QUANTIFIED IMPACT OF GEOMEMBRANE WRINKLES ON LEAKAGE THROUGH COMPOSITE LINERS

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ABSTRACT

Equations show that geomembrane wrinkle size depends on the following parameters: the coefficient of thermal expansion of the geomembrane; the geomembrane temperature difference between installation and wrinkle formation; the geomembrane bending modulus; the geomembrane mass per unit area (linked to the geomembrane thickness); and the interface friction angle between the geomembrane and the underlying material. Other equations show that the rate of leakage through composite liners depends on the following parameters: the quality of contact between the geomembrane and the underlying low-permeability material (which is related to wrinkles); the size and number of geomembrane holes; the hydraulic head on top of the geomembrane; and the thickness and hydraulic conductivity of the material underlying the geomembrane. In this paper, equations for wrinkle size and leakage rate are combined. Numerical examples lead to interesting conclusions on the importance of the parameters mentioned above.

1. INTRODUCTION

A composite liner consists of two components, a geomembrane and an underlying low-permeability material. Typically, the low-permeability material is a layer of compacted clay or a geosynthetic clay liner (GCL). Giroud & Bonaparte (1989b) showed that the efficacy of a composite liner depends on intimate contact between the geomembrane and the underlying low-permeability material. When a geomembrane exhibits wrinkles, there is no intimate contact between the geomembrane wrinkles and the underlying low-permeability material; however, there may be intimate contact between the underlying low-permeability material and the portions of geomembrane located between wrinkles (Figure 1).

Fig. 1. Composite liner including a geomembrane that exhibits wrinkles.
Thiel & Giroud (2011) indicate that intimate contact between the geomembrane and the underlying material requires ballasting of the geomembrane, and they recommend a ballast thickness of the order of 0.3 to 0.6 m. It is important to note that such a ballast thickness, while being sufficient to ensure intimate contact between a flat geomembrane and the underlying material, may not be sufficient to flatten high wrinkles present in the geomembrane before the placement of the ballasting layer. Therefore, unless wrinkles have been completely eliminated during geomembrane installation, a number of wrinkles are likely to remain when the geomembrane is covered. The purpose of this paper is not to describe measures that can be taken to minimize wrinkles during geomembrane installation (which is of course good practice) but to provide a design method applicable to cases where wrinkles are present.

This paper presents a methodology to quantify the impact of geomembrane wrinkles on the rate of leakage. The approach consists in combining equations for the development of geomembrane wrinkles and equations for evaluating the rate of leakage through a composite liner due to holes in the geomembrane.

2. ANALYSIS

2.1 Wrinkle development

It is recognized that, in the field, the presence of seams and manufacturing folds can affect the location and the development of wrinkles. However, the analysis of wrinkle development presented below assumes an ideal geomembrane with no seams and no manufacturing folds that could affect the pattern of wrinkles (Figure 2).

![Figure 2. Geomembrane wrinkle geometry.](image)

The theoretical analysis of wrinkle development presented by Giroud & Morel (1992) is complex, but these authors have proposed an approximate equation that is relatively simple to calculate the height of wrinkles, $H_w$:

$$H_w = \frac{1}{2} \left( \frac{\alpha \Delta T E t_{GM}^2}{\rho g \tan \delta} \right)^{1/3}$$

where: $H_w$ is the wrinkle height; $\alpha$ is the geomembrane coefficient of thermal expansion; $\Delta T$ is the difference between the temperature of the geomembrane when the wrinkles are observed and the temperature of the geomembrane when it is installed; $E$ is the geomembrane bending modulus; $t_{GM}$ is the geomembrane thickness; $\rho$ is the geomembrane density; $g$ is the acceleration due to gravity; and $\delta$ is the interface friction angle between the geomembrane and the underlying material. Basic SI units are: $H_w$ (m), $\alpha$ (°C⁻¹), $T$ (°C), $E$ (Pa), $t_{GM}$ (m), $\rho$ (kg/m³), $g$ (m/s²), and $\delta$ (°).
Equation 1 is only applicable to homogeneous geomembranes (i.e. non-reinforced geomembranes made of one material only). These include, for example, non-reinforced geomembranes made of polymeric compounds based on polyvinyl chloride (PVC), high density polyethylene (HDPE), linear low density polyethylene (LLDPE), polyethylene (PP), and ethylene propylene diene monomer (EPDM).

