude of side shear resistance along vertical or near vertical sides of a translational slide mass. Results of the parametric study show use of  $K_0$  for approximating the shear resistance overestimates the 3D/2D FS ratio obtained from FE and FD analyses. However, use of an earth pressure coefficient ( $K_T$ ) that is in-between at-rest ( $K_0$ ) and active ( $K_A$ ) earth pressure provides a reasonable estimate of the side shear resistance and 3D/2D FS ratios that are in agreement with FE and FD analyses. Based on these findings, the charts provided by Arellano and Stark (2000) showing the influence of shear resistance on 3D/2D FS ratios for various slope inclinations and geometries are updated herein because they were developed using  $K_0$ .

### CONSIDERATION OF SIDE FORCES IN 3D LE SLOPE STABILITY SOFTWARE

In 3D slope stability software, a user defines the grid extent in x and y-directions. The user also specifies the number of rows and columns, which essentially determines the size of the individual vertical 3D columns. These vertical columns are the 3D equivalent of vertical slices in a 2D analysis. Similar to a 2D analysis, the resisting force is computed at the base of each column, instead of vertical slice, using the shear strength of the material through which the column base rests. The resisting forces due to the earth pressure and Mohr-Coulomb shear strength parameters applied to the vertical sides of the columns along the ends of the slide mass are not computed by existing 3D software because only the base is considered. To overcome this limitation, different techniques have been suggested to include the shear resistance along the vertical or near vertical sides of the slide mass, which are briefly described below.

**Stark and Eid (1998).** Stark and Eid (1998) suggest using a shear force equal to the side resistance to calculate the 3D FS. This is accomplished by assuming an “imaginary” material layer surrounds the sides of the slide mass not the back scarp. The material properties of the imaginary layer only affect the shear strength along the vertical sides and not the base or the back scarp of the slide mass. The soil parameters of the imaginary layer are:

- Unit weight of the imaginary layer equals that of the upper layer,  $\gamma'_{\text{imaginary}} = \gamma'_{\text{upper}}$
- Imaginary layer is frictionless,  $\phi'_{\text{imaginary}} = 0$
- The cohesion of the imaginary layer is equal to the shear strength due to  $K_0$ , acting on the vertical sides of the slide mass,  $c'_{\text{imaginary}} = K_0 \sigma'_v \tan \phi'_{\text{upper}}$ , where,  $\sigma'_v$  is the average vertical effective stress over the depth of the sliding mass side, and  $K_0 = 1 - \sin \phi'_{\text{upper}}$ .

In addition, each vertical side of the sliding mass is assigned a slight (less than  $5^\circ$ ) outward inclination to include a single row of columns so the analysis and software can calculate the effect of cohesion in its resisting force calculations.

**Arellano and Stark (2000).** Arellano and Stark (2000) use a rectangular slide mass without rounded or a curved head scarp in their parametric study. To include side resistance, an external horizontal and vertical side force equivalent to the shear resistance due to at-rest earth pressure

( $K_0$ ) acting on the vertical sides at the centroid of the two parallel sides is included. The shear resistance acting on the vertical sides is the same as used in Stark and Eid (1998), i.e.,  $c' = K_0 \sigma'_v \tan \phi'_{upper}$  and  $\phi' = 0$ . (1)

The side shear force,  $S'$ , acting on the vertical sides is estimated by multiplying  $c'$  by the cross-sectional area of the vertical side. For simplicity, the side resistance of only the upper layer is used and the small side area between the interface of upper material and lower material and the base of failure surface is neglected when estimating the cross-section centroid. Additionally, it is assumed that  $S'$  acts parallel to the base of the failure surface at a slope of 3% down slope.