The following new development in the wrinkle theory makes it possible to establish a relationship between the height of wrinkles, \( H_w \), and the spacing between wrinkles, \( S_w \) (Figure 2). A typical wrinkle is represented in Figure 3.

\[ \frac{W_w^' - W_w}{W_w} = \frac{8}{3} \left( \frac{H_w}{W_w} \right)^2 = \frac{8}{3} R_{WH}^2 \]  

[2]

with the ratio between the width and height of wrinkle, \( R_{WH} \), defined as follows:

\[ R_{WH} = \frac{W_w}{H_w} \]  

[3]

Equation 2 can also be written as follows:

\[ W_w^' - W_w = \frac{8}{3} \left( \frac{H_w}{W_w} \right) H_w = \frac{8 H_w}{3 R_{WH}} \]  

[4]

The thermal expansion of the geomembrane, which is \( W_w^' - W_w \), can be expressed as follows:

\[ W_w^' - W_w = \alpha \Delta T \ S_w \]  

[5]

where: \( S_w \) is the center-to-center spacing between wrinkles (Figure 2).

Combining Equations 4 and 5 gives the following expression:

\[ S_w = \frac{8 H_w}{3 R_{WH} \alpha \Delta T} \]  

[6]
Based on numerical calculations performed by Giroud & Morel (1992), the ratio $R_{WH}$ is typically between 2 and 6. This is in agreement with a value $R_{WH} \approx 4$, which may be inferred from experimental data by Chappel et al. (2012).

The total length of wrinkles in one direction in a given geomembrane area, $A_{GM}$, can be calculated using the straightforward equation that follows:

$$L_w = \frac{A_{GM}}{S_w}$$  \[7\]

Therefore, the total length of wrinkles per unit area of geomembrane can be calculated using the following equation, derived from the preceding equation:

$$\frac{L_w}{A_{GM}} = \frac{D}{S_w}$$  \[8\]

where: $D$ is a dimensionless factor equal to 1 if wrinkles are present only in one direction and equal to 2 if wrinkles are present in two perpendicular directions, as shown in Figure 4.

![Fig. 4. Example of geomembrane wrinkles in two perpendicular directions [Photo J.P. Giroud.]](image)

The basic SI unit for the total length of wrinkles per unit area, $L_w / A_{GM}$, is m$^{-1}$. However, a practical SI unit is km/ha, with:

$$L_w / A_{GM} \text{ in km/ha} = 10 \frac{L_w}{A_{GM}} \text{ in m}^{-1}$$  \[9\]

The surface area occupied by wrinkles, $A_w$, can be calculated using the following equation:

$$A_w = L_w \cdot W_w$$  \[10\]

The relative surface area occupied by wrinkles, $A_w_{relat}$, can be calculated by the following equation:

$$A_w_{relat} = \frac{A_w}{A_{GM}} = \frac{L_w \cdot W_w}{A_{GM}}$$  \[11\]
Combining Equations 8 and 11 gives the following alternative equation for the relative surface area occupied by wrinkles:

\[ w_{rel} = \frac{A_{w}}{A_{GM}} = \frac{D W_{w}}{S_{w}} \]  

[12]

Combining Equations 3, 6 and 12 gives a third equation for the relative surface area occupied by wrinkles:

\[ w_{rel} = \frac{A_{w}}{A_{GM}} = \left( \frac{3}{8} \right) D R_{w}^2 \alpha \Delta T \]  

[13]

The fact that there are three equations for calculating the same quantity makes it possible to cross-check numerical calculations. Equation 13 shows that the area occupied by wrinkles does not depend on the physical and mechanical properties of the geomembrane that govern the wrinkle height, width and spacing, i.e. the thickness, density, modulus and interface friction angle. Equation 13 shows that the area occupied by wrinkles is essentially governed by the thermal expansion, whereas the height of wrinkles is, to a great extent, governed by the bending modulus (see Equation 1). Thus, the wrinkles of PVC geomembranes are much smaller than the wrinkles of HDPE geomembranes because the bending modulus of PVC geomembranes is two orders of magnitude lower than that of HDPE geomembranes, whereas the coefficients of thermal expansion of these two geomembranes are of the same order of magnitude (the coefficient of thermal expansion of HDPE geomembranes being only two to three times higher than that of PVC geomembranes).

To be complete, the analysis should include a quantification of the flattening of wrinkles under load. This is a complex matter that would require extensive research. In this paper, only approximate reduction factors are tentatively proposed. As suggested by Rowe (2012, p. 152), wrinkles with a height smaller than 30 mm are likely to be flattened when the geomembrane is covered, whereas higher wrinkles are likely to remain. In fact, several independent publications cited by Rowe (2012) show that wrinkles typically higher than 30 mm are reduced but not eliminated by overlying soil layers. Also, based on test results published by Gudina & Brachman (2006), the width of relatively high wrinkles, such as HDPE geomembranes wrinkles, is reduced by a factor of approximately 2 under a load of 250 kPa. Combining these data, the following reduction factors, \( F_\sigma \) and \( F_H \), are proposed to reduce the height of wrinkles:

\[ F_\sigma = \frac{1}{1 + 0.004 \sigma} \quad \text{with } \sigma \text{ in kPa} \]  

[14]

where: \( \sigma \) is the normal stress exerted on the geomembrane by the overlying material; and

\[
F_H = \begin{cases} 
0 & \text{if } H_w \leq 0.03 \text{ m} \\
H_w - 0.03 & \text{if } 0.03 \text{ m} \leq H_w \leq 0.05 \text{ m} \\
0.02 & \text{if } 0.05 \text{ m} \leq H_w \leq 0.07 \text{ m} \\
1 & \text{if } H_w \geq 0.07 \text{ m}
\end{cases} 
\]  

[15]

As a result, it is possible to calculate a reduced height of wrinkle, \( H_{w\ reduced} \), as follows:

\[ H_{w\ reduced} = F_\sigma F_H H_w \]  

[16]

If a reduction is applicable, the reduced height should be used to calculate the wrinkle width (using the ratio \( R_{win} \) defined by Equation 3). However, the spacing between wrinkles must be calculated using Equation 6 with the non-reduced
winkle height, and the wrinkle length must be calculated using Equations 7 and 8, with the spacing between wrinkles calculated using Equation 6 with the non-reduced wrinkle height. Indeed, wrinkle reduction due to normal stress affects the height and width of wrinkles, but not the spacing and the length of wrinkles.

The relative area occupied by wrinkles should be calculated using Equation 11 or 12 with the reduced width, and Equation 13 should be written as follows to account for the wrinkle height reduction:

\[ A_{w \text{ relat}} = \frac{A_w}{A_{GM}} = \left(\frac{3}{8}\right) F F_H D R_{w_h} \alpha \Delta T \]  

[17]

2.2 Leakage rate calculation

As shown in Figure 1, the geomembrane is not in contact with the underlying low-permeability material in areas where the geomembrane exhibits wrinkles. In this case, the following equation (Rowe 2012) can be used to calculate the rate of leakage for a given length of wrinkle due to geomembrane holes located in the wrinkle:

\[ Q_{w \text{ max}} = L_w \left[ W_i k \left( \frac{h}{H} \right) + 2h \sqrt{k \theta} \right] \]  

[18]

where: \( Q_{w \text{ max}} \) is the rate of leakage associated with wrinkles; \( L_w \) is the considered length of wrinkles; \( W_w \) is the width of wrinkles; \( k \) is the hydraulic conductivity of the low-permeability material underlying the geomembrane; \( H \) is the thickness of the low-permeability material underlying the geomembrane; \( h \) is the hydraulic head over the flat portion of the geomembrane; and \( \theta \) is the hydraulic transmissivity of the interface between the geomembrane and the low-permeability material underlying the geomembrane. Basic SI units are: \( Q_w \) (m³/s), \( L_w \) (m), \( W_w \) (m), \( h \) (m), \( k \) (m/s), and \( \theta \) (m²/s). Values of \( \theta \) depend on the underlying material. Typical values are: \( 8.3 \times 10^{-8} \) m²/s for \( k = 1 \times 10^{-8} \) m/s and \( 1.6 \times 10^{-8} \) m²/s for \( k = 1 \times 10^{-9} \) m/s, for compacted clay; and \( 1 \times 10^{-11} \) m²/s to \( 1 \times 10^{-10} \) m²/s for GCLs.