Arellano and Stark (2000) use this side resistance formulation to investigate effect of side shear resistance on the ratio of 2D/3D FS for three different slope inclinations, i.e., 1H:1V, 3H:1V, and 5H:1V. After including the shear resistance along vertical sides, Arellano and Stark (2000) present a relationship between 3D/2D FS and W/H ratio for different slope inclinations. For a slope of 1V:1H with a W/H ratio of one, the 3D/2D FS ratio is about 1.30, which indicates a 30% increase in 2D FS. On the contrary, a slope of 5V:1H with the same W/H ratio has a 3D/2D FS ratio of about 3.2. This high 3D/2D FS ratio for a 5H:1V slope is due to the use of at-rest earth pressure, instead of an earth pressure between active and at-rest, and is revised in this paper because a 3D/2D FS ratio of 3.2 does not match field observations or FE and FD analyses Chugh (2003).

**Eid et al. (2006); and Eid (2010).** Eid et al. (2006) and Eid (2010) include the shear resistance along the two vertical sides of a slide mass by imposing a “group” of external horizontal and vertical forces ( $S_y$  and  $S_z$ ) that are the components of the shear resisting force ( $S$ ). Calculation of the resisting force is the same as used by Stark and Eid (1998) and Arellano and Stark (2000) except the forces generated by at-rest earth pressure and pore water pressure are calculated separately and then imposed at the centroids of the corresponding areas on the vertical sides of slide mass. In Arellano and Stark (2000) the earth pressure forces are approximated using the average vertical effective stress over the depth of the sliding mass and applied at the centroid of the vertical sides of the slide mass.

## MAGNITUDE OF SIDE RESISTANCE

To investigate the actual magnitude of side shear resistance to modify existing 3D LE methods, the slide mass model used by Arellano and Stark (2000) was analyzed in 2D and 3D using LE, FE, and FD software. 2D and 3D LE analyses were performed using CLARA-W (Hungry 2001) and a 3D extension of Janbu's (1954) procedure. 2D and 3D FE analyses were performed using PLAXIS 3D Tunnel V.2 (Brinkgreve and Broere 2004). 2D and 3D FD analyses were also performed using FLAC (Itasca 2000) and FLAC3D (Itasca 2002), respectively.

The objectives of these analyses are to: (1) determine the magnitude of 3D/2D FS ratios computed by FE and FD procedures; (2) determine magnitude of 3D/2D FS ratios computed using  $K_0$  and  $K_A$  for the side shear resistance in 3D LE analyses; and (3) develop recommendations for the coefficient of earth pressure that should be used to estimate the side shear resistance in 3D LE analysis of translational landslides with vertical or near vertical sides.

## PARAMETRIC SLOPE MODEL

For comparison purposes, the slope model used by Arellano and Stark (2000) was reanalyzed herein using LE, FE and FD procedures. Detailed information about the model and material properties is given in Arellano and Stark (2000). Slope inclinations of 1H:1V, 3H:1V, and

5H:1V with a height (H) of 10 m were analyzed. For FE and FD analyses, each slope inclination was analyzed with W/H ratios of 1, 2, 5, and 10 with friction angles of 30° and 8° for upper and lower materials, respectively, to model a translational sliding situation. LE analyses were performed for W/H ratios of 1, 1.5, 2, 4, 5, 6, 8, and 10 with four combinations of  $\phi'_{upper}/\phi'_{lower}$  values. The friction angle of upper material ( $\phi'_{upper}$ ) was constant at 30° while the friction angle of the lower material ( $\phi'_{lower}$ ) was assigned values of 8°, 10°, 20°, and 30°.

To simulate a natural bedding plane or a weak geosynthetic interface in a landfill liner system, the lower material was assumed to slope at 3% down slope. The groundwater or leachate level was placed at a height of H/2 as measured at a distance L from the toe and linearly decreasing to a height of zero at the toe.

LE, FE, and FD are different procedures and use different solution strategies. In continuum analysis, the slope models were extended past the locations where the slope failure is likely to occur. Therefore, slope models in FE and FD analyses are wider than models used for the same inclination in LE analysis. Also, the lower material is represented by a layer of 0.8 m thick, which is followed by a bottom block. The presence of the bottom block restricts the failure surface to the weaker layer to simulate a translational failure.