In the Equation 18 bracket: (1) the first term quantifies the rate of infiltration into the low-permeability material through the wrinkle footprint (rectangular area of length \( L_w \) and width \( W_w \)); and (2) the second term quantifies the amount of liquid that flows in the interface between the flat portion of geomembrane and the underlying low-permeability material on each side of the wrinkle and eventually infiltrates into the low-permeability material. Equation 18 is based on the assumption that the wrinkle contains liquid under the same head as the head applied on the flat portion of geomembrane. This assumes that holes in the wrinkle are sufficient (in number and/or size) to allow liquid to flow into the wrinkle at a rate at least equal to the rate of infiltration into the underlying low-permeability material. Therefore, the leakage rate calculated using Equation 18 is the maximum rate of leakage associated with a given wrinkle, hence the symbol \( Q_{w \text{ max}} \).

The maximum rate of leakage per unit area of geomembrane associated with wrinkles can be calculated as follows using the following equation derived from Equation 18:

\[ \frac{Q_{w \text{ max}}}{A_{GM}} = \frac{L_w}{A_{GM}} \left[ W_w k \left( \frac{h}{H} \right) + 2h \sqrt{k \theta} \right] \]  

[19]

where the ratio \( L_w/A_{GM} \) is given by Equation 8.
The leakage rate obtained using Equation 18 has a first upper boundary, because the flow that exits a wrinkle by infiltrating into the underlying low-permeability material cannot be greater than the flow that enters the wrinkle through geomembrane holes located in the wrinkle. This boundary is the leakage rate for free flow through an orifice:

\[ Q_{lim1} = n_w \times 0.6a \sqrt{2gh_w} \]  

[20]

where: \( n_w \) is the number of holes in the considered wrinkle; and \( a \) is the assumed hole area. Basic SI units are: \( Q_{lim} \) (m³/s), \( a \) (m²), \( g \) (m/s²), and \( h_w \) (m); while \( n_w \) is dimensionless.

A hole in a wrinkle can be located anywhere between the apex and the edge of a wrinkle. An approximate average height for a hole in a wrinkle is the mid-height of the wrinkle. Therefore, the head \( h_w \) can be calculated as follows:

\[ h_w = \max \left( 0, h - \frac{H_w}{2} \right) \]  

[21]

where: \( h \) is the hydraulic head above the flat portion of the geomembrane. The head, \( h_w \), is not used in Equation 18, because that equation gives, in fact, the rate of leakage at the level of the base of a wrinkle. Therefore, the full hydraulic head, \( h \), is used in Equations 18 and 19. However, this may be conservative as it is possible that the hydraulic head in the wrinkle is less than the full hydraulic head, \( h \). More work is needed on this aspect of the method.

The number of holes in a wrinkle of length \( L_w \) and width \( W_w \) is given by the following equation:

\[ n_w = \frac{N_w L_w W_w}{A_w} \]  

[22]

where: \( N_w \) is the number of holes per unit area of wrinkle (measured in horizontal projection).

Combining Equations 11, 20 and 22 gives the upper boundary leakage rate per unit area of geomembrane as follows:

\[ \frac{Q_{lim1}}{A_{GM}} = N_w A_{w, rel} 0.6a \sqrt{2gh_w} \]  

[23]

The value of \( A_{w, rel} \) can be calculated using any one of Equations 11, 12 and 17.