For a 3D LE analysis, shear resistance along the parallel sides of the slide mass was incorporated by adding external horizontal and vertical side forces using two separate techniques:

- Application of one set of external horizontal and vertical side forces at overall centroids of the two parallel sides as done by Arellano and Stark (2000).
- Adding maximum possible sets of external horizontal and vertical side forces at the centroids of the active columns on the two parallel sides. Maximum number of loads that can be specified in CLARA-W is 100, i.e., 50 sets (horizontal and vertical) of loads may be applied on either parallel sides of slope model.

The 3D slope model in the FE and FD analyses include 6m wide end blocks and displacement condition of fully fixed ( $u=0$ ,  $v=0$ , and  $w=0$ ) at the boundaries. In addition, the 3D analysis in FLAC3D uses side blocks with higher strength and an interface between the slope and end blocks to allow relative movement at the slope-block contact. The material properties used in the CLARA-W, PLAXIS, and FLAC analyses are shown in Table 1.

**Table 1. Material Properties for Stability Analyses of Slope Model.**

Parameter	Upper	Lower	Bottom	End	Interface <sup>3</sup>
	Material	Material	Block	Blocks <sup>2</sup>	
Unit weight <sup>1</sup> , $\gamma$ (kN/m <sup>3</sup> )	17	18	18	25	-
Cohesion, $c'$ (kPa)	0	0	0	0	0.05
Friction angle, $\phi'$ (°)	30	8,10,20,30	40	45	30
Dilatation angle, $\psi$ (°)	0	0	0	0	-
Young's modulus (kN/m <sup>2</sup> )	$3 \times 10^4$	$3 \times 10^3$	$3 \times 10^5$	$3 \times 10^6$	-
Poisson's ratio, $\nu$	0.35	0.35	0.35	0.35	-
Bulk Modulus (kN/m <sup>2</sup> )	$3 \times 10^4$	$3 \times 10^3$	$3 \times 10^5$	$3 \times 10^6$	-
Shear Modulus (kN/m <sup>2</sup> )	$1 \times 10^4$	$1 \times 10^3$	$1 \times 10^5$	$1 \times 10^6$	-
Normal stiffness (kN/m <sup>2</sup> )	-	-	-	-	$1 \times 10^4$
Shear stiffness (kN/m <sup>2</sup> )	-	-	-	-	$1 \times 10^3$

<sup>1</sup>Density  $\rho$  (kg/m<sup>3</sup>) = Unit weight x 1000/9.81; <sup>2</sup>End blocks in PLAXIS analysis use same material properties as slope; <sup>3</sup>Only used in FLAC analysis

## EFFECT OF SHEAR RESISTANCE ALONG VERTICAL SIDES

To verify the magnitude of shear resistance along the vertical sides of the slide mass, 3D/2D FS ratios obtained from FE and FD analysis for three slope inclinations are plotted for different W/H ratios. For the 3D LE analysis, horizontal and vertical side force equivalents were computed using the shear resistance due to at-rest earth pressure ( $K_0=1-\sin\phi'$ ), active earth pressure ( $K_A=1-\sin\phi'/1+\sin\phi'$ ), and an earth pressure coefficient ( $K_\tau$ ) that is in-between the  $K_0$  and  $K_A$  values. For simplicity/consistency in the analyses, a value of  $K_\tau=0.5*(K_0+K_A)$  was used for the in-between case or the average between  $K_0$  and  $K_A$ . Thereafter, 3D/2D FS ratios computed for the LE analysis were compared with results of FE and FD analysis to determine the optimal earth pressure coefficient to use to incorporate 3D side resistance in LE analyses.

## RESULTS FROM CONTINUUM ANALYSIS

The FE and FD analyses shows that 3D/2D FS ratios for all W/H combinations are greater than unity, i.e., 3D FS is always greater than 2D FS. The highest value of 3D/2D FS ratio corresponds to the 5H:1V slope for W/H=1. The highest values of 3D/2D FS ratio is 2.04 and 2.05 obtained from FE and FD analyses, respectively instead of 3.2 as reported by Arellano and Stark (2000). The values of 2.04 and 2.05 are in agreement with 2.05 reported by Chugh (2003).

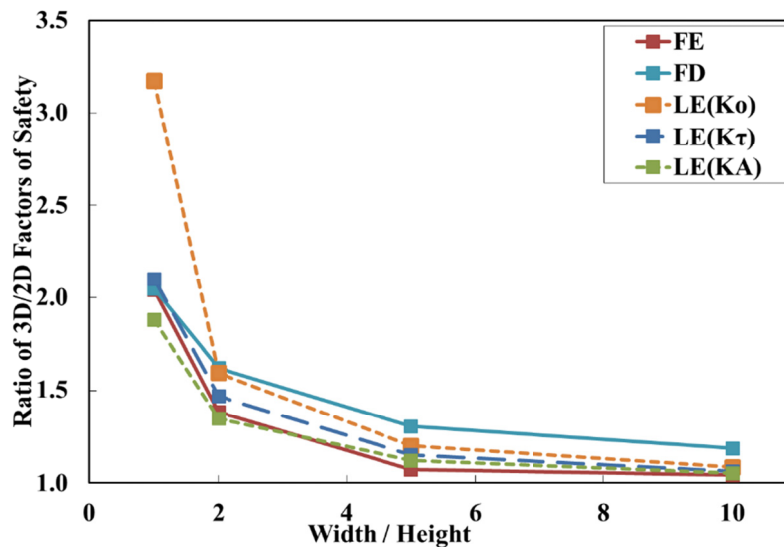
In summary, 3D/2D FS ratios from continuum procedures (FE and FD) show similar trends. For example, 3D/2D FS ratios increase with decreasing W/H ratios and for a given W/H ratio and flatter slopes have higher 3D/2D FS ratios. 3D/2D FS ratios obtained from the FD analysis are slightly higher than FE. Therefore, the FD and FE analyses are used as upper and lower bounds, respectively, for 3D/2D FS ratios for each slope inclination of model geometry.

## RESULTS FROM LE ANALYSIS

For illustration, 3D/2D FS values for 5H:1V slope obtained from FE, FD, and LE using external side forces estimated using  $K_0$ ,  $K_A$ , and  $K_\tau$  (applied at overall centroids of the two parallel sides) are compared in Figure 1.

Figure 1 shows using  $K_0$  produces 3D/2D FS ratios that are greater than those produced by  $K_A$ . This is caused by  $K_0$  being almost 50% greater than  $K_A$ , i.e., for a  $\phi'_{upper}=30^\circ$ ,  $K_0=0.5$  and  $K_A=0.33$ , and the resulting side resistance being included via the cohesion parameter in the FS calculation. Figure 1 also shows that using  $K_0$  to estimate side shear resistance results in 3D/2D FS ratios that are higher than the upper limit set by FD analysis for the same slope inclination. The 3D/2D FS ratio for the 5H:1V slope and W/H=1 is the highest. This ratio (3D/2D FS=3.2) is significantly higher than the FD 3D/2D ratio because  $K_0$  produces an excessive side resistance. However, using  $K_A$  in all three slope inclinations underestimates 3D/2D FS ratios for W/H ratios less than 2. Therefore, the optimal earth pressure to estimate field side resistance appears to be in-between  $K_0$  and  $K_A$ .

In summary, for all three slope inclination studied, 3D/2D FS ratios are within the upper and lower FD and FE bounds when an earth pressure coefficient ( $K_\tau$ ) equal to the average of  $K_\theta$  and  $K_A$  is used.