The leakage rate obtained using Equation 18 has a second upper boundary, \( Q_{lim2} \), which is the leakage rate that corresponds to interference between two adjacent wrinkles. This happens when the interface between the flat portion of the geomembrane and the underlying low-permeability material is saturated with liquid flowing from two adjacent wrinkles. In other words, this happens when liquid flowing from one wrinkle meets liquid flowing from the adjacent wrinkle. In this case, the leakage rate can be expressed as follows:

\[ Q_{lim2} = L_w \left[ W_w k \left( \frac{h}{H} \right) + k (S_w - W_w) \right] \]  

[24]

Combining Equations 7, 10, 11 and 24 gives:

\[ \frac{Q_{lim2}}{A_{GM}} = k \left[ 1 + A_{rel, w} \left( \frac{h}{H} - 1 \right) \right] \]  

[25]
The resulting rate of leakage associated with wrinkles per unit area of geomembrane, $Q_w/A_{GM}$, can be calculated as follows:

$$
\frac{Q_w}{A_{GM}} = \min \left( \frac{Q_{w,\text{max}}}{A_{GM}}, \frac{Q_{w,1}}{A_{GM}}, \frac{Q_{w,2}}{A_{GM}} \right)
$$

[26]

The geomembrane between the wrinkles is assumed to be in intimate contact with the underlying low-permeability material (as illustrated in Figure 1). Overburden conditions for intimate contact were mentioned above in Section 1. The leakage rate in this case is calculated using the following equation (Giroud 1997, Touze-Foltz et al. 2008):

$$
Q_{\text{contact}} = C a^{0.1} h^{0.9} k^{0.74} \left[ 1 + 0.1 \left( \frac{h}{H} \right)^{0.95} \right] \text{ with } C = 0.21 \text{ for compacted clay and } C = 0.0024 \text{ for GCL}
$$

[27]

The leakage rate per unit area of geomembrane due to the portion of geomembrane in intimate contact with the underlying low-permeability material is then calculated as follows:

$$
\frac{Q_{\text{contact}}}{A_{GM}} = N \left(1 - A_{\text{relat}}\right) Q_{\text{contact}}
$$

[28]

where: $N$ is the number of holes per unit area of geomembrane. The same value can be used for $N$ and $N_w$ (e.g. 5 holes per hectare) in the case of a simple calculation, but a higher value may be used for $N_w$ if it is considered that wrinkles are more exposed to construction damage and stresses in service than the flat portion of the geomembrane. The basic SI unit for $N_w$ is m$^{-2}$, but, typically, the number of holes per hectare is considered. When the number of holes is given per hectare (which is generally the case) it should be multiplied by 0.0001 to generate the number of holes per m$^2$ used in calculations.

Finally, the leakage rate per unit area calculated for geomembrane holes located in the area where the geomembrane is in intimate contact with the underlying low-permeability material is added to the leakage rate per unit area calculated for the geomembrane holes located in wrinkles. These two leakage rates can be added, because the area considered in the determination of the “unit area” is the total geomembrane area, including area with wrinkles and area without wrinkles.

3. NUMERICAL APPLICATION

3.1 Numerical results for wrinkle quantification

First, a numerical calculation was done to compare the theoretical analysis presented in this paper with the data from full scale field experiments published by Chappel et al. (2012) and Rowe (2012). Among the many results obtained by these authors, the following values appear to be typical: wrinkle height of the order of 80 mm, spacing between wrinkles of the order of 3.5 m, wrinkle length between 5 and 10 km/ha, and surface area occupied by wrinkles of the order of 20% of the geomembrane surface area. The experimental conditions were: 1.5 mm thick smooth HDPE geomembrane with no wrinkles on a “cool October morning” in Canada and wrinkles observed at a geomembrane temperature of 53ºC. The following values of the parameters were used in the numerical calculation: $\Delta T = 45^\circ\text{C}$, $\alpha = 4 \times 10^{-4} \text{ °C}^{-1}$, $\rho = 940 \text{ kg/m}^3$, $t_{GM} = 1.5 \text{ mm}$, $E = 250 \text{ MPa}$ (modulus at approximately 50ºC from experimental data (Giroud 1995)), $\delta = 10^\circ$, and $D = 2$
for wrinkles in two perpendicular directions. The value $R_{WH} = 4$ suggested by the observations reported by Chappel et al. (2012), as indicated above after Equation 6, was used in the numerical calculation. The following values were obtained: wrinkle height, 92 mm; wrinkle spacing, 3.41 m; wrinkle length per unit area of geomembrane, 5.9 km/ha; and surface area occupied by wrinkles, 21.6% of the geomembrane surface area. It appears that the numerical calculation based on the theoretical analysis is in agreement with the experimental data. This shows that it is possible to model wrinkles.