**Figure 1. Comparison of 3D/2D FS ratios from FD, FE, and LE (with side forces) for 5H:1V slope and various W/H ratios.**

The use of an intermediate value of  $K_\tau$  for calculation of side forces is supported by field slide mass observations where, generally the slide mass is cracked near the ground surface and the cracks decrease in width with depth. Therefore, near the ground surface the side resistance may agree better with  $K_A$  and near the base of the slide mass it may agree better with  $K_\theta$ . Based on triaxial compression tests, Lambe and Whitman (1969) report horizontal strain of less than 0.5% is required to change the stresses from at-rest to active earth pressure. Therefore, it is possible that after the slip surface develops and movement begins, the at-rest earth pressure transitions to an active pressure.

The effect of point(s) of application of external horizontal and vertical side forces at the overall centroids of two parallel sides or centroids of individual active columns is shown in Figure 2. Dotted lines show the 3D/2D FS ratios obtained by applying side shear forces at overall centroids of the two parallel sides, while solid lines show 3D/2D FS ratios which were obtained by applying side shear forces at maximum number of active columns on the two parallel sides. The results show the variation in FS computations is about 5% which is less than the acceptable error of 12% reported by Duncan (1996). Therefore, application of one set of external horizontal and vertical side forces at overall centroids of the two parallel sides as done by Arellano and Stark (2000) is considered sufficient for practical application.

Figure 3 presents a relationship between the 3D/2D FS ratios and W/H for the three slope inclinations considered in the parametric study. Dotted lines show the 3D/2D FS ratios obtained using side shear resistance estimated using  $K_\theta$ , while the solid lines represent 3D/2D FS ratios obtained using  $K_\tau$ , i.e., in-between  $K_\theta$  and  $K_A$ . Figure 3 shows for the 1H:1V slope there is little

difference between 3D/2D FS ratios obtained using  $K_\tau$  or  $K_\theta$ . However, the difference in 3D/2D FS ratios is greater for flatter slopes, i.e., 5H:1V slope (W/H=1), using  $K_\tau$  and  $K_\theta$  (2.10 and 3.2, respectively). The maximum value of 3D/2D FS obtained using  $K_\tau$  to calculate side resistance is about 2% higher than 3D/2D FS ratio obtained from FE and FD analyses, i.e., 2.05 vs 2.10.

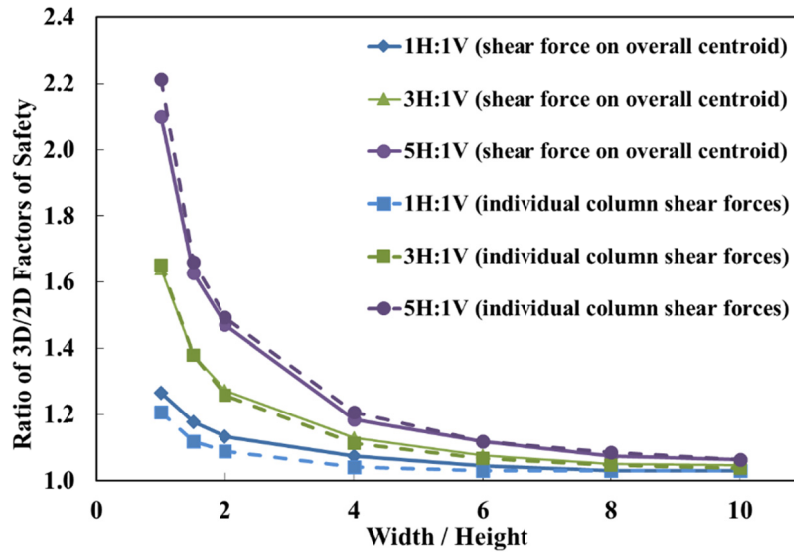


Figure 2. Effect of single or multiple points of application of side shear resistance.