Another calculation was done for a PVC geomembrane. The calculated wrinkle height (Equation 1) was 12 mm; but, after application of the reduction factor (Equation 15), the reduced wrinkle height was zero. As a result, the area covered by wrinkles was calculated to be zero. Based on this calculation, the entire PVC geomembrane can be expected to be in intimate contact with the underlying low-permeability material (provided it is ballasted, as indicated in Section 1).

3.2 Numerical results for leakage quantification

The rate of leakage associated with the geomembranes considered in Section 3.1 was calculated using the equations presented in Section 2.2. A hydraulic conductivity of $1 \times 10^{-9}$ m/s was used for the low-permeability material underlying the geomembrane, which is a typical value, and a hydraulic head of 0.3 m was assumed. A number of holes in the geomembrane of 5 per hectare and a hole area of 1 cm$^2$ were assumed (Giroud & Bonaparte (1989a p. 64)). The following values were obtained for the rate of leakage per unit area of geomembrane:

- $9.0 \times 10^{-10}$ m/s (780 liters per hectare per day (lphd)) for the HDPE geomembrane with wrinkles; and
- $3.3 \times 10^{-12}$ m/s (3 lphd) for a perfectly flat geomembrane (e.g. an HDPE geomembrane installed with extreme care, or a PVC geomembrane).

It appears that the impact of wrinkles on the calculated leakage rate is considerable. It is interesting to compare the above values with the calculated rate of leakage through a geomembrane alone on a highly permeable material obtained with the equation for free flow through an orifice: $7.3 \times 10^{-8}$ m/s (63,000 lphd), with same holes and same hydraulic head as above. In this comparison, the geomembrane lying flat on a low-permeability material is 20,000 times more effective than a geomembrane alone on a highly permeable material (which is consistent with generally recognized performance of composite liners) while the geomembrane with wrinkles resting on a low-permeability material is only 80 times more effective than a geomembrane alone on a highly permeable material. Furthermore, an HDPE geomembrane (with 5 holes per hectare having an area of 1 cm$^2$, and resting on a low-permeability material) has a performance, when it exhibits wrinkles, only slightly better than the performance of a low-permeability soil liner alone. This shows that it is imperative to eliminate wrinkles to fully benefit from a composite liner.

Calculations were also performed for a hole area of 0.1 cm$^2$, as this is a maximum hole size that can be expected in the case of geomembranes installed with extensive construction quality assurance including electric leak location survey. In this case, the calculated rate of leakage is $8.9 \times 10^{-10}$ m/s (770 lphd). This calculated leakage rate is approximately equal to the leakage rate calculated for the case of the 1 cm$^2$ holes mentioned above. This is because, in both cases, most of the leakage rate is due to flow at the bottom of the wrinkles, which is controlled by the hydraulic conductivity of the material underlying the geomembrane and not by the geomembrane holes.
4. DISCUSSION AND CONCLUSION

Some aspects of the methodology can be improved, such as: (i) the impact of geomembrane seams and folds on wrinkle development; and (ii) two assumptions that may cause an overestimation of the leakage rate (1 – the hydraulic head in the wrinkles assumed to be equal to the hydraulic head applied between wrinkles, and, 2 – the entire wrinkle network assumed to be filled with liquid flowing through a small number of geomembrane holes located in the wrinkles). Even though the methodology needs to be refined, it already provides a tool that makes it possible to evaluate to which extent geomembrane wrinkles are detrimental to leakage control with composite liners. For the first time, equations that govern wrinkle formation and equations for leakage rate evaluation are combined. This makes it possible to perform parametric studies to evaluate the relative importance of the various parameters. Preliminary results presented in this paper show that it is imperative to eliminate wrinkles to fully benefit from a composite liner. More results will be published.

REFERENCES