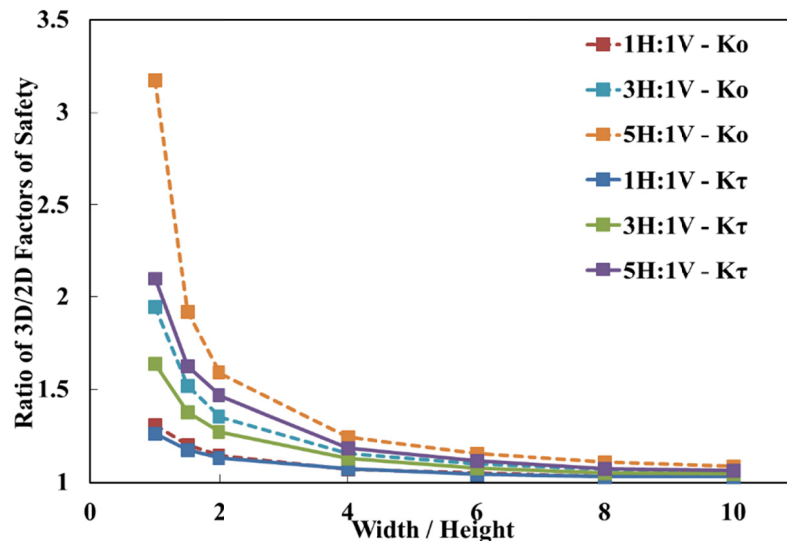


Figure 3. Effect of shear resistance along vertical sides of slide mass using  $K_\theta$  and  $K_\tau$  procedure

Based on these analyses and  $K_\tau$ , updated relationships between ratio of 3D/2D FS and W/H for the three slope inclinations are presented in Figure 4. These relationships are for friction angles of  $30^\circ$  and  $8^\circ$  for the upper and lower materials, respectively, and supercede the relationships presented in Arellano and Stark (2000).

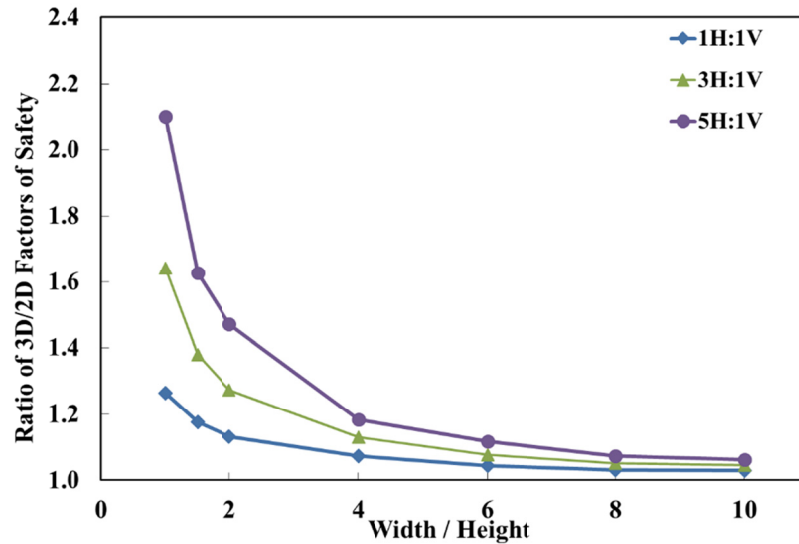


Figure 4. Effect of shear resistance along vertical sides of slide mass using  $K_{\tau}$ .

## CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendation are presented based on the FE, FE, and LE analyses presented above:

- 3D/2D FS ratios for all slope inclinations analyzed using FE, FD, and LE procedures are greater than unity.
- Shear resistance along the two vertical sides that parallel the direction of movement in translational slides can be applied as external forces in LE software to calculate the 3D FS.
- For translational slides with vertical or near vertical sides, continuum procedures (FE and FD) provide better estimates of 3D/2D FS ratios than LE but special attention is required to slide mass boundary conditions.
- Comparison of LE with FD and FE analyses indicate using  $K_0$  to estimate an empirical shear resistance along two vertical sides in LE method overestimates 3D/2D FS ratios for flat (5H:1V) slopes.
- An earth pressure coefficient ( $K_{\tau}$ ) equal to the average of  $K_0$  and  $K_A$  provides a better estimate of shear resistance acting along two vertical sides and results in 3D/2D FS ratios that are in agreement with FE and FD analyses. For simplicity and to be conservative,  $K_A$  could be used for  $K_{\tau}$ .
- For practical purposes, application of one set of external horizontal and vertical side forces at overall centroids of the two parallel sides as done by Arellano and Stark (2000) provides reasonable accuracy in LE analyses.
- LE, FE, and FD analyses show that side resistance is a function of W/H and slope inclination. For a given slope inclination, 3D effects are higher for lower W/H ratios, which tends to become constant for W/H ratios greater than six.
- Flatter slopes have higher 3D/2D FS values because of a larger side area.
- 3D values of FS should not be compared to 2D regulatory value of FS, e.g., 1.5, because of a lower level of safety due to inclusion of the side resistance. A FS greater than 1.5 should be used to create the same level of safety.



## REFERENCES

- Arellano, D., and Stark, T. D. (2000). "Importance of three dimensional slope stability analysis in practice." *Proc., Slope Stability 2000 Specialty Conference, Denver, Colorado, 5-8 August 2000*, ASCE, Geotechnical Special Technical Publication No. 101, 18-32.
- Brinkgreve, R. B. J., and Broere, W. (2004). "PLAXIS 3D Tunnel V 2 Professional Version." *PLAXIS bv*, P.O.Box 572, 2600 AN Delft, The Netherlands.
- Cavounidis, S. (1987). "On the ratio of factors of safety in slope stability analysis." *Geotechnique*, 37(2), 207-210.
- Chugh, A. K., (2003). "On the boundary conditions in slope stability analysis." *Int. J. for Numerical and Analytical Methods in Geomechanics*, 27, 905-926.
- Duncan, J. M (1996). "State of the Art: Limit equilibrium and finite element analysis of slopes." *J. Geotech. Eng*, ASCE, 122, 577-596.
- Eid, H. T. (2010). "Two and three dimensional analyses of translational slides in soils with nonlinear failure envelopes." *Canadian Geotech. J.*, 47(4), 388-399.
- Eid, H. T., Elleboudy, A. M., Elmarsafawi, H. G., and Salama, A. G. (2006). "Stability analysis and charts for slopes susceptible to translational failure." *Canadian Geotech. J.*, 43(12), 1374-1388.
- Hutchinson, J. N., and Sarma, S. K. (1985). "Discussion: Three-dimensional limit equilibrium analysis of slopes by Chen, R. H., and Chameau, J. L." *Geotechnique*, 35(2), 215-216.
- Hungr, O. (2001). User's Manual CLARA-W: Slope Stability Analysis in Two or Three Dimensions for Microcomputers. *O. Hungr Geotech. Research Inc*, West Vancouver B.C., Canada.
- Itasca Consulting Group (2000). "FLAC-Fast Lagrangian Analysis of continua". *Itasca Consulting Group*, Minneapolis, Minnesota.
- Itasca Consulting Group (2002). "FLAC3D-Fast Lagrangian Analysis of continua in three dimensions". *Itasca Consulting Group*, Minneapolis, Minnesota.
- Janbu, N. (1954), "Stability Analysis of Slopes with Dimensionless Parameters", *Harvard University, Soil Mechanics Series*, No. 46. Harvard University, Cambridge, Mass.
- Lambe, T. W., and Whitman, R. V. (1969). *Soil mechanics*, John Wiley and Sons, NY.
- Stark, T. D., and Eid, H. T. (1998). "Performance of three-dimensional slope stability methods in practice." *J. Geotech. Eng Division*, ASCE, 124(11), 1049-1060.